Interferometric Noise Reduction in Fiber-Optic Links by Superposition of High Frequency Modulation

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Abstract— In this paper, the reduction of interferometric noise by superposition of high frequency modulation is analyzed. It is shown that the nature of this reduction is due to a redistribution of noise energy from baseband to higher frequencies where it can be discarded by low-pass filtering. Detailed analysis revealed the dependence of the noise reduction factor on the product $f_0\tau$, and the modulation index of the high frequency superimposed modulation. The proper choice of parameters can lead to complete elimination of the converted phase noise from the system.

I. Introduction

It is well known that phase noise fluctuations in the output of a semiconductor laser can produce intensity noise fluctuations upon transmission through a fiber-optic link due to interferometric phase-to-intensity conversion [1]–[4]. In a single-mode fiber link, the interferometric conversion occurs when multiple reflections occur between a pair of fiber interfaces (Fig. 1). Even in the absence of such fiber discontinuities, Rayleigh scattering in a sufficiently long piece of fiber can cause similar effects [5]. If a laser source is used in a multimode fiber link, the different transverse modes of the fiber interfere with one another and produce the well known "modal noise." In this paper, we shall deal mostly with the former case (multiple reflections in single-mode fiber link) although it is straight-forward to extend the formalism to the case of modal noise in multimode fiber. The nature of the interferometric noise has been studied in [1]–[3]. It was shown that this excess noise can cause bit-error-rate floors [6], and the system performance has been evaluated as a function of the number of magnitude of the reflections [7].

To the extent that such interferometric noises arise from interference of the laser output with a delayed version of itself, it is obvious that reduction of the laser coherence can eliminate these noises. Indeed, there have been proposals and early demonstrations that by applying a high frequency modulation to the laser, the coherence of the laser can be reduced, which leads to a reduction of the type of interferometric noise mentioned above. However, except in the case of a very deep modulation (where the laser output is pulse-like and each pulse is incoherent with the previous ones), one does not expect the laser output to be rendered totally incoherent by the applied modulation, but instead one simply "chirps" the lasing frequency sinusoidally at the modulation frequency. Can one expect interferometric noises to be reduced (or even totally eliminated) under this situation? The following analysis will show that the answer is positive with proper choices of modulation format and parameters.

Manuscript received July 1, 1991; revised January 13, 1992. This work was supported in part by IBM TJW Research Center and an NSF-PYI award.

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IEEE Log Number 9200478.

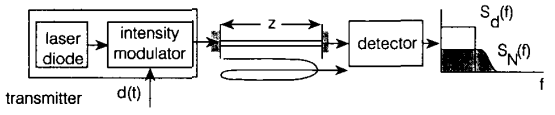

Fig. 1. Fiber-optic system exhibiting multiple reflections between a pair of fiber interfaces.

We shall first describe the interferometric noise in Section II. These results are already well known [1]–[3] but are repeated here for the sake of defining the symbols used in this paper. Next, we shall consider a number of high frequency superimposed modulation formats. The order of the sequence is chosen to best illustrate the nature of the mechanisms responsible for the interferometric noise reduction. These will be discussed in detail in the following sections.

II. Interferometric Noise

As we mentioned before, the results derived in this section are well known [1]–[3], but are repeated here for the sake of defining the symbols used in the paper.

Consider the intensity noise generated in a single-mode (SM) fiber optic link through interferometric FM-AM conversion due to, for example, double reflection between two pairs of connectors (Fig. 1). The laser is assumed to be single-mode, and it is also assumed that the data is intensity modulated onto the optical output using an external chirp-free modulator. The electric field at the input of the fiber is given by

$$E(t) = \sqrt{P(t)}\, e^{j\Omega_0 t + \varphi(t)} \tag{1}$$

where $P(t)$ is the output laser power, Ω_0 is the optical carrier frequency and $\varphi(t)$ is the laser phase noise. The field at the output of the fiber is represented by the superposition of the original input field with delayed version of itself

$$E_{\text{out}}(t) = \psi_1 E(t - t_1) + \psi_2 E(t - t_2) \tag{2}$$

where ψ_1 and ψ_2 are the relative field intensities, and t_1 and t_2 are delays. Without loss of generality, we can assume that $\psi_1 = 1$, $\psi_2 = \psi$ $t_1 = 0$ and $t_2 = \tau$.

The optical power of the laser can be written as

$$P(t) = P_0 d(t) \tag{3}$$

where P_0 is the average output laser power and $d(t)$ is data which is band limited. We neglect intrinsic laser intensity noise

in the following analysis, since it is much smaller than the other noises considered.

