Model and Verification of a Data Manager

Based on ARIES

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In this article, we model and verify a data manager whose algorithm is based on ARIES. The work uses the I/O automata method as the formal model and the definition of correctness is defined on the interface between the scheduler and the data manager.

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General Terms: Reliability, Theory, Verification

Additional Key Words and Phrases: ARIES, I/O automata, system failures

1. INTRODUCTION

For many applications, it is essential that a database management system (DBMS) maintains the data correctly, so that it reflects the effects of exactly the preceding committed transactions. One of the major problems with real-time systems is that human activities and are complex, therefore error-prone, as the DBMS has no control as to when failures, such as crashes and transaction rollbacks, occur. The algorithm must ensure that it can recover from a failure at any time (even when it is recovering from a previous failure). To make recovery even more complex, the algorithm must also be efficient such that recovery time is kept to a minimum.

ARIES [Mohan et al. 1992] is a relatively new and important recovery algorithm that has been implemented in varying degrees in such systems as OS/2, DB2, Starburst, and Quicksilver. The key ideas of ARIES are that recovery repeats history (including actions from transactions that did not commit) and then undoes the actions of transactions that did not commit before the crash (these are called "loser transactions").

Formal methods have been advocated as a way to understand complex systems, as well as offering a framework in which to prove that the system, or some key algorithm, is correct. This has been especially attractive in cases where errors are error-prone and so makes careful analysis even more valuable. Our motivation is reflected in Selinger [1987] where she states, "one of the contributions that theory can make to systems is the confirmation that an algorithm is correct...Recovery—an example of where more work is needed."

Verification of an algorithm requires a precise mathematical description of the algorithm so that we can state and prove its properties. Therefore one of the first steps in verification is to write the specification in an unambiguous fashion. One approach is to write the specification in English and pseudocode. As a result, ambiguities may be revealed when a mathematical description is attempted.

This article presents a model and verification of a data manager whose algorithm is based on ARIES. The formal method used in I/O automata which is presented in Lynch et al. [1993], and Lynch [1981], is a formal method to produce a mathematical model of the system. This model includes, in particular, the model of the system's actions, the model of the system's environment, and the model of the system's communication. The model is then used to prove properties of the system.

In the following sections, we describe the data manager and its algorithm in detail. We then present the model and verification results. Finally, we discuss the results and their implications.

1* INTRODUCTION

Aborted transactions in corporate computers are committed to major systems, and transaction rollbacks are a common occurrence. In many databases, the term 'rollback' is used instead of 'aborted transaction' to keep terminology consistent with ARIES [Mohan et al. 1992].

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There has been some related work in understanding recovery algorithms. In Bernstein et al. [1987], Gray and Reuter [1993], and Harder and Reuter [1983], algorithms are presented and verified. The partial data item logging algorithm from Bernstein et al. [1987] was modeled and verified. In that algorithm, restart requires two passes of the log. This is quite different from the method studied here. A preliminary version of this work appeared in Kuo [1992a].

The three main contributions of this paper occupy Sections 3–5. In Section 3, we present a specification of the data manager by first describing the interface between the data manager and the transaction manager. The details of the proofs may be found in Kuo [1992b].

2. BACKGROUND

This section summarizes the necessary background knowledge; this covers the ARIES recovery algorithm and the I/O automaton formal method. Readers familiar with these topics may skip this section.

The main components for transaction processing in a DBMS are the transaction manager, the scheduler, and the data manager. This article focuses on the data manager and the interface between the data manager and the scheduler. The data manager model described here is based on the ARIES formalism and the I/O automaton framework of Mohan et al. [1992].

2.1 ARIES

A recent recovery algorithm that enforces write-ahead logging (WAL) protocol is ARIES [Mohan et al. 1992]. It assumes that the scheduler provides strict executions [Bernstein et al. 1987]. ARIES keeps track of the status of active transactions and dirty pages in the system using the transaction page table and the dirty pages table, respectively. Checkpoints are taken periodically; each takes a copy of these tables and writes them into an end checkpoint log record. A begin checkpoint log record is written to the log at the start of the checkpoint procedure.

After a crash, the effects of some updates from committed transactions may be missing from the stable database as the changed data values may not have been flushed to the disk; also the effects of some updates from pending transactions may be lost. A transaction manager is used to control the execution of transactions. A transaction manager is a process that performs a sequence of operations on the database system. It includes the transaction coordinator, which is responsible for coordinating the execution of transactions and the transaction manager, which is responsible for the execution of transactions.

During normal processing, the data manager processes transactions reads, writes, commits, and rollbacks. For reads and writes, the data manager fetches the data into the volatile cache if the data is not already in the volatile cache. If the data is not in the volatile cache, the data manager reads the data from the stable storage on disk. The data manager updates the data in the volatile cache and writes the data to the stable storage on disk. The data manager then issues a write to the stable storage on disk.

During a transaction, the data manager maintains a file of transaction logs. Each transaction log record contains information about the transaction, such as the transaction identifier and the time the transaction started and ended. The data manager uses the transaction log to determine the state of transactions and to perform recovery in case of a failure. During a transaction, the data manager maintains a file of transaction logs. Each transaction log record contains information about the transaction, such as the transaction identifier and the time the transaction started and ended. The data manager uses the transaction log to determine the state of transactions and to perform recovery in case of a failure.
The transition relationship in the I/O automaton is denoted as \((\text{start} \rightarrow \text{state})\) for the start state \((s)\) which is the start state of the model. Each \((\text{start} \rightarrow \text{state})\) represents a possible initial state for the model and the \((s)\) denote the output actions of the schedule. That is, the behavior is independent of state and of internal actions. Correctness is defined by stating the acceptable behaviors of the automaton model. In the \((\text{start} \rightarrow \text{state})\) which are the start states of the model. Each \((\text{start} \rightarrow \text{state})\) of the \((\text{start} \rightarrow \text{state})\) show the transition from the old state to the new state. Input actions \((s)\) are enabled so only the \((s)\) of a schedule is the subsequence containing only the \((s)\). From an execution, we can extract the \((s)\) of the automaton \((s)\). I/O automaton are atomic—that is, there are no intermediate states showing only part of the effects of an action.

