Retiming

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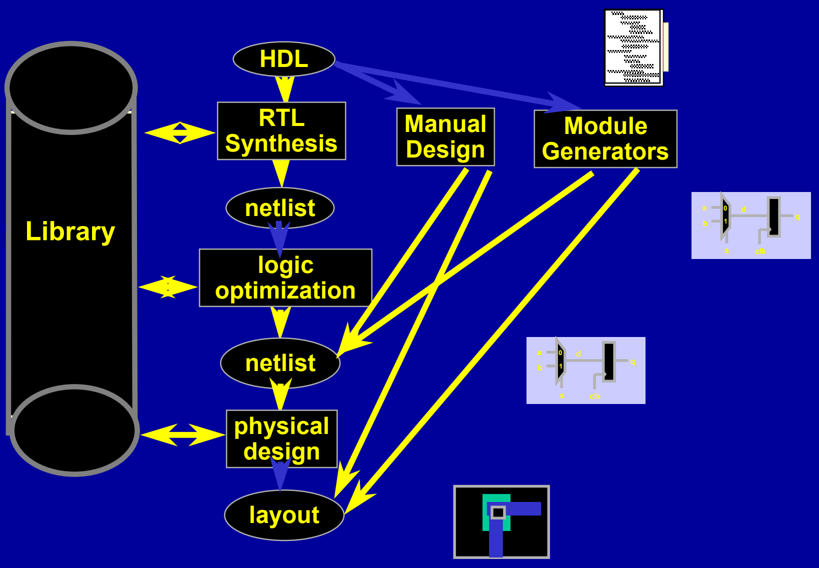
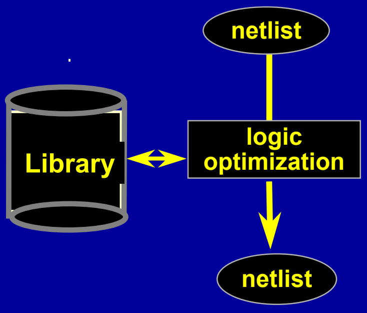
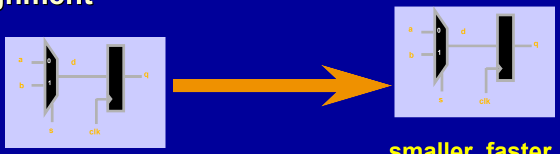
RTL Design Flow



Logic Optimization

- Perform a variety of transformations and optimizations
  - Combinational transformations
    - Technology independent
    - Technology dependent
  - Sequential transformations
    - FSM state assignment
    - Retiming





pre-optimized

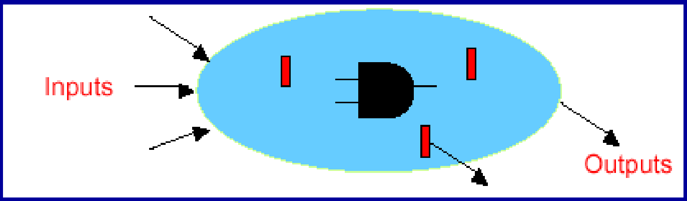
smaller, faster less power

Sequential Optimization

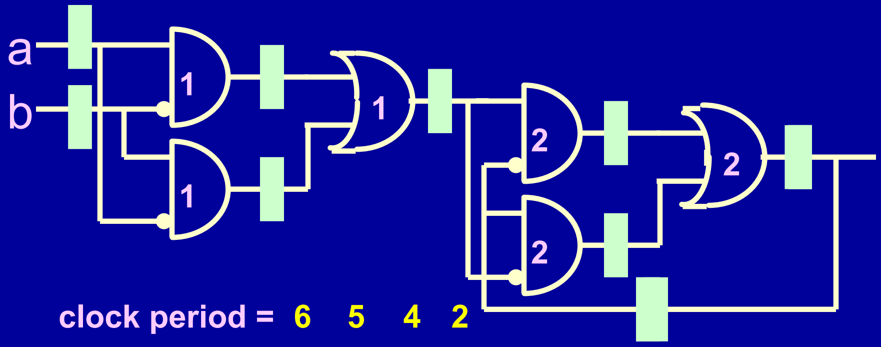
- Architectural Restructuring
- System-Level Optimizations
- Clock skew scheduling
  - balancing combinational circuit delay by adjusting clock schedule of individual registers
- Retiming
  - balancing of path delays by moving registers within circuit topology
  - interleaving with combinational optimization techniques

What About the Register Placement?