The electric field at the fiber output will be

$$E_{\text{out}}(t) = \sqrt{P_0 d(t)}\, e^{j\Omega_0 t} e^{j\varphi(t)} + \psi\sqrt{P_0 d(t-\tau)}\, e^{j\Omega_0(t-\tau)+j\varphi(t-\tau)} \quad (4)$$

Then, the intensity at the output of the fiber is:

$$i(z,t) = |E(t)|^2 + \psi^2|E(t-\tau)|^2 + 2\psi\,\text{Re}\{E(t)E^*(t-\tau)\}. \quad (5)$$

We can identify the signal $(i_s(z,t))$ and the noise part $(i_N(z,t))$ of $i(z,t)$ as

$$i_S(z,t) = P_0\left[d(t) + \psi^2 d(t-\tau)\right] \approx P_0 d(t)$$

where we have assumed $\psi \ll 1$ and

$$i_N(z,t) = 2\psi P_0\sqrt{d(t)d(t-\tau)}\,\cos[\Omega_0\tau + \varphi(t) - \varphi(t-\tau)]. \quad (6)$$

The laser phase noise $\varphi(t)$ is modeled to follow Gaussian probability density function and $\varphi(t)$ and $\varphi(t-\tau)$ are correlated in such a way that [1]:

$$\left\langle(\varphi(t) - \varphi(t-\tau))^2\right\rangle = \frac{|\tau|}{\tau_c} \quad (7)$$

where τ_c is the laser coherence time, and $\langle\rangle$ denotes statistical averaging. In this case, (7) corresponds to a Lorentzian lineshape of the spectral distribution. The expression (6) is a general result, which will be used to derive the noise spectrum in several cases. With the above assumptions, we ignore the fine structure of the spectrum due to relaxation oscillations [8] because it is of secondary importance in this work.

It is straightforward to find the signal and the noise spectrum. We need to find the corresponding autocorrelation functions. They are:

$$R_S(\delta\tau) = E\{i_S(z,t)i_S(z,t+\delta\tau)\} = (1+\psi^2)P_0^2 R_d(\delta\tau) \quad (8)$$

$$R_N(\delta\tau) = E\{i_N(z,t)i_N(z,t+\delta\tau)\} = 2\psi^2 P_0^2 R_{dd}(\delta\tau)[R_-(\delta\tau) + R_+(\delta\tau)\cos(\Omega_0\tau)] \quad (9)$$

where $R_d(\delta\tau)$ is the autocorrelation function of $d(t)$, and

$$R_{dd}(\delta\tau) = E\left\{\sqrt{d(t)d(t+\delta\tau)d(t-\tau)d(t+\delta\tau-\tau)}\right\}. \quad (10)$$

The corresponding power spectral densities are denoted by $S_d(f)$ and $S_{dd}(f)$. To compute $R_{dd}(\delta\tau)$ one needs to specify the data statistics. However, for our purpose we do not need the explicit knowledge of $R_{dd}(\delta\tau)$, but we only need to know the bandwidth of the Fourier transform of R_{dd}. If we assume that $d(t)$ consists of ideal rectangular pulses, then the bandwidth of $S_{dd}(f)$ is equal to the bandwidth of $S_d(f)$.

The expression in [] in (9) is recognized as that due to the interferometrically converted laser phase noise of a CW laser in the absence of data modulation. The expressions R_+ and R_- are given by:

$$R_-(\delta\tau) = \langle\cos[\varphi(t) - \varphi(t+\tau) - \varphi(t+\delta\tau) + \varphi(t+\delta\tau+\tau)]\rangle \quad (11)$$

$$R_+(\delta\tau) = \langle\cos[\varphi(t) - \varphi(t+\tau) + \varphi(t+\delta\tau) - \varphi(t+\delta\tau+\tau)]\rangle \quad (12)$$

and have been previously calculated [1], [2]. The variations of the term $R_+(\delta\tau)\cos(\Omega_0\tau)$ are of the order of the laser wavelength. We are interested in the macroscopic variations, which are on much bigger scale than those due to the term involving $R_+(\delta\tau)$. For this reason the term including $R_+(\delta\tau)$ will be neglected.

$R_-(\delta\tau)$ is given by [1]

$$R_-(\delta\tau) = \exp\left[-\frac{1}{2\tau_c}(2|\tau| - |\tau-\delta\tau| - |\tau+\delta\tau| + 2|\delta\tau|)\right]. \quad (13)$$

For sufficiently long τ, the corresponding power spectral density of $R_-(\delta\tau)$ (denoted by $S_-(f)$, given by the Fourier transform of $R_-(\delta\tau)$) assumes the Lorentzian lineshape of the lasing field down-converted to baseband, with a typical width of 10 to 100 MHz.