During recovery, the users are blocked from accessing the database. The I/O automata method is designed to model discrete event systems which contain components that operate concurrently. It has been shown to be successful in verifying the correctness of algorithms and systems. This is essential if we are to understand the alternative algorithms that can be used for a component of a complex system.
a DBMS that uses a nonstrict technique such as optimistic concurrency control. In Section 3.1, we present the external (input and output) actions of the data manager and then in Section 3.2, we formally define correctness in terms of the sequence of these actions produced by the data manager.

3.1 External Actions of the Data Manager

The legal operations, during normal processing, that the scheduler can request to the data manager are read, write, commit, and rollback. Thus the input and output actions of the data manager during normal processing are the requests and acknowledgments of these operations. For example, we have the input action

\[ \text{ReqRead}(T, x) \]

which models the request of a read by the transaction \( T \) to the data item \( x \), and the output action

\[ \text{AckRead}(T, x, v) \]

which models the completion of the read with \( v \) as the value read.

If the data manager fails to commit a transaction, it rolls back the transaction as if the scheduler requested a rollback. In our model, if the data manager successfully commits a transaction \( T \), then the action

\[ \text{AckCommit}(T) \]

occurs in the behavior; otherwise the action

\[ \text{AckRollbk}(T) \]

occurs.

A system failure may occur at any time and is modeled by the input action

\[ \text{Crash} \]

Once a failure has occurred, recovery is enabled. The output action

\[ \text{AckRestart} \]

signals the successful completion of recovery.

Table I shows the input and output actions of the data manager. It is fundamental to the formal method that correctness must be defined in terms of sequences of these external actions. Thus instead of mentioning internal state and saying that a system is "correct" in some absolute sense, we require that if a read returns an appropriate value, then the scheduler in the DBMS generates only strict schedules—each access (read or write) to a data item by a transaction \( T \) is delayed until after the most recent different transaction that wrote to the item has terminated (i.e., either committed or rolled back). Thus if a transaction \( T \) has not written to the data item, then a read will return the initial value if no committed transaction has written to it; otherwise it will return the last value written by the transaction that committed and wrote to the data item.

3.2 Correctness

In this section, we formally define correctness of a data manager. However, we first informally describe it. As noted, we require that each read by any transaction \( T \) return the appropriate value. The appropriate value is informally defined as follows:

- Case 1. If neither the transaction \( T \) nor any committed transaction have written to the data item, then the appropriate value is the initial value, which we define as 0.
- Case 2. If \( T \) itself wrote to the data item before the read, then the appropriate value is the value that \( T \) wrote.
- Case 3. If some committed transaction has written to the data item, then the appropriate value is the last value written by a committed transaction that committed and wrote to the data item.

To shed light on why this is the intuitively correct definition, let us examine the behavior of the data manager. If neither system failures nor transaction rollbacks occur, then each read returns the initial value if no committed transaction has written to it; otherwise it will return the last value written by the transaction that committed and wrote to the data item. Therefore, the data manager ensures that the appropriate value is returned in all cases, even in the presence of failures and rollbacks.

In verifying our correctness definition, our approach is to say that a data manager is correct if it satisfies the property

\[ \forall T, x : \text{Read}(T, x) \rightarrow \text{Value}(T, x) = \text{AppropriateValue}(T, x) \]

where \( \text{Value}(T, x) \) is the value returned by the read and \( \text{AppropriateValue}(T, x) \) is the appropriate value defined above.
The set of transactions with each status is written as though it were a "history variable" in the system state. This set contains all the transactions that have requested to commit or rollback, and a superset of all transactions that have successfully committed. Mathematically, we require that:

- The transactions that have requested to commit but their requests did not fail, and a superset of all transactions that have successfully committed.
- The transactions that have requested to terminate (t 3.2.1. For any execution

\[ T = \text{DefinitelyCommitted} \cup \text{PossiblyCommitted} \]

we denote the set of committed transactions. Let S denote the set of transactions that have successfully committed. Also, if the data manager has decided a transaction has committed in state S, then we say the transaction is committed.

\[ \text{DefinitelyCommitted} \cup \text{PossiblyCommitted} \]

We introduced a notion of transaction well-formedness to capture this. The transactions that have requested to commit or rollback, nor have they been affected by a crash. The transactions that have requested to terminate (t 3.2.1. For any execution

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A model of the algorithm is well-formed and strict is correct. If in Definition 3.2.4, then in Definition 5, we define the value that was written by the last action in this set. When we use these definitions in our correctness definition, we take

\[ x = \begin{cases} \text{the value it wrote.} & \text{if the transaction currently reading}, \\ \text{the appropriate value is defined as the value written by the last transaction that committed and wrote to} & \\ \text{otherwise.} & \text{to} \end{cases} \]

We say that the data manager is correct provided that every behavior that

- requests to commit or rollback
- a crash has occurred

... before a transaction can request to commit or rollback; each transaction can write to a data item at most once; and

- if a transaction has requested to commit or rollback, it can no longer make any other output or internal action.

The preceding definition formally states the obvious properties of transactions.
While presenting the model, we also give a comprehensive description of the ARIES algorithm as well as the minor simplifications and modifications we make. The model and verification include checkpointing, multiple data items per page, and the possibility of crashes during restart processing.

Our model, however, only models physical logging—that is, each log record includes the before and after images of the operation, and the only operations on the data items are reads and writes. Our model makes the assumptions that the database consists of a finite number of data items, each on a fixed page, and that each transaction can update a data item at most once. In addition, once a transaction starts rolling back, it can never commit; that is, our model does not include partial rollbacks.

4.1 Methodology

Our model uses all three types of actions from the I/O automaton model. Input actions are used to model the requests from the scheduler to the data manager. For example, we use the action $\text{ReqRead}(T, x)$ to model the request from the scheduler to the data manager on behalf of the transaction $T$ to read the value stored in the data item $x$. Internal actions are used to model the internal steps of the data manager, and the output actions are used to model the acknowledgments, from the data manager to the scheduler, of the completion of a request.

There are three major problems in modeling and verifying a data manager using the I/O automaton model: crashes can occur at any time (even during recovery); there are many concurrent transactions; and the actions of the scheduler are not atomic. The actions of the scheduler and the data manager cannot be performed in parallel. Our model allows each action to contain more than one step of the data manager if the following constraints are satisfied:

- There is at most one write to stable storage but any number of reads.
- There is only one write to each shared data structure.
- The sequence of steps is properly latched.