- Pure combinational optimization can be suboptimal since relations across register boundaries are disregarded
- Optimize a sequential circuit by optimally placing registers. Move register(s) so that
  - clock cycle decreases, or number of registers decreases and
  - input-output behavior is preserved
- Also, can combine retiming with combinational optimization techniques
  - Move latches out of the way temporarily
  - optimize larger blocks of combinational



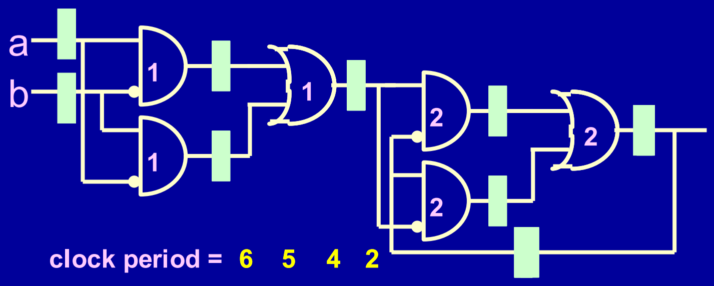
Retiming - tradeoffs



clock period =	6	5	4	2
# registers =	4	4	3	4

Retiming - Introduction

- Move registers
- Goals
  - clock period (min-period retiming)
  - number of registers (min-area retiming)
  - number of registers for a target clock period (constrained min-area retiming)



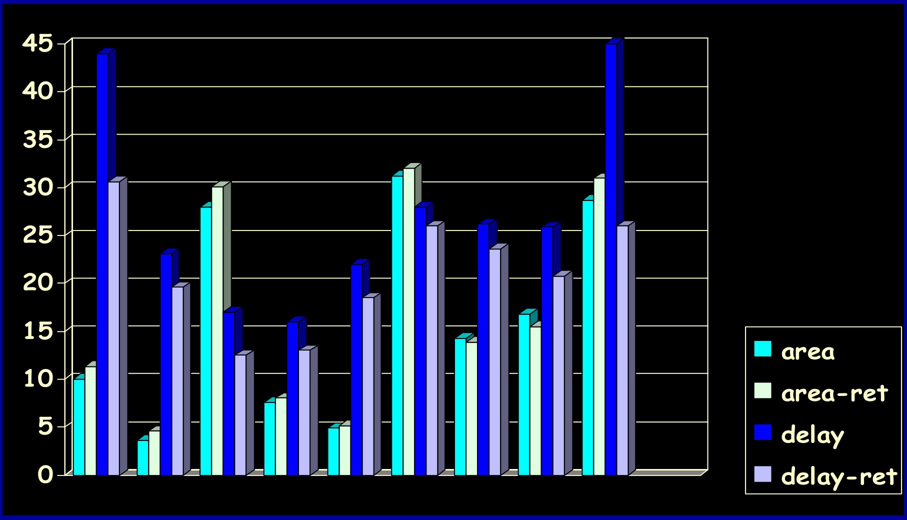
clock period = 6 5 4 2

# registers = 4 4 3 4

Importance of Retiming

- Practical sequential optimization
- Global optimality for clock period and register positioning
- Must for HDL synthesis
  - lowers dependency on user description
  - ease of specification
- Low power strategy
  - decrease #registers with no loss in performance

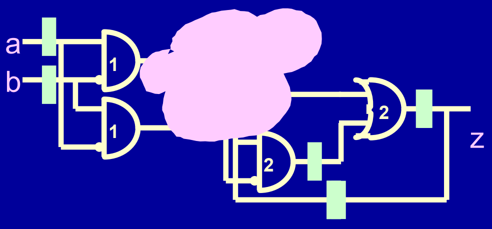
Practical Importance of Retiming

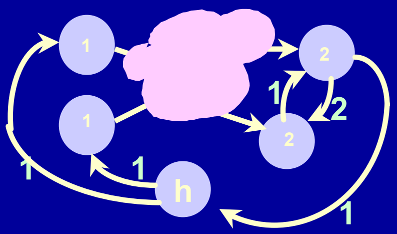


Retiming - Problem Definition

- Circuit ⟷ graph
  - gate ⟷ vertex
  - wire ⟷ edge
  - environment ⟷ host vertex and host edges

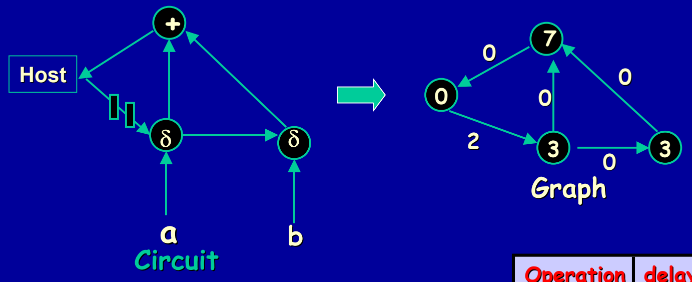
V = set of gates
E = set of edges
d(v) – delay of gate (vertex), $d(v) \geq 0$
w(e) – # of registers on edge e, $w(e) \geq 0$





Circuit Representation

Example: Correlator



Operation	delay
δ	3
+	7

δ(x, y) = 1 if x=y
0 otherwise

- Every cycle in Graph has at least one register i.e. no combinational loops.