Then, the noise autocorrelation function becomes:

$$R_N(\delta\tau) = 2\psi^2 P_0 R_{dd}(\delta\tau) R_-(\delta\tau). \quad (14)$$

Its power spectral density is given by

$$S_N(f) = 2\psi^2 P_0 S_{dd}(f) \bigstar S_-(f) \quad (15)$$

where $\bigstar$ denotes convolution.

The power spectral densities are schematically illustrated in Fig. 3. Note that the S/N ratio cannot be increased by increasing the data signal power, since the noise power also increases correspondingly as per (15). It illustrates clearly the deleterious effect of interferometric FM-IM noise on the maximum achievable S/N ratio of the transmitted data.

A. Superimposed High-Frequency Modulation—External Phase Modulation

Consider the case in Fig. 2(b) which is similar to Fig. 2(a) but for an external phase modulator placed at the output of the laser. The modulator is driven by a single RF tone at a "high" frequency (higher than the data bandwidth, to be specified later). The optical field at the input to the fiber is then

$$E(t) = \sqrt{d(t)P_0}\, e^{j\Omega_0 t} e^{j\varphi(t)} e^{ja\cos(\omega_0 t)} \quad (16)$$

where a is the phase modulation index and $f_0 = \omega_0/2\pi$ is the RF modulation frequency. Following a similar procedure as in the last section, we obtain for the output signal intensity

$$i_S(z,t) = d(t)P_0 \quad (17)$$

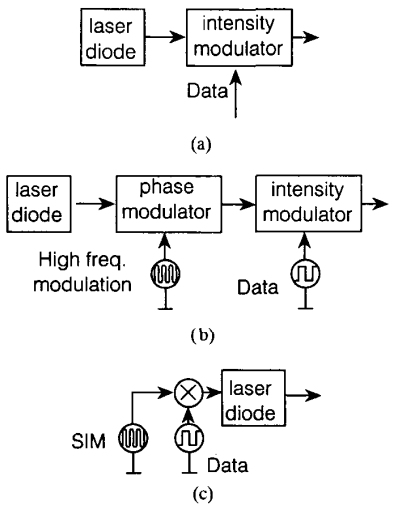

Fig. 2. High frequency modulation formats: a) no high frequency modulation; b) external phase modulation; c) direct modulation.

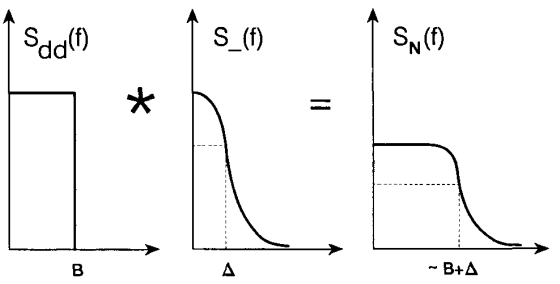

Fig. 3. Power density spectra $S_{dd}(f)$, $S_{-}(f)$, and $S_N(f)$.

where $d(t)$ and P_0 are the same as before, and the noise intensity at the fiber output is, according to (6)

$$\begin{aligned} i_N(z,t) &= 2\psi\,\mathrm{Re}\{E(t)E^*(t-\tau)\} \\ &= 2\psi P_0\sqrt{d(t)d(t-\tau)} \\ &\quad\cdot\cos\left[\Omega_0\tau + \Delta\varphi(t,\tau) + A\sin\left(\omega_0 t - \frac{\omega_0\tau}{2}\right)\right] \end{aligned} \tag{18}$$

where

$$A = -2a\sin\left(\frac{\omega_0\tau}{2}\right) \tag{19}$$

and $\Delta\varphi(t,\tau) = \varphi(t) - \varphi(t-\tau)$. The noise power spectral density is the Fourier transform of the noise autocorrelation function $R_N(t,\delta\tau)$. In this case it will be necessary to perform both time and statistical averaging to properly model the nonstationary conditions [9].

The autocorrelation function of the noise term $i_N(z,t)$ is given by

$$\begin{aligned} R_N(t,\delta\tau) &= \langle i_N(z,t)i_N(z,t+\delta\tau)\rangle \\ &= 2\psi^2P_0^2R_{dd}(\delta\tau)R_{-}(\delta\tau) \\ &\quad\cdot\cos\left[2A\sin\left(\frac{\omega_0\delta\tau}{2}\right)\right. \end{aligned}$$

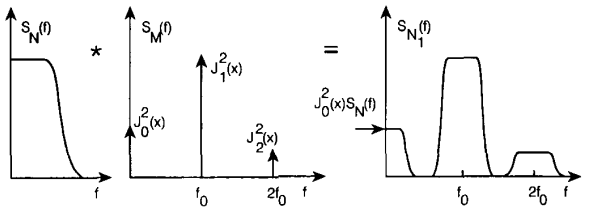

Fig. 4. Distribution of the noise power among the different harmonics. Notice how the noise power is transferred from baseband to the higher frequencies where it can be easily filtered.