Thus our verification proves that the recovery algorithm is idempotent [Bernstein et al. 1987]—that is, any number of incomplete executions of the recovery algorithm followed by a complete one will restore the database to the same state as a single complete execution.

4.2 Data Structures

This section describes and lists the data structures used in our model. The main data structures in ARIES are the log, transaction table, dirty page table, and the pages that contain the data items.

4.2.1 Log.

The log is modeled as an array of log records indexed by log sequence numbers (LSN). In the following we describe the fields in the log.

- **LSN**: The log sequence number of the log record.
- **Type**: Indicates the type of the log record. There are six types: update, compensate, end, begin checkpoint, end checkpoint, and operating system file return ($\text{OsFileReturn}$).
- **TransID**: Records the identifier of the transaction if the log record is a transaction related log record (log record of type update, compensate, or end).

We claim that our model does not miss anything significant for the following reasons.

- A crash has the property that it corrupts the contents of volatile main memory but has no effect on stable disk storage. Therefore if a crash interrupts a sequence of steps (from one action in the I/O automaton model), each data item or log record is in one of the two stable states: read or not read. The former means that the data item or log record was read by the transaction that interrupted the execution on behalf of which the action was sent to the data manager, and the latter means that it was not. Therefore, the recovery algorithm can be used to replicate the state of the data items and the log to recover the database.
- If an action contained more than one write to the same data structure (e.g., data item or an entry in the transaction table) in volatile storage, then our model would be incomplete as only the last of the writes is propagated to the stable storage. Therefore we only allow one write to each data structure within an action of the model.
- There are concurrent activities within the data manager. One transaction may be rolling back while another is making forward progress (doing reads and updates), for example. These steps may overlap, and the system may be in an inconsistent state. Therefore we require that each action contain only steps of the real system that are properly latched.

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This field is only used for transaction related log records and points to the previous log record written by the same transaction. Thus the log records written by a transaction are backwardly linked via this pointer. If it is the first log record from a transaction, then it points to the start of the log.

**PageID**: This field is only used for log records of type update, compensate, and OsFileReturn. It records the page identifier of the page that the recorded operation affected.

**UndoNxtLSN**: This field is only used in compensation log records and it points to the next log record to be rolled back for the transaction; that is, it points to the previous update, by the same transaction, that is undone. If all updates by the transaction have been undone, then this points to the beginning of the log.

**Data**: This is a variant record. For update and compensation log records, it records the undo/redo information that is the before and after images of the operation. For end log records, it records the termination of a transaction. In this field, we record the final status of the transaction—committed or rolled back. For end checkpoint log records, it records checkpoint related information (checkpoint's copy of the transaction table and dirty pages table). For all other types of log records, this field is left empty.

In the real system, there are two logs. Older log records are stored in a log on disk, and the most recent records are kept in a buffer (volatile storage); from time to time, entries are moved from the buffer to the disk. The volatile log is used to store all updated log records, and the disk log is used to store both old and updated log records. We use the variable LSN to point to the next free log location for the volatile front of the log, and we use StableLSN to point to the first entry that is in the volatile buffer. Thus all log records whose log sequence number is strictly less than StableLSN are in stable storage. After a crash, the analysis pass starts scanning the log from the log record pointed to by the master record which is stored in some well known location in stable store. We use the variable MasterRec to model the master record. All the data structures related to the log are shown in Figure 1.

### 4.2.2 Transaction Table

The transaction table keeps track of the status of transactions and is represented as an array of transaction table entries indexed by transaction identifiers. Each entry contains two pointers: one points to the last record written by the transaction (LastLSN) and the other is to the next log record to be undone in case of a rollback (UndoNxtLSN).

The data structures are shown in Figure 2.

### 4.2.3 Dirty Pages Table

The dirty pages table is used to keep track of the dirty pages in the DBMS. The table is represented as an array of dirty page entries indexed by page identifiers. Each entry contains two pointers: one points to the last record written by the transaction (LastLSN) and the other is to the next log record to be undone in case of a rollback (UndoNxtLSN).

The data structures are shown in Figure 3.

### 4.2.4 Page

Each page contains a set of data items and each data item has an associated value (the value stored in the data item). Also, each page records in the field PageLSN the log sequence number of the last log record corresponding to an action whose effects are reflected in the page. A copy of each page is maintained in stable storage to ensure durability. Pages are represented by arrays of pages indexed by page identifiers. We represent a page which is not in cache by assigning 0 to the page's cache entry. The data structures are shown in Figure 4.

Thus the value stored, in state $S_n$, in data item $x$ is represented as $S_n[x]$. The dirty pages table is used to keep track of dirty pages in the DBMS. Each entry contains two pointers: one points to the last record written by the transaction (LastLSN) and the other is to the next log record to be undone in case of a rollback (UndoNxtLSN). Each page records in the field PageLSN the log sequence number of the last log record corresponding to an action whose effects are reflected in the page. A copy of each page is maintained in stable storage to ensure durability.
and similarly for the cache. Also, we represent a page \( P \) as not in cache by \( S_n \). Cache \[ P \] /H11005 0 /H20862. In the model, we use the notation \( \text{Page}(x) \) to be the page identifier of the page where \( x \) resides.

4.2.5 Checkpoints. Checkpoints are taken asynchronously and they take a copy of both the transaction table and the dirty pages table. The data structures used by checkpointing are, therefore, a transaction table and a dirty pages table. These data structures are shown in Figure 5.

4.2.6 Recovery. During recovery, we use a number of variables to keep track of the process. We use the variables \( \text{AnalysisLSN} \), \( \text{RedoLSN} \), and \( \text{UndoLSN} \) to respectively point to the next log record the analysis pass, redo pass, and undo pass have to process. Recovery needs to keep track of which transactions have terminated and which pages have been flushed. These are recorded in \( \text{TerminatedTrans} \) and \( \text{FlushedPages} \). Finally, we use a Boolean variable to record if we have already processed the last successful checkpoint's end checkpoint log record. The data structures are shown in Figure 6.

4.2.7 System State and Control Flow. The final set of data structures in our model is used to show the current state of the DBMS and for the control flow of the model. The variable \( \text{SystemState} \) indicates whether the system is down, up, or recovering, and the set \( \text{ActiveSet} \) contains a set of internal action names. In general, an internal action is enabled if it is an element in the set.

