In general, we want to place a distribution over sentences. Basic / classic solution: n-gram models

$$P(w) = \prod_i P(w_i | w_{i-1} \ldots w_{i-k})$$

Question: how to estimate conditional probabilities?

$$P(w|w') = \frac{c(w', w)}{c(w')} + \alpha(w')P(w)$$

Problems:
- Known words in unseen contexts
- Entirely unknown words
- Many systems ignore this – why?
- Often just lump all new words into a single UNK type

What’s wrong with add-d smoothing?
- Let’s look at some real bigram counts [Church and Gale 91]:

<table>
<thead>
<tr>
<th>Count in 22M words</th>
<th>Actual c* (held 22M)</th>
<th>Add-one c*</th>
<th>Add-0.00000037 c*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.446</td>
<td>0.446</td>
<td>0.444</td>
</tr>
<tr>
<td>2</td>
<td>3.24</td>
<td>3.26</td>
<td>3.26</td>
</tr>
<tr>
<td>3</td>
<td>2.32</td>
<td>2.32</td>
<td>2.32</td>
</tr>
<tr>
<td>4</td>
<td>4.21</td>
<td>4.23</td>
<td>4.23</td>
</tr>
</tbody>
</table>

**Big things to notice:**
- Add-one vastly overestimates the fraction of new bigrams
- One solution: use held-out data to predict the map of c to c*

We’d like to not need held-out data (why)?

Idea: leave-one-out validation
- Nc: number of types which occur k times in the entire corpus
- Take each of the c tokens out of corpus in turn
- c “training” sets of size c-1, “held-out” of size 1
- How many “held-out” tokens are seen in “training”?
  - Nc
- How many held-out tokens are seen k times in “training”? (k+1)Nc
- There are Nc words with training count k
- Each should occur with expected count (k+1)Nc/Nc
- Each should occur with probability: (k+1)Nc/Nc (k+1)Nc

Hypothesis: counts of k should be k* = (k+1)Nc/Nc

Problem: what about “the”? (say k=4417)
- For small k, Nc > Nc
- For large k, too jumpy, zeros wreck estimates

Simple Good-Turing [Gale and Sampson]: replace empirical Nc with a best-fit power law once count counts get unreliable

Hypothesis: counts of k should be k* = (k+1)Nc/Nc

Katz Smoothing
- Use GT discounted bigram counts (roughly – Katz left large counts alone)
- Whatever mass is left goes to empirical unigram

$$P_{Katz}(w|w') = \frac{c(w', w)}{c(w')} + \alpha(w')P(w)$$
Kneser-Ney: Discounting

- Kneser-Ney smoothing: very successful estimator using two ideas
- Idea 1: observed n-grams occur more in training than they will later:

<table>
<thead>
<tr>
<th>Count in 22M Words</th>
<th>Avg in Next 22M</th>
<th>Good-Turing κ*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.446</td>
<td>0.446</td>
</tr>
<tr>
<td>2</td>
<td>1.24</td>
<td>1.20</td>
</tr>
<tr>
<td>3</td>
<td>2.24</td>
<td>2.24</td>
</tr>
<tr>
<td>4</td>
<td>3.23</td>
<td>3.24</td>
</tr>
</tbody>
</table>

- Absolute Discounting
  - Save ourselves some time and just subtract 0.75 (or some d)
  - Maybe have a separate value of d for very low counts

\[ P_{\text{ad}}(w|w') = \frac{c(w',w) - d}{c(w')} + \alpha(w')P(w) \]

Kneser-Ney

- Kneser-Ney smoothing combines these two ideas
  - Absolute discounting
    \[ P(w|w') = \frac{c(w',w)}{c(w')} + \alpha(w')P_c(w) \]
  - Lower order continuation probabilities
    \[ P_c(w) \propto |w'| : c(w',w) > 0 | \]

- KN smoothing repeatedly proven effective (ASR, MT, …)
  - Teh, 2006 shows KN smoothing is a kind of approximate inference in a hierarchical Pitman-Yor process (and better approximations are superior to basic KN)

What Actually Works?

- Trigrams and beyond:
  - Unigrams, bigrams generally useless
  - Trigrams much better (when there’s enough data)
  - 4-, 5-grams really useful in MT, but not so much for speech

- Discounting
  - Absolute discounting, Good-Turing hold-out estimation, Witten-Bell

- Context counting
  - Kneser-Ney construction of lower-order models

- See [Chen+Goodman], reading for tons of graphs!

Data >> Method?

- Having more data is better…

- … but so is using a better estimator

- Another issue: N > 3 has huge costs in speech recognizers

Tons of Data?