$$\left.\cdot\cos\left(\omega_0 t - \frac{\omega_0\tau}{2} + \frac{\omega_0\delta\tau}{2}\right)\right]. \tag{20}$$

In deriving (20) the $R_{+}(\delta\tau)$ term is neglected as before. Also, there is a cross term $E\{i_S(t+\delta\tau)i_N(t)\}$ which can be shown to be proportional to $\cos(\Omega_0\tau)$. Using the same arguments as before we can neglect this term. Then, the desired expression for $R_{N_1}(\delta\tau)$, which is obtained by time averaging of $R_N(t,\delta\tau)$ can be written in the following form using Bessel functions expansion:

$$\begin{aligned} R_{N_1}(\delta\tau) &= \overline{R_N(t,\delta\tau)} \\ &= 2\psi^2P_0^2R_{dd}(\delta\tau)R_{-}(\delta\tau) \\ &\quad\cdot\left[J_0^2(A) + 2\sum_{n=1}^{\infty}J_n^2(A)\cos(n\omega_0\delta\tau)\right] \end{aligned} \tag{21}$$

The noise power spectral density is

$$\begin{aligned} S_{N_1}(f) &= J_0^2(A)S_N(f) \\ &\quad+\left\{\sum_{n=1}^{\infty}J_n^2(A)[S_N(f-nf_0)+S(f+nf_0)]\right\} \end{aligned} \tag{22}$$

where $S_N(f)$ is the noise spectrum without superimposed modulation given with (15). The expression for $S_{N_1}(f)$ is our primary result. It shows that the noise now is distributed among the different harmonics (Fig. 4). Note that in the absence of high-frequency phase modulation, $A = 0$ and the noise power spectral density will be $S_{N_1}(f) = S_N(f)$. With phase modulation, the noise becomes modulated at multiples of the superimposed frequency, with carrier intensities equal to $J_n^2(A)$.

The part of the noise spectrum of concern to us is that near baseband, which is given by $J_0^2(A)S_N(f)$. This expression is valid if the phase modulation frequency is higher than at least twice the bandwidth of the interferometric noise spectrum $S_N(f)$, which we assume to be band-limited. The baseband noise is thus reduced by a factor $J_0^2(A)$, which we define as the noise reduction factor (NRF). Fig. 5 shows a plot of NRF vs normalized modulation frequency $f_0\tau$. It is obvious that NRF is a periodic function of $f_0\tau$ and the maximum reduction depends on the phase modulation index a. It implies that the NRF is also function of the distance ($\tau = \text{const.} \times z$).

It is seen from Fig. 5 that for very strong phase modulation, the interferometric noise can by-and-large be eliminated except for the unfortunate situation when $f_0\tau = k$, where k is integer.

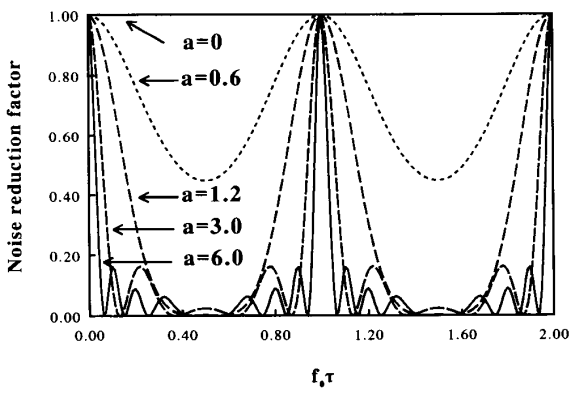

Fig. 5. Noise reduction factor versus $f_0\tau$ for external phase modulation. The phase modulation index a is parameter. The NRF is periodic with period 1.

This is somewhat undesirable since a transmitter incorporating a phase modulator at one frequency may work for a particular fiber link, but may not work for another one. We shall show in Section II-C that this situation can be remedied if the applied phase modulation consists NOT of a single high frequency tone but is rendered "noisy" instead, such that its spectrum centers at a high frequency and with a width much larger than $1/\tau$.

