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Computer Science Division — EECS

Spring 1999 John Kubiatowicz

Homework Quiz (HW #3)

March 1, 1999
CS152 Computer Architecture and Engineering

This quiz covers one of the problems from homework #3.
Good Luck!

Your Name:	
SID Number:	
Discussion Section:	

Total:	

Combinational Multipliers built with Carry Save Adders

In your homework, you learned about using Carry Save Adders to perform multiplication. Below we review using carry save adders to add four 3-bit numbers to produce a single 5-bit number:

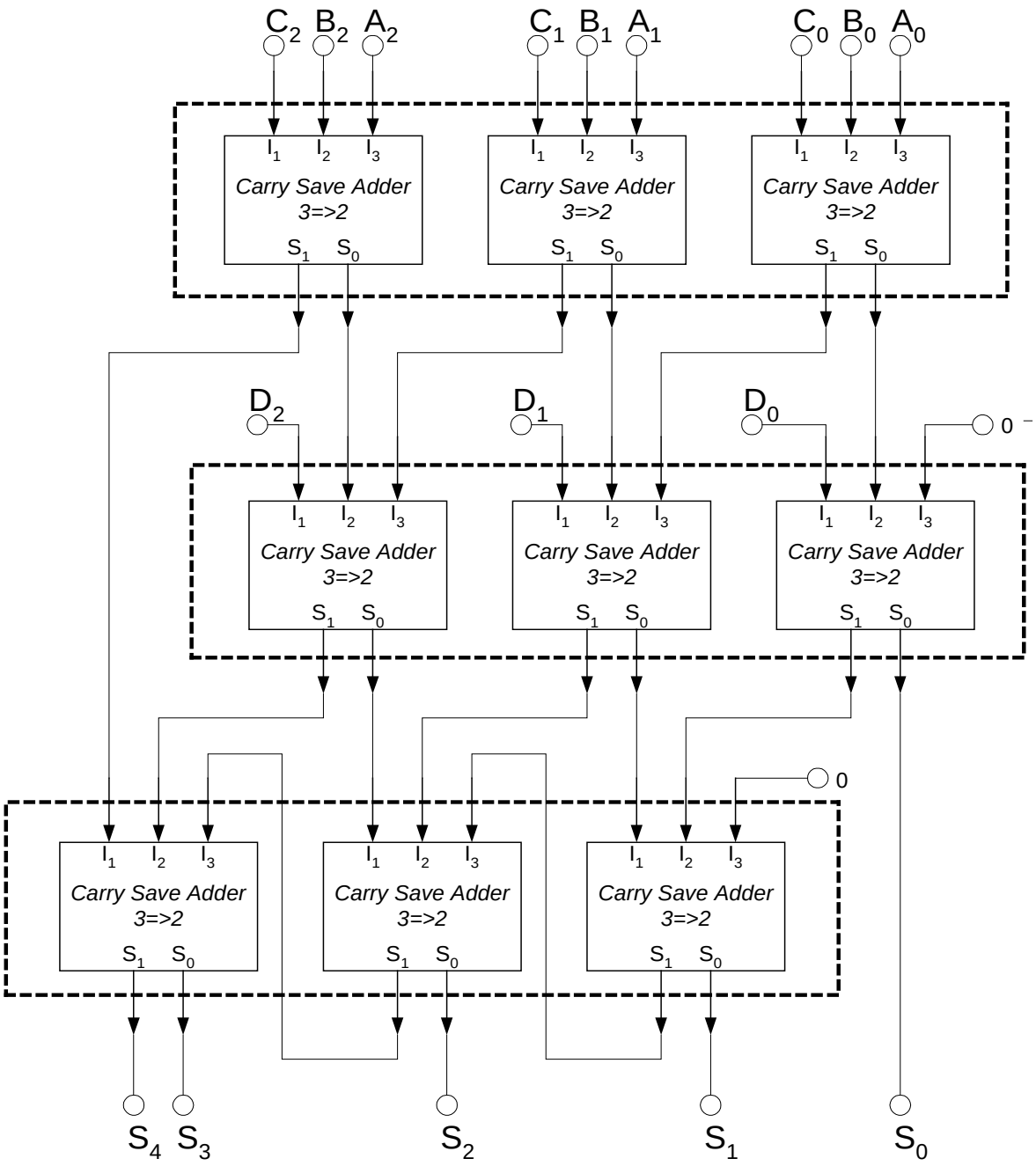
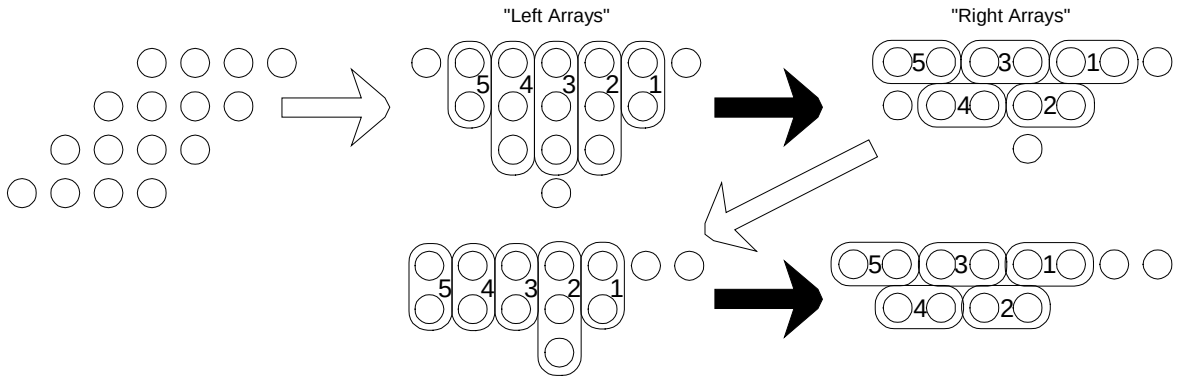