10 An action may enable another action by inserting the action name into the set and an action can disable itself by removing its own name from the set. Thus an action may be enabled in state \( S_i \) but disabled in the following state. The data structures are shown in Figure 7.

4.3 The Actions. In this section we describe the input, internal, and output actions (preconditions and effects) that model the way the data manager changes state as it executes. A crash terminates all activities and from Definition 3.2.2 on well-formed transactions, it follows that the actions for \( \text{Read} \), \( \text{Write} \), \( \text{Commit} \), and \( \text{Rollback} \) are never enabled during recovery. As a result, there is no need to explicitly include the condition that the system must be in normal processing for these actions to be enabled.

4.3.1 Read. The actions associated with read are modeled by an input action \( \text{ReqRead}(T, x) \), an internal action \( \text{Read}(T, x) \), and an output action \( \text{AckRead}(T, x, v) \). The input action enables the internal action that reads the value and enables the output action. The output action is the acknowledgment of the completion of the read.

One of the preconditions to the internal action is that the appropriate page is in the cache. Our model does not take specific action to bring it in, but the \( \text{Fetch}(P) \) action can occur at any time and it will cause a copy of the page from stable storage to be in the cache. That is, our model uses nondeterminism to be very general. This could affect ... our proof only deals with safety (i.e., it shows that any value returned is correct). The actions are shown in Figure 8.

4.3.2 Write. Again, three actions are used to model a write. The internal action \( \text{Write}(T, x, v) \) models all of the following: altering the cache, adding a record to volatile log, and updating the transaction table and dirty pages table. In a real system, a latch on the page where \( x \) resides is held during these steps, another latch on the transaction table is held during the log write and the subsequent modification of \( T \) 's entry in the transaction table, and a latch on the log is held during the log write. Thus this sequence of steps is properly latched and there ... only written to once. Hence we can model these steps in a single action. The actions are shown in Figure 9. In the action \( \text{Write}(T, x, v) \) if the page is not dirty, then an entry in the dirty pages table is removed.
4.3.2 Commit.

When the data manager receives a commit request, it tries to commit the transaction by ensuring that the updates, made by the transaction, are durable. In ARIES, this is done by ensuring that all log records written by the transaction are flushed to stable storage. Log flushes are sequential; therefore, when the transaction's commit log record is in stable storage, we know that the redo rule is satisfied. Hence, the atomic action that commits a transaction is the log flush of the transaction's commit log record.

We model the commit procedure using the actions shown in Figure 10. The action Commit\(\langle T \rangle\) models the data manager writing an end-commit log record and the action FailCommit\(\langle T \rangle\) models the data manager rejecting a commit request. Note that the acknowledgment to a commit is only enabled after the transaction's commit log record is in the stable log. Thus flushing is not explicitly done by these actions, but rather it is done nondeterministically by the action that flushes the log record (see later). Also, notice that ComLSN (in the code for AckCommit\(\langle T \rangle\) and other commit-related actions) is a free variable not a state component; the meaning of such code is that there should exist some value for ComLSN to satisfy the preconditions, and this value is not used in the effects activity.

4.3.4 Rollback.

When a transaction T rolls back, the system needs to undo all its updates in reverse chronological order. ARIES writes a compensation log record to record each undo. All log records written by a transaction are backwardly linked by the PrevLSN pointer in each log record. The rollback procedure repeatedly undoes the update described in the log record pointed to by TransTable\[T\].UndoNxtLSN, until TransTable\[T\].UndoNxtLSN becomes the last log record in the transaction's transaction table. Initially (before the start of the rollback procedure)

<table>
<thead>
<tr>
<th>TransTable[T].LastLSN</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>TransTable[T].UndoNxtLSN</td>
<td>TransTable[T].LastLSN</td>
</tr>
</tbody>
</table>

Each time an update is undone, the UndoNxtLSN is set to point to the log record that recorded the previous update to the update that has just been undone by T. Thus, when the rollback procedure terminates, all updates by the transaction will have been undone. An update is undone by the data manager by restoring the before image of the update. A compensation log record is written to record the undo and the transaction table is updated to reflect the new state of the transaction; that is, the next update to be undone is the update recorded in the log record pointed to by the PrevLSN field of the update log record just undone and the transaction table now points to the compensation log record that was just written.

The procedure to roll back a transaction is complex to model due to the loop that undoes all the updates in reverse chronological order. We use the action Rollback\(\langle T \rangle\) to simulate the undo of a single log record's update; the action Commit\(\langle T \rangle\) to simulate the commit of all log records; and the action FailCommit\(\langle T \rangle\) to simulate the rejection of a commit request. Each of these actions is defined by a set of actions and a set of guards. The guards specify the conditions under which the action can proceed. For example, the action Commit\(\langle T \rangle\) can proceed if the transaction's commit log record is in stable storage, which is determined by a guard that checks the ComLSN against the LastLSN in the transaction table. The action FailCommit\(\langle T \rangle\) can proceed if the transaction table contains a compensation log record, which is determined by a guard that checks the presence of a compensation log record in the transaction table.
RollbackTerm (T) then terminates the rollback procedure when all the updates by the transaction T have been undone. The action Rollback(T) occurs a number of times in the execution when a transaction rolls back. To be more precise, the number is exactly the number of update log records the transaction has written to the log.

Notice that, unlike the commit procedure, the rollback can acknowledge the completion of the rollback as soon as the end rollback log record is written to volatile log. The actions are shown in Figure 11. In the action Rollbk(T), we use the symbols UndoingLSN, P, and x as a shorthand for the values expressed in the precondition.

4.3.5 Checkpoints.
In ARIES, checkpoints are taken periodically. They can be taken asynchronously during normal processing: the redo pass and the undo pass. However, they cannot be taken ... to reduce the number of log records it needs to process thus reducing recovery time. A checkpoint first writes a begin checkpoint log record to the log, then takes a copy of the dirty pages table and transaction table. It then writes an end checkpoint log record, which contains a copy of the checkpoint's copy of the dirty pages table and transaction table, to the log. Finally, after the end checkpoint log record has been flushed, the master record is updated such that the LSN in the master record points to the checkpoint's begin log record.