B. Directly Modulated Laser Diode

The effectiveness of a superimposed high frequency phase modulation in reducing interferometrically generated noise has been shown in the above section. However, the arrangement shown on Fig. 2(b) where external phase and intensity modulators are used is not very practical. In reality, it is far more preferable to apply the data and high frequency modulation directly to the laser, as shown in Fig. 2(c). Note that the data and the high frequency signal are multiplied together. In this section, we show that apart from slight quantitative differences, this scheme achieves a similar noise reduction effect as in the idealistic case considered previously. For the analysis, we use the fact that when directly modulating a laser diode, the phase and amplitude of the output lasing field are related by [10]

$$\dot{\varphi}_m = \frac{\alpha}{2\pi}\frac{1}{P(t)}\frac{\partial P(t)}{\partial t}. \tag{23}$$

We assume that the high frequency modulation current applied to the laser diode is sinusoidal, at frequency f_0. Using the result of a large signal analysis [11], the output intensity is given by

$$P_m(t) = \frac{P_0}{I_0(a)} e^{a\cos(\omega_0 t)} \tag{24}$$

where $I_0(a)$ is the modified Bessel function of zero order and a is a parameter describing the modulation depth parameter which depends both on the frequency and the modulation amplitude [11]. The phase modulation is given by (23) and (24) which is, assuming $\alpha = 2\pi$, $\varphi_m(t) = a\cos(\omega_0 t)$. The field at the laser output is

$$E(t) = \sqrt{P(t)}\, e^{ja\cos(\omega_0 t)} \tag{25}$$

where $P(t) = P_m(t)d(t)$ and we have assumed that the phase modulation due to the data input is negligible compared to that due to the high frequency superimposed modulation. This is justified by the fact that the latter is applied at a much higher frequency than the former and the phase modulation frequency is to first order proportional to the modulation frequency, as evident from the relationship in (23). The influence of laser phase noise was neglected. Using the previous results, we easily derive the expression for the signal and noise intensity at the output of the fiber

$$i_S(z,t) = d(t)P_m(t) + \psi^2 d(t-\tau)P_m(t-\tau) \approx d(t)P_m(t) \tag{26}$$

$$i_N(z,t) = 2\psi\sqrt{d(t)d(t-\tau)}\,\sqrt{P_m(t)P_m(t-\tau)} \cdot \cos\left[\Omega_0\tau + \Delta\varphi(t,\tau) + A\sin\left(\omega_0 t - \frac{\omega_0\tau}{2}\right)\right]. \tag{27}$$

The autocorrelation function of the signal is

$$R_S(\delta\tau) = R_d(\delta\tau)\overline{P_m(t)P_m(t+\delta\tau)}. \tag{28}$$

It can be shown, after some algebra, that the autocorrelation of the noise is:

$$\langle i_N(z,t)i_N(z,t+\delta\tau)\rangle = 2\psi^2 R_{dd}(\delta\tau) \cdot \overline{\sqrt{P_m(t)P_m(t-\tau)P_m(t+\delta\tau)P_m(t+\delta\tau-\tau)}} \cdot R_-(\delta\tau)\cos\left[2A\sin\left(\frac{\omega_0\delta\tau}{2}\right)\cos\left(\omega_0 t - \frac{\omega_0\tau}{2} + \frac{\omega_0\delta\tau}{2}\right)\right]. \tag{29}$$

The term involving $R_+(\delta\tau)$ was neglected as before. If we limit our discussion only to the small signal case, we can write:

$$\overline{\sqrt{P_m(t)P_m(t-\tau)P_m(t+\delta\tau)P_m(t+\delta\tau-\tau)}} = \frac{P_o^2}{I_0^2(a)} \cdot \left[I_0(B) + I_1(B)\cos\left(\omega_0 t - \frac{\omega_0\tau}{2} + \frac{\omega_0\delta\tau}{2}\right)\right] \tag{30}$$

where $B = 2a\cos\left(\frac{\omega_0\tau}{2}\right)\cos\left(\frac{\omega_0\delta\tau}{2}\right)$. After time averaging, we get for the autocorrelation function:

$$\begin{aligned} R_{N_2}(\delta\tau) &= \overline{\langle i_N(z,t)i_N(z,t+\delta\tau)\rangle} \\ &= 2\frac{(\psi P_0)^2}{I_0^2(a)} R_{dd}(\delta\tau)R_-(\delta\tau)I_0(B) \\ &\quad \cdot J_0\left[2A\sin\left(\frac{\omega_0\delta\tau}{2}\right)\right]. \end{aligned} \tag{31}$$

This function is periodic, with period $T = \frac{1}{f_0}$, where f_0 is the modulation frequency. Again, the noise spectrum has the same form, as in the case of external high frequency modulation. The only term that is of interest is (as in the

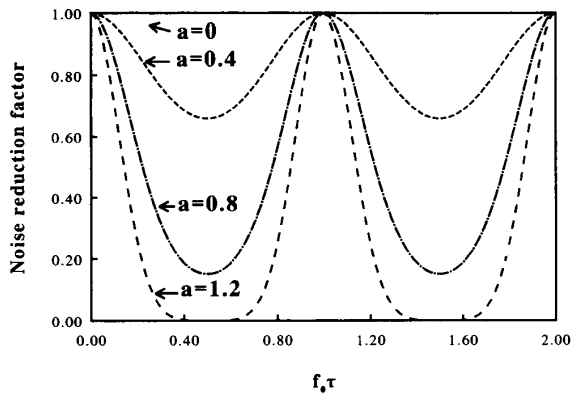

Fig. 6. Noise reduction factor versus $f_0\tau$ for directly modulated LD. a is parameter. The results are very close to those on Fig. 5.