Figure 1: Adding four 3-bit numbers with carry-save adders.

1. Draw a complete 4 x 4 multiplier that takes two four-bit numbers (call the input bits A3…A0 and B3…B0), and which outputs an 8-bit result (M7 … M0). Use a carry-save architecture for this, use as little hardware as possible, and *assume that the numbers are unsigned*:



Extra Credit

One way to view a 4x4 carry-save multiplier is as a sequence of transformations on arrays of dots. This diagram is given above. Each dot is a non-zero bit. The white arrows are renaming of bits, while the black arrows are carry-save logic. Number these arrays 1-5. Array 1 contains the input bits. We push all the bits straight up (filling in the empty spots which had zeros) to get array 2.

Each subsequent *pair* of arrays (called "left" and "right") represents one level of carry-save logic. The circled dots in a *column* of a left arrays are bits that are summed with a 3⇒2 adder. The circled dots in a *row* of a right array are the results. Thus, for instance, the two bits labeled "1" in array 3, are the result of adding the 2 bits labeled "1" in array 2. The long white arrow represents output of the first level going into inputs of the next. ETC. We continue until we have only two rows of bits, which will be added with a standard adder (possibly a carry-lookahead adder).

Representing each level by the maximum number of bits in any one column, we get a sequence that starts with the number of multiplier bits (4 in this case) and ends with the number 2:

$$4 \Rightarrow 3 \Rightarrow 2$$

Note that we could have more than one adder per column in a left array if there are more bits. For a 6x6 multiplier, for instance, array 2 would have *two* adders in the middle column, producing *four* rows of result bits in array 3. The first level would thus take 6 bits ⇒ 4 bits, etc.

2. How many levels of carry-save addition would be needed for a 53 x 53 bit multiplier (this is the number of mantissa bits in IEEE floating point)? The answer for 4x4 multiplier would be "2":