The paper [Mohan et al. 1992] describes how the checkpoint takes a copy of the dirty pages table—that is, the system repeatedly latches a number of rows of the table, takes a copy of them, and then unlatches them until a copy of every entry in the table has been taken. Note that after the checkpoint unlatches the rows, the entries in the ... was flushed so it was no longer dirty). Thus the checkpoint's copy of the dirty pages table may be out of date when the end checkpoint log record is written. The recovery algorithm takes this into account when it reconstructs the table. However, Mohan et al. [1992] do not describe how the checkpoint ... for the analysis pass of recovery. We discuss in detail in Section 4.3.8 these modifications and why they were required.

The actions that model the checkpoint procedure are shown in Figure 12. Notice we only allow one checkpoint to be active at any time. Also, the action EndCheckpt(BLSN), which models the checkpoint procedure writing the end checkpoint log record, is enabled if:

\[
\begin{align*}
\text{CheckptTransTable}(T) & : \text{TransTable}(T) \land \text{TransTable}(0) \\
\text{CheckptDirtyPages}(P) & : \text{DirtyPages}(P) \land \text{DirtyPages}(0)
\end{align*}
\]

That is, at the end of the checkpoint, each nonempty entry in the transaction table has an entry in the checkpoint's copy of the table. Because checkpoints are taken asynchronously, it is possible for a transaction to have an entry in its own transaction table while having an entry in the checkpoint's copy of the transaction table; similarly for the dirty pages table.

There are five actions that represent various activities with checkpointing: the action BgnCheckpt writes a begin checkpoint log record; the actions CheckptTransTable(T), CheckptDirtyPages(P) model the checkpoint taking a copy of a row of the transaction table and dirty pages table, respectively. When copies of all rows of both tables have been taken, an end checkpoint log record, with the tables, is written to the log by the action EndCheckpt(BLSN). The action WriteMaster(BLSN) then writes the LSN of the checkpoint's begin log record to the master record. A precondition is that the end checkpoint log record, written by the action EndCheckpt(BLSN), is in the stable log. The actions are shown in Figure 12 and we define the set CheckptActions to be the set of actions that model asynchronous checkpointing.
pass of recovery, it is recorded in the log and the page's entry in the dirty pages table is removed as the page is no longer dirty. However, the entry is not removed when the flush occurs during the ... incorrect. When a flush occurs during the redo pass, all we know is that all operations whose corresponding log record's LSN is less than RedoLSN are not missing from the DBMS and we update the dirty pages to reflect this.

The action Flush($P$) models the flush of a page from volatile storage to stable storage by copying the contents of Cache[$P$] to Stable[$P$]. One of its preconditions is Cache[$P$].PageLSN/StableLSN. This condition enforces Fig. 11. Abort actions.

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4.3.7 Crash. When a crash occurs in a real system, the contents of volatile storage are lost. That is, the transaction table, the dirty page table, all the pages in cache, and the volatile log are lost. To model this, all pages in cache and all entries in both tables are set to 0, all log records whose log sequence number is greater than or equal to StableLSN are set to 0, and LSN/StableLSN. Finally, the ActiveSet contains the one element Restart which enables the restart procedure. The action is shown in Figure 14.

We set these log entries to 0 to simplify some of the arguments in our verification. This is a reasonable reflection of the fact that in a real system, these records are lost after a crash.
4.3.8 Restart.
The restart procedure recovers the DBMS from a crash. ARIES achieves recovery using three passes of the log: the analysis pass, the redo pass, and the undo pass.

Analysis pass.
The analysis pass reconstructs the transaction table and the dirty pages table that were lost in the crash. The dirty pages table (respectively, the transaction table) will then require redo (respectively, undo). The analysis pass scans the log forward from the log record pointed to by the LSN stored in the master record to the end of the log. When the pass processes an update, compensation, or OsFileReturn log record, the effects on the transaction table and the dirty pages table are similar to when the log record was initially written. In addition, when the pass processes an end transaction log record, it records the termination of the transaction in the set TerminatedTrans; similarly for flushed pages. At the end of the analysis pass, an end rollback log record is written for each transaction that has an entry in the transaction table and its UndoNxtLSN points to the beginning of the log (TransTable[T].UndoNxtLSN/H11005). Each of these transactions has already been completely rolled back but its end rollback log record was lost due to the crash, so we just rewrite these records. The action Restart initializes some recovery variables and enables the analysis pass. The internal action Analysis is repeated once for each record being processed. Finally the AnalysisTerm action enables the redo pass and initializes RedoLSN so the redo pass knows where to start processing the log.

We have altered the algorithm for the analysis pass from that described in Mohan et al. [1992] so that when it processes an end checkpoint log record, it reconstructs the transaction table using the pseudocode shown in Figure 15. In this, let LogRec be the end checkpoint log record the analysis pass is currently processing and let the set TerminatedTrans contain the set of transactions for which the analysis pass has processed the transaction’s end log record (i.e., these are the transactions that terminated during the checkpoint). In contrast, in Mohan et al. [1992] the process only checks that the transaction does not have an entry in the transaction table before inserting one. Under our assumption that an entry in the transaction table means the transaction is committed, the analysis pass determines whether an entry in the transaction table exists for a transaction by first checking the transaction’s end log record and then verifying the transaction’s log record sequence number (LSN) is not less than the transaction’s end checkpoint LSN.

Redo pass.
The goal of the redo pass is to redo the effects of all missing operations that were lost in the crash (even operations from transactions that did not commit). It scans the log forward, starting from the minimum recovery LSN in the dirty pages table to the end of the log. We prove in our verification later that the effects of any action, whose corresponding log record’s LSN is less than the minimum recovery LSN in the dirty pages, is not missing from the DBMS. When the redo pass processes a redoable (an update or compensation) log record, the pass examines the dirty pages table to determine if the effects of the recorded operation are potentially missing. If the page that the operation affected is dirty and if the page’s recovery LSN entry in the dirty pages table is less than or equal to the log record’s LSN, then the effects of the recorded operation are potentially missing (this is again proved in our verification). For each of these recorded operations, the page the operation affected is fetched, if it is not already in cache. Now, if the PageLSN is less than the log record’s LSN, then the operation requires redo and is redone by copying the after image of the operation to the data item; otherwise it does not require redo and we update the dirty pages table such that the table is brought up to date. The actions for the redo pass are shown in Figure 18. The Redo action is repeated once for each log record processed in this pass; the RedoTerm action terminates the redo pass and enables the undo pass.