[Graphs from Brants et al., 2007]
Large Scale Methods

- Language models get big, fast.
  - English Gigawords corpus: 2G tokens, 0.3G trigrams, 1.2G 5-grams
  - Need to access entries very often, ideally in memory
- What do you do when language models get too big?
  - Distributing LMs across machines
  - Quantizing probabilities
  - Random hashing (e.g. Bloom filters)

[Image: Talbot and Osborne 07]

Beyond N-Gram LMs

- Lots of ideas we won’t have time to discuss:
  - Caching models: recent words more likely to appear again
  - Trigger models: recent words trigger other words
  - Topic models
- A few recent ideas
  - Syntactic models: use tree models to capture long-distance syntactic effects [Chelba and Jelinek, 98]
  - Discriminative models: set n-gram weights to improve final task accuracy rather than fit training set density [Roark, 05, for ASR; Liang et al., 06, for MT]
  - Structural zeros: some n-grams are syntactically forbidden, keep estimates at zero if the look like real zeros [Mohri and Roark, 06]
  - Bayesian document and IR models [Daume 06]

Overview

- So far: language models give P(s)
  - Help model fluency for various noisy-channel processes (MT, ASR, etc.)
  - N-gram models don’t represent any deep variables involved in language structure or meaning
  - Usually we want to know something about the input other than how likely it is (syntax, semantics, topic, etc)
- Next: Naïve-Bayes models
  - We introduce a single new global variable
  - Still a very simplistic model family
  - Lets us model hidden properties of text, but only very non-local ones...
  - In particular, we can only model properties which are largely invariant to word order (like topic)

Text Categorization

- Want to classify documents into broad semantic topics (e.g. politics, sports, etc.)
- Obama is hoping to rally support for his $825 billion stimulus package on the eve of a crucial House vote. Republicans have expressed reservations about the proposal, calling for more tax cuts and less spending. GOP representatives seemed doubtful that any deals would be made.
- California will open the 2009 season at home against Maryland Sept. 5 and will play a total of six games in Memorial Stadium in the final football schedule announced by the Pac-10 Conference Friday. The original scheduled called for 12 games over 12 weekends.
- Which one is the politics document? (And how much deep processing did that decision take?)
  - One approach: bag-of-words and Naïve-Bayes models
  - Another approach later...
  - Usually begin with a labeled corpus containing examples of each class

Naïve-Bayes Models

- Idea: pick a topic, then generate a document using a language model for that topic.
- Naïve-Bayes assumption: all words are independent given the topic.

\[
P(c, w_1, w_2, \ldots, w_n) = P(c) \prod_{i=1}^{n} P(w_i | c)
\]

We have to smooth these!

\[
P(c \mid w_1, w_2, \ldots, w_n) = \frac{P(c) \prod_{c'=c} P(w_i \mid c)}{\sum_{c'} P(c') \prod_{i=1}^{n} P(w_i \mid c')}
\]

What about totally unknown words?
- Can work shockingly well for text (especially in the wild)
- How can unigram models be so terrible for language modeling, but class-conditional unigram models work for text?
- Numerical / speed issues
- How about NB for spam detection?

Using NB for Classification

- We have a joint model of topics and documents

\[
P(c, w_1, w_2, \ldots, w_n) = P(c) \prod_{i=1}^{n} P(w_i | c)
\]

Gives posterior likelihood of topic given a document

\[
P(c \mid w_1, w_2, \ldots, w_n) = \frac{P(c) \prod_{c'} P(w_i \mid c)}{\sum_{c'} P(c') \prod_{i=1}^{n} P(w_i \mid c')}
\]
Two NB Formulations

- Two NB event models for text categorization
  - The class-conditional unigram model, a.k.a. multinomial model
    - One node per word in the document
    - Driven by words which are present
    - Multiple occurrences, multiple evidence
    - Better overall – plus, know how to smooth
  - The binominal (binary) model
    - One node for each word in the vocabulary
    - Incorporates explicit negative correlations
    - Know how to do feature selection (e.g., keep words with high mutual information with the class variable)

Example: Barometers

<table>
<thead>
<tr>
<th>Reality</th>
<th>P(+,+,r) = 1/8</th>
<th>P(+,+,s) = 3/8</th>
<th>P(+,+,x) = 3/8</th>
<th>P(−,−,s) = 1/8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunny</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raining</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NB Model

- NB FACTORS:
  - \( P(s) = 1/2 \)
  - \( P(+|s) = 1/4 \)
  - \( P(+|r) = 3/4 \)
  - \( P(-|s) = 9/10 \)
  - \( P(−|−) = 1/10 \)

PREDICTIONS:

- \( P(r|h,h) = 9/10 \)
- \( P(s|h,h) = 1/10 \)

(Non-)Independence Issues

- Mild Non-Independence
  - Evidence all points in the right direction
  - Observations not entirely independent
  - Results
    - Inflated confidence
    - Deflated Priors
  - What to do? Boost priors or attenuate evidence (hack)

\[
P(c; w_1, w_2, \ldots, w_n) = \prod_j P(w_j | c) \prod_j P(c)^{\text{boost}}
\]

- Severe Non-Independence
  - Words viewed independently are misleading
  - Interactions have to be modeled (‘not bad’)
  - What to do?
    - Change your model!

Language Identification

- How can we tell what language a document is in?
  - The 38th Parliament will meet on Monday, October 4, 2004, at 11:00 a.m.
  - The first item of business will be the election of the Speaker of the House of Commons. Her Excellency the Governor General will open the First Session of the 38th Parliament on October 5, 2004, with a Speech from the Throne.

- How to tell the French from the English?
  - Treat it as word-level textual?
    - Overkill, requires a lot of training data
    - You don’t actually need to know about words
  - How to tell the French from the English?
    - Sume lempérance et enfin de stabilità e di crescita
    - Patto di stabilità e di crescita
  - Option: build a character-level language model

Class-Conditional LMs

- Can add a topic variable to other language models

\[
P(c, w_1, w_2, \ldots, w_n) = P(c) \prod P(w_i | w_{i-1}, c)
\]