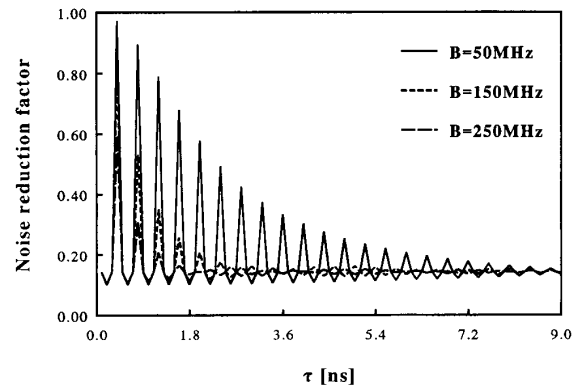

Fig. 7. Noise reduction factor versus τ for external phase modulation with Gaussian noise of various bandwidth B. The phase modulation index is $a^2 = 4$ and $f_0 = 1$ GHz

previous cases) the baseband term. If we use Fourier series expansion of Bessel functions [12], we get for the baseband term

$$R_{N0}(\delta\tau) = 2\frac{(\psi P_0)^2}{I_0^2(a)} R_{dd}(\delta\tau) R_-(\delta\tau) \cdot \left[I_0^2\left(a\cos\left(\frac{\omega_0\tau}{2}\right)\right) J_0^2\left(2a\sin\left(\frac{\omega_0\tau}{2}\right)\right) + 2I_1^2\left(a\cos\left(\frac{\omega_0\tau}{2}\right)\right) J_1^2\left(2a\sin\left(\frac{\omega_0\tau}{2}\right)\right)\right]. \tag{32}$$

Fig. 6 shows plot of the NRF versus normalized frequency $f_0\tau$. The results are very similar to the previous cases (Fig. 5).

C. Superimposed Modulation with Band-pass Gaussian Noise

We have seen in Section II-A and II-B that for a single tone high frequency phase modulation, the interferometric noise can be reduced except for the unfortunate situations when the modulation frequency and the round trip delay is related by $f_0\tau = k$, where k is integer. To achieve suppression of interferometric noise under all situations independent of τ, we consider broadening the single tone modulation into a noise band, centered at an arbitrary high frequency. This noise can be generated, for example, by ordinary diode.

To analyze the situation, consider the ideal case where phase modulator is applied externally to the laser output (Fig. 2(b)). Let the modulation applied to the phase modulator of Fig. 2(b) be bandpass noise $n(t)\cos(\omega_0 t)$. The electric field at the input of the fiber is

$$E(t) = \sqrt{d(t)P_0}\, e^{j\Omega_0 t} e^{j\varphi(t)} e^{jan(t)\cos(\omega_0 t)}. \tag{33}$$

Then, following the procedure described in Section II-A we get for the autocorrelation function of the noise R_{N_3}

$$\begin{aligned} R_{N_3}(\delta\tau) = {} & 2(\psi P_0)^2 R_{dd}(\delta\tau) R_-(\delta\tau) \\ & \cdot \exp\{-a^2[2R_n(0) - 2R_n(\delta\tau)\cos(\omega_0\delta\tau)]\} \\ & \cdot \exp\{-a^2[-2R_n(\tau)\cos(\omega_0\tau) + R_n(\tau+\delta\tau) \\ & \cdot \cos(\omega_0(\tau+\delta\tau)) + R_n(\tau-\delta\tau) \\ & \cdot \cos(\omega_0(\tau-\delta\tau))]\} \end{aligned} \tag{34}$$

where $R_n(\delta\tau)\cos(\omega_0\delta\tau)$ is the autocorrelation function of the noise at the input of the phase modulator. In principle, the noise power spectral density can then be computed from the autocorrelation function (Eq. 34). It consists of a large number of terms.