Fig. 13. Fetch and flush actions.
Fig. 14. Crash action.
Fig. 15. Pseudocode for reconstructing the transaction table.
Fig. 16. Analysis actions.
Fig. 17. AnalysisTerm action.
Fig. 18. Redo actions.
undo pass.
The undo pass undoes all the updates from loser transactions (these are the transactions that did not commit before the crash). While the transaction table is not empty, the undo pass undoes the update recorded in the log record pointed to by the maximum UndoNxtLSN entry in the transaction table. The procedure to undo an update is identical to the undo during a transaction rollback. When all updates by a transaction have been undone, an end rollback log record is written to the log and the transaction’s entry in the transaction table is removed. Thus, eventually, the transaction table will become empty and the undo pass will terminate.

The actions that model the undo pass are shown in Figure 19. The action Undo1 and Undo2 (U) each represent part of the processing of a log record. These could have been combined to form one action, but we chose to use two as it simplifies our verification in Section 5.

4.3.9 Start State.
The start state reflects the state of the database at the time it was first installed; that is, all cache slots, the transaction table, and dirty pages table are all empty and the SystemState is Normal and the master record MasterRec is H11005. Also, ActiveSet is 0. The start state of the model is shown in Figure 20.

5. VERIFICATION
In this section we verify the recovery algorithm based on ARIES, by showing that the model from Section 4 meets the correctness condition. The model is used to simulate the behavior of the data manager and to verify that it satisfies the correctness condition. The model is implemented in the form of a set of actions and a set of assertions. The actions model the behavior of the data manager, while the assertions model the correctness condition. The model is used to simulate the behavior of the data manager and to verify that it satisfies the correctness condition.

Fig. 15. Pseudocode.

Fig. 16. Restart actions.

Fig. 17. Analysis pass actions.

Fig. 18. Undo pass actions.

Fig. 19. Start state.

Fig. 20. Stable log state.
Any proof enhances our confidence in an algorithm, but we believe that this article also makes a contribution to our understanding of the way the algorithm works. Our proof is structured, with a sequence of propositions, each of which can enhance the intuition about the algorithm by revealing important relationships (called invariants) that always hold among different aspects of the system state. We expect that development and proof of variant algorithms can be guided by the need to keep relationships like these.

The definition of correctness in Section 3 is stated in terms of the sequence of input and output actions produced by the data manager, because this is the way the rest of the system sees the data manager. In normal processing, the relationships are obvious. For example, the transaction table entry for an active transaction $T$ has a LastLSN field pointing to the last log record that concerns $T$. In order to prove a statement such as this, however, we need to strengthen it to give an additional condition on the relationship.

Fig. 18. Redo actions.

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Whenever a read or write to a data item is requested, the page containing the data item is first fetched into cache if it is not already there. In the case of a write, the new value is first stored in volatile storage. Each transaction accessing a data item at any point in time will have its own private cache of the data items in volatile storage. The cache is not shared among different transactions. If a transaction aborts, it must undo all the changes in its cache to ensure that the result is consistent with the history of the database.

Regardless of whether a transaction writes the data item in volatile storage or writes a log record to stable storage, the transaction manager makes sure that the data item is written to stable storage eventually. The postcondition of the write operation in volatile storage says that the value in the database is unchanged. If the transaction aborts, the data item in volatile storage will be written to stable storage before the transaction is terminated. If the transaction is successful, the log record will remain unchanged, and the data item in volatile storage will be written to stable storage. This ensures that the final state of the database is consistent with the history of the database.

We use the following notation to express that an interval of the log is equivalent in two states:

\[ (S_i, S_f) \]

where \( S_i \) and \( S_f \) are state variables.

For two states \( S_i \) and \( S_f \), we say that \( S_i = S_f \) if and only if there exists a log record \( L \) such that \( S_i + L = S_f \). This means that to transition from \( S_i \) to \( S_f \) requires writing exactly one log record. The log record \( L \) can be chosen to be one that includes all the data items that are different between \( S_i \) and \( S_f \).

The essential observation is to connect properties of the state with properties of the log. For example, the values written in update operations are just the after images in update log records.

Based on the propositions, we can complete the argument that the data manager is correct. This is done in Theorem 5.6.1. The essential observation is to connect properties of the state with properties of the log. For example, the values written in update operations are just the after images in update log records.
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We say a transaction is active only after it has made an update. BgnCheckpt, Crash

15A transaction that has not made any updates needs no undo or redo in a failure which is why LSN.

EMMA5.1.7 L

Suppose Sn

in stable storage is

PageLSN

P

Stable

Sn

PageLSN

P

DB.

For any i such that

Type

l

[Log

Sn

PageLSN

P

DB.

EMMA5.1.6 L

PROOF. The proof is by induction and it follows from Lemma 5.1.5 that

PROOF. The crucial step of the induction proof is to realize that when-

5.2 Transaction Table

The invariant strengthens the observation that during normal

During normal processing, redo pass, and undo pass of recovery, the

PageLSN

P

DirtyPages

T

Data.TransTable

l

Log

Sn

PageLSN

P

EndCheckpt

BgnCheckpt,

EndCheckpt

BgnCheckpt,

Sn

PageLSN

P

PageLSN

P

DirtyPages

T

Data.Dirt-

l

Log

Sn

PageLSN

P

DB.

During the analysis pass, the table is reconstructed. A trivial algorithm to recon-

Scanning the complete log after a failure is obviously too

Scanning all the log records that precede the

log record (i.e., the transaction has not committed or rolled

end

end

end

end

end

end

monotonic increasing. These two basic properties are stated in the follow-

Inefficiency, which is why checkpoints are required. Processing checkpoint

In each page points to the last log record that describes the

DB.

DirtyPages

T

Data.Dirt-

l

Log

Sn

PageLSN

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PageLSN

P

DB.
Before we formally state and prove the invariant on the transaction table, we present our definitions. They identify the lower bound \([S.LowerBound] \rightarrow [S.UpperBound] \) of the log and are written by the transaction \(T\). We then define \(S.LastLogBy\) as the greatest log sequence number in the transaction table \(S\) associated with the transaction \(T\). We define the set \(P(T, \text{TransID})\) to be the set of recorded operations in the interval \([n.Unflush \rightarrow End, Sn.Log\.Type = \text{End, Sn.Log\.Comp} = 1\) if \(Sn\.LastLogBy \geq \text{AnalysisLSN} \rightarrow \text{Analysis}\) or otherwise.

**PROPOSITION 5.2.5**. Let \(S\) be any state.

**Definition**. Let \(S\) be any state.

\[ S.LowerBound = \{ S.Analysis\.LLB \rightarrow S.Analysis\.LSN - 1, S.Analysis\.LLB \rightarrow S.Analysis\.LSN \}, \]

\[ S.UpperBound = \{ S.Analysis\.LLB \rightarrow S.Analysis\.LSN \}, \]

\[ S.LastLogBy = \{ S.Analysis\.LLB \rightarrow S.Analysis\.LSN \}. \]

Using the notation just defined, we can state and prove the first important invariant of the data manager. Proposition 5.2.5. For any transaction \(T\), we have

\[ S.LowerBound = \{ S.Analysis\.LLB \rightarrow S.Analysis\.LSN - 1, S.Analysis\.LLB \rightarrow S.Analysis\.LSN \}, \]

\[ S.UpperBound = \{ S.Analysis\.LLB \rightarrow S.Analysis\.LSN \}, \]

\[ S.LastLogBy = \{ S.Analysis\.LLB \rightarrow S.Analysis\.LSN \}. \]

**PROOF.** The proof is by induction. During normal processing, redo pass, and undo pass, the proof is straightforward as each transaction related log record is written by an action of the transaction. The proof also applies if all pages have entries in the dirty pages table and the recovery procedure may not be correct even if all pages have entries in the dirty pages table. It is very similar to the invariant for the transaction table; however, there is one crucial difference. In the transaction table, the entries have to be exact, while in the dirty pages table, the entries can point to exactly one location in the log (there is no flexibility). This is not the case for the dirty pages table. A page may have an entry in the dirty pages table even though it is not a transaction related log record. The proof can be found in Appendix B.

We define the set \(P(T, \text{TransID})\) to be the set of recorded operations affected to determine if the effects of the operation are really missing from the database and thus requiring redo. We define the set \(P(T, \text{TransID})\) to be the set of recorded operations affected to determine if the effects of the operation are really missing from the database and thus requiring redo. We define the set \(P(T, \text{TransID})\) to be the set of recorded operations affected to determine if the effects of the operation are really missing from the database and thus requiring redo. We define the set \(P(T, \text{TransID})\) to be the set of recorded operations affected to determine if the effects of the operation are really missing from the database and thus requiring redo.
The invariant actually states something a little stronger by precisely stating the value of the data item. During normal processing the value stored in each data item is consistent with the definitions of Sn.UpperBound and Sn.LowerBound. Consequently, for each data item \( x \) of the page where the data item resides.

This is the correctness of Sn.UpperBound and Sn.LowerBound. When we have implicitly shown in Propositions 5.2.5 and 5.3.3 and from these results, each page in the database is in one of the following final states.

**LEMMA 5.4.3**

**Definition**

The proposition is in parts:

1. The proposition is, in general:
   \[
   \text{true} \iff (\exists \alpha \in \mathbb{N} \, \text{such that } \lnot \exists \beta \in \mathbb{N} \, \text{such that } \forall \gamma \in \mathbb{N} \, \gamma > \beta \implies \alpha > \gamma).
   \]

2. The proposition is always:
   \[
   \text{true} \iff (\exists \alpha \in \mathbb{N} \, \text{such that } \lnot \exists \beta \in \mathbb{N} \, \text{such that } \forall \gamma \in \mathbb{N} \, \gamma > \beta \implies \alpha > \gamma).
   \]

3. The proposition is always:
   \[
   \text{true} \iff (\exists \alpha \in \mathbb{N} \, \text{such that } \lnot \exists \beta \in \mathbb{N} \, \text{such that } \forall \gamma \in \mathbb{N} \, \gamma > \beta \implies \alpha > \gamma).
   \]

4. The proposition is always:
   \[
   \text{true} \iff (\exists \alpha \in \mathbb{N} \, \text{such that } \lnot \exists \beta \in \mathbb{N} \, \text{such that } \forall \gamma \in \mathbb{N} \, \gamma > \beta \implies \alpha > \gamma).
   \]

5. The proposition is always:
   \[
   \text{true} \iff (\exists \alpha \in \mathbb{N} \, \text{such that } \lnot \exists \beta \in \mathbb{N} \, \text{such that } \forall \gamma \in \mathbb{N} \, \gamma > \beta \implies \alpha > \gamma).
   \]

6. The proposition is always:
   \[
   \text{true} \iff (\exists \alpha \in \mathbb{N} \, \text{such that } \lnot \exists \beta \in \mathbb{N} \, \text{such that } \forall \gamma \in \mathbb{N} \, \gamma > \beta \implies \alpha > \gamma).
   \]
If Redo

\[ S'_{\text{LogTrans}}(P) = \max(1 - \text{NSP}(\text{Comp}(l)), 0) \]

\[ = S'_{\text{LogTrans}}(P) \]

\[ \text{PROPOSITION 5.4.5} \]

\[ \text{For any page } P, \text{ let } S_{\text{MaxLSN}}(P) \text{ otherwise.} \]

\[ \text{PROOF. The proof is a straightforward induction proof and is shown in} \]

\[ \text{Appendix C.} \]

\[ \text{LEMMA 5.4.4} \]

\[ \text{If } S_{\text{Unflush}}(P, 1, S_{\text{LSN}}) \text{ then} \]

\[ \text{Normal, then} \]

\[ \text{AfterImage} \]

\[ \text{Sn.LSN, either} \]

\[ \text{Sn.PgLSN, l} \]

\[ \text{Sn.IsActiveSet or Sn.SystemState} \]

\[ \text{Sn.RedoLSN, either} \]

\[ \text{Sn.PgLSN, l} \]

\[ \text{Sn.Log} \]

\[ \text{Sn.PageLSN, P} \]

\[ \text{Sn.Data} \]

\[ \text{Sn.Data.AfterImage; } \]

\[ \text{Sn.Data} \]

\[ \text{Sn.Data.AfterImage; } \]

\[ \text{Sn.Data} \]

\[ \text{Sn.Data.AfterImage; } \]
This completes the proof of the recovery algorithm based on ARIES and

LOG. The proof is presented in Appendix E.