If we assume that τ is much larger than the width of the autocorrelation function $R_n(\delta\tau)$, we can simplify the calculation of the noise reduction factor in (34). The assumption is valid if, for example, $\tau \geq 50$ ns and at the same time the bandwidth of the noise is in the order of 100 MHz. This is applicable in a system with length of at least 10 m. In this case, in (34) all terms involving τ can be neglected since $R_n(\delta\tau)$ is very small for large τ and we get

$$\begin{aligned} R_{N_3}(\delta\tau) \approx {} & 2(\psi P_0)^2 R_{dd}(\delta\tau) R_-(\delta\tau) \\ & \cdot \exp\{-2a^2[R_n(0) - R_n(\delta\tau)\cos(\omega_0\delta\tau)]\}. \end{aligned} \tag{35}$$

A minimum value for the noise reduction factor was calculated by summing the power in the baseband contributed by the modulating noise (this is equivalent to the assumption of having a single DC-component, as in the case of single tone high-frequency modulation). Two different power spectral densities for the modulating noise were assumed: flat (band-limited) and Lorentzian, both with the same equivalent bandwidth. On Fig. 7 we show the noise reduction factor versus τ. It can be observed that the NRF decreases as the bandwidth of the modulating noise is increased. For large values of τ, the NRF becomes independent of the bandwidth.

The noise reduction factor versus a is shown on Fig. 8. Note that the spectral shape of the band-pass noise is insignificant for the overall interferometric noise reduction. Thus, for large τ, the noise reduction depends only on the total power of the modulating band-pass noise.

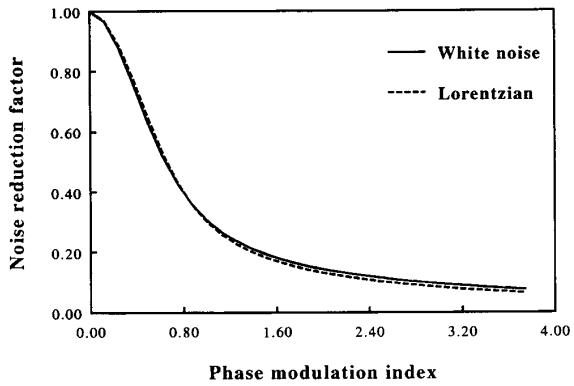

Fig. 8. Noise reduction factor with Gaussian noise superimposed modulation for two different power spectral densities (flat and Lorentzian), versus a, for large τ and $R_n(0) = 1$.

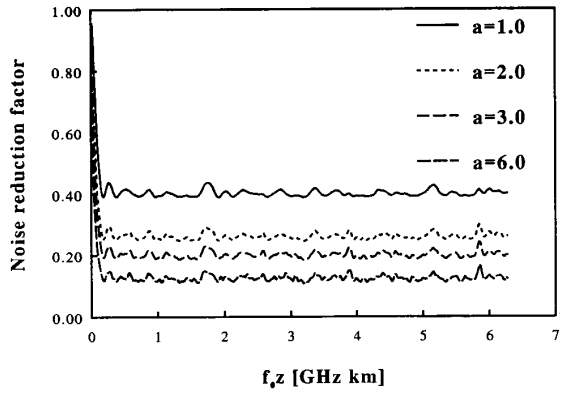

Fig. 9. Noise reduction factor versus $f_0 z$ (z is the fiber length) for direct phase modulation with single tone and multimode fiber exhibiting modal noise. The number of modes is assumed to be 50 which are equally excited. The large number of modes makes the NRF aperiodic, as in the case of modulation with band-pass Gaussian noise.

Thus, by picking a noise generator for driving the phase modulator, we can be assured of the elimination of the interferometric noises originating from any multiple reflections. With a noise bandwidth of several hundred megahertz, centered for example at 1 GHz, interferometric noises resulting from reflections from interfaces longer than 1 m can be practically eliminated (Fig. 7).

III. Multimode Fiber: The Modal Noise

In the previous sections we derived the noise reduction characteristics for the case of double reflection in a single-mode fiber. The result can be extended to multimode fibers which exhibit modal noise. Extensive analysis of the modal noise phenomenon and the steps to take to prevent it was done in [13].

The modal noise is the result of two causes: first, any mechanical distortion of the fiber due to vibrations, bending etc., will produce phase changes between fiber modes. Second, any source wavelength change will produce changes in the relative mode delays. We shall not consider long term changes due to wavelength drift of the laser, but instead consider short term wavelength fluctuations, arising from the finite linewidth of the laser. Here we will assume that the modal noise is caused exclusively by the laser wavelength fluctuations, although the extension to both wavelength fluctuations and mechanical distortions can be easily incorporated.

Let the electric field at distance z be

$$E_{\text{out}}(t) = \sum_{i=1}^{M} \psi_i E(t - t_i) \tag{36}$$

where t_i are the delay times for fiber modes. To simplify the analysis, we will assume that all fiber modes are equally (uniformly) excited, which represents, in the case of no superimposed modulation, the worst case as far as modal noise is concerned [14]. This approximation does not affect the results significantly. The exact excitation will produce the same results as with uniform excitation, but with smaller number of modes. Since the number of modes in a multimode fiber is usually very large, the effect of this approximation is negligible.