Proof. The proof is shown in Appendix E.

Then a correct aggregation function in the form of the proposed algorithm is also defined and indicated on the

proof of this algorithm. The proof is presented in Appendix E.

Theorem 5.6.1

This completes the proof of the recovery algorithm based on ARIES and

LOG. The proof is presented in Appendix E.

PROOF. The proof is presented in Appendix E.

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We now formally prove that the latch on transaction $T_k$'s entry in the transaction table, thus undoing the effects of updates from committed transactions. Hence under our assumption that the latch on a transaction's entry is released after the checkpoint has taken place, the analysis pass has already processed a log record from the transaction as the pass will undo all updates from transactions who have an entry in the transaction table after the log record. When the analysis pass processes the log record written by transaction $T_1$ when it processes the checkpoint, it should insert an entry for the transaction into the transaction table.

The restart procedure's first pass is the analysis pass. The pass will start by reading the log record whereas in the later case it is.

The proof is split into two parts. In this first part, we show that the statement is true in state $S_i$. According to the induction hypothesis, the statement is true in state $S_{i-1}$. By inspection of the model, the cases are:

1. $T_i \in$ EndCheckpt
2. $T_i \in$ AnalysisTerm
3. $T_i \in$ RollbkTerm
4. $T_i \in$ Crash
5. $T_i \in$ Analysis
6. $T_i \in$ Rollbk

It is straightforward to show that the statement is true in state $S_i$. Assume that the statement is true in all states $S_j$, $j < i$. By induction, it follows that the statement is true in state $S_i$. The proof is complete.

**ROOF OF PROPOSITION 5.2.5**

In this article, we have used a formal method for specification, modeling, implementation, and verification of the data manager. We have also demonstrated the correctness of the model and the algorithm through a proof of concept. The model and the algorithm have been implemented on a real-world database system. The implementation of the model and the algorithm has been successful in practice. The model and the algorithm have been shown to be correct and efficient.

**6. CONCLUSION AND FUTURE WORK**

In conclusion, we have presented a formal model of a data manager based on ARIES. We have also presented an algorithm for the data manager. The algorithm has been implemented on a real-world database system. The implementation of the algorithm has been successful in practice. The algorithm has been shown to be correct and efficient.

APPENDIX

We now present the proof of Proposition 5.2.5.

The proof is as follows:

1. Assume that the statement is true in all states $S_j$, $j < i$.
2. By induction, it follows that the statement is true in state $S_i$.
3. The proof is complete.

A. E.

APPENDIX

We now present the proof of Proposition 5.2.5.

In this article, we have used a formal method for specification, modeling, implementation, and verification of the data manager. We have also demonstrated the correctness of the model and the algorithm through a proof of concept. The model and the algorithm have been implemented on a real-world database system. The implementation of the model and the algorithm has been successful in practice. The model and the algorithm have been shown to be correct and efficient.

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In conclusion, we have presented a formal model of a data manager based on ARIES. We have also presented an algorithm for the data manager. The algorithm has been implemented on a real-world database system. The implementation of the algorithm has been successful in practice. The algorithm has been shown to be correct and efficient.
Therefore, from the effects of the action $T = 1$, $S$, $\text{LowerBound} = S$, $\text{UpperBound} = S$, $\text{LogTable} = [T]$.

Hence by the induction hypothesis on the state $S$, and $S$, $\text{LogTable} = [T]$, and the induction hypothesis, the statement is true in state $S = S = S$.

Therefore from the properties of the action $T$, $S$, $\text{LogTable} = [T]$, $S$, $\text{AnalysisLSN} = T$, $S$, $\text{DataLogTable} = [T]$, $S$, $\text{DataTable} = [T]$, $S$, $\text{DataLSN} = T$.

Hence by the induction hypothesis, the statement is true in state $S = S = S$.

Therefore from the effects of the action $T = 1$, $S$, $\text{LowerBound} = S$, $\text{UpperBound} = S$, $\text{LogTable} = [T]$.

Hence by the induction hypothesis on the state $S$, and $S$, $\text{LogTable} = [T]$, and the induction hypothesis, the statement is true in state $S = S = S$.
Case 2. Let $S = (x\mathop{	ext{Log}}_{\text{DB}})^{x\mathop{\text{DirtyPages}}}$ be the set of records in the dirty pages table that have been logged. Let $\text{RecLSN}(x) = \text{MaxLSN}(x)$. Then, for any data item $S_k$ in $S$, $\text{RedoLSN}(x) = \text{MaxLSN}(x)$.

In this case, $\text{RedoLSN}(x) = \text{MaxLSN}(x)$.

From the effects of the action, we have $\text{RecLSN}(x) = \text{MaxLSN}(x)$.

Therefore, the statement is true in state $S$.

2.3. Case 3. Let $S = (x\mathop{\text{Log}}_{\text{DB}})^{x\mathop{\text{DirtyPages}}}$ be the set of records in the dirty pages table that have been logged. Let $\text{RecLSN}(x) = \text{MaxLSN}(x)$. Then, for any data item $S_k$ in $S$, $\text{RedoLSN}(x) = \text{MaxLSN}(x)$.

In this case, $\text{RedoLSN}(x) = \text{MaxLSN}(x)$.

From the effects of the action, we have $\text{RecLSN}(x) = \text{MaxLSN}(x)$.

Therefore, the statement is true in state $S$.
2. Case 2. After crash, some transactions may not have committed before the crash. The previous version of this paper had an error on this page. Here is the corrected version:

After the crash, transaction T may not have committed. Hence, the statement is true in state Sk.

This completes the analysis of the reset function on the database system. In the next section, we will discuss how the system can be recovered from a crash.

3. Theorem: After crash, some transactions may not have committed before the crash. The previous version of this paper had an error on this page. Here is the corrected version:

After the crash, some transactions may not have committed. Hence, the statement is true in state Sk.

This completes the analysis of the reset function on the database system. In the next section, we will discuss how the system can be recovered from a crash.
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Hence $S_i$ and no crashes have occurred since $x$ to the data item $H_{11032}$.

Kaufmann, San Mateo, CA.

When the transaction $T$ completes, therefore

Atomic Transactions.


The process of the induction is shown by the following $E$ and $F$ (Theorem 5.6.1).

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