A high frequency superimposed modulation is applied directly to the laser as illustrated in Fig. 2(c). Then, the autocorrelation function becomes (for direct modulation of LD with a single laser mode):

$$\begin{aligned} R_{N0}(\delta\tau) = {} & \frac{2P_0^2}{I_0^2(a)} R_-(\delta\tau) \sum_{i=1}^{M}{}' \sum_{j=1}^{M}{}' (\psi_i \psi_j)^2 \\ & \cdot \left[I_0^2\left(a\cos\left(\frac{\omega_0\tau_{ij}}{2}\right)\right) J_0^2\left(2a\sin\left(\frac{\omega_0\tau_{ij}}{2}\right)\right) \right. \\ & \left. + 2I_1^2\left(a\cos\left(\frac{\omega_0\tau_{ij}}{2}\right) J_1^2\left(2a\sin\left(\frac{\omega_0\tau_{ij}}{2}\right)\right)\right) \right] \end{aligned} \tag{37}$$

where $\tau_{ij} = t_i - t_j$ is the relative mode delay between fiber modes and the prime denotes that the summation excludes the cases when $i = j$.

Fig. 9 shows simulation results using (37) for the NRF due to directly superimposed modulation. We notice that the increased number of fiber modes removed the periodicity that was observed in the case of double reflections in a single-mode fiber (Fig. 5). After the inital decay, NRF remains almost constant and independent of the frequency of the superimposed modulation. The reduction is dependent on the parameter a, which represents the modulation depth parameter.

IV. Conclusion

The analysis of this paper provides a theoretical framework for the suppression of interferometric noise by superimposing high frequency modulation. The fundamental mechanism for this reduction process is the redistribution of noise energy to high frequencies due to a superimposed phase modulation, as illustrated by the simple, but idealistic case of Fig. 2(b). The more practical situation in which the high frequency modulation is applied directly to the laser diode modulation can be interpreted as a manifestation of the ideal phase modulation scheme through frequency chirping of the lasing emission. Modulation with a single tone produces noise suppression for most situations except when the modulation frequency and the

inverse round-trip delay of the reflective interfaces are related by integer multiples. This situation is avoided if modulation is applied with multiple tones, or preferably a bandpass filtered white Gaussian noise source, as long as the bandwidth of the noise source is larger than the inverse of the shortest τ anticipated in the fiber link.

In the above analysis the fiber chromatic dispersion due to the chirping has not been considered. It will manifest itself as additional power penalty, however the noise floor due to the multiple reflections observed in [6] will be avoided. In any event, the chromatic dispersion is not a dominant effect in multimode or short fiber links.

It is interesting to note that the high frequency modulation occurs naturally for a self-pulsating laser diode [16]. Although the self-pulsating mechanism originates from an undamped relaxation oscillation process due to the presence of a saturable absorber in the laser (one but not the only means to generate self-pulsation), and is quite different from the case of an externally applied modulation, the characteristics of the laser emission is practically very similar for both cases. Self-pulsation can be regarded, from the point of view of its output characteristic, as a directly modulated nonself-pulsating laser with a near 100% modulation depth. The results obtained in this paper are thus applicable to those lasers as well.

On the contrary to single-mode fibers, for multimode fiber, the NRF does not show the periodic peaking even for a single tone phase modulation. This is attributed to the large number of modes, which introduces yet another degree of randomness with consequences similar to the case of multitone modulation.

In the simulation uniform excitation of fiber modes was assumed, which gives, according to the literature, the largest modal noise in the absence of superimposed modulation. When we apply the high frequency superimposed modulation, the uniform excitation becomes the best case, for the reasons explained earlier. If combined with other existing techniques, superimposed modulation can considerably reduce the level of phase (and modal) noise in optical communication systems using multimode fiber.

Acknowledgment

The authors thank R. Bates and J. Crow at IBM, and N. K. Shankaranarayanan at Columbia University for valuable discussions. While this work was in progress S. W. Wu and A. Yariv of Caltech have independently analyzed the effect of phase modulation on the interferometric noise due to Rayleigh backscattering. One of the authors (K. Lau) acknowledges useful discussions with A. Yariv on this subject, and would like to thank H. Blauvelt of Ortel Corporation for suggesting the use of direct modulation for reducing interferometric noise in single-mode fiber-optic systems.

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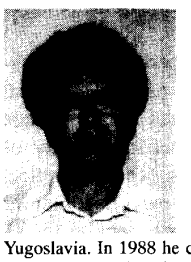

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