Preliminaries

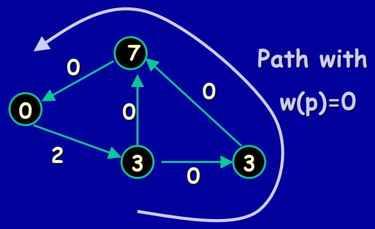
- For a path p: $V_0 \to$

$$d(p) = \sum_{i=0}^{k} d(v_i) \quad \text{(includes endpoints)}$$

$$w(p) = \sum_{i=0}^{k-1} w(e_i)$$

- Clock cycle

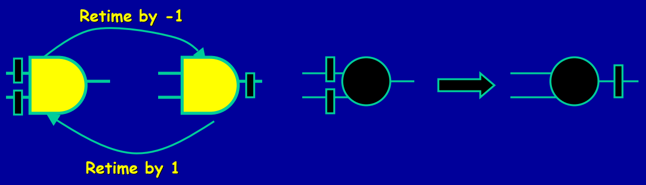
$$c = \max_{p:w(p)=0} \{d(p)\}$$



For correlator c = 13

Basic Operation

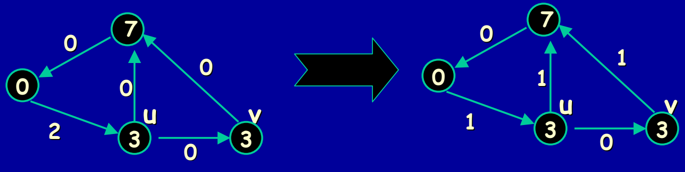
- Movement of registers from input to output of a gate or vice versa



- Does not affect gate functionalities
- A mathematical formulation: Retardation
  - r: $V \rightarrow Z$, an integer vertex labeling
  - $w_r(e) = w(e) + r(v) - r(u)$ for edge e= (u,v)

Basic Operation

- Thus in the example, r(u) = -1, r(v) = -1 results in



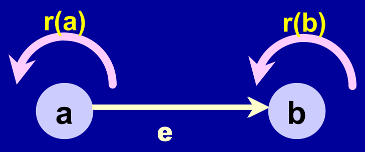
- For a path p: $s \rightarrow t$, $W_r(p) = w(p) + r(t) - r(s)$
- Retiming
  - r: $V \rightarrow Z$, an integer vertex labeling
  - $w_r(e) = w(e) + r(v) - r(u)$ for edge e= (u,v)
  - A retiming r is legal if $w_r(e) \geq 0, \forall e \in E$

Retiming - Assumptions

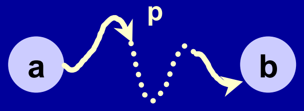
- Each loop in circuit contains at least one register
- Circuit uses single clock and edge-triggered elements (identical skew)
- Gate delay is constant (and non-negative)
- Registers are ideal (set-up, drive independent of load)
- Any power-up state of the design can be safely handled by the environment (initial state assumption)

Retiming - Formulation

- Assign integers to each vertex so that objective is met
- Valid retiming constraints



$w_r(e) = w(e) + r(b) - r(a) \geq 0$



$w_r(p) = w(p) + r(b) - r(a)$

Retiming for Minimum Clock Cycle

– **Problem Statement**: (Minimum cycle time)

– Given G(V, E, d, w), find a Legal retiming r so that

$$c = \max_{p:W_r(p)=0} \{d(p)\} \qquad \text{(A)}$$

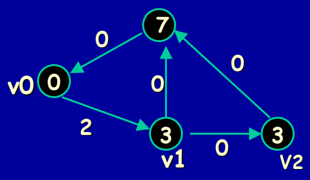
is minimized

– **Retiming: 2 important matrices**

- Register weight matrix
  $W(u,v) = \min\{w(p) : u \xrightarrow{p} v\}$
- Delay matrix
  $D(u,v) = \max\{d(p) : u \xrightarrow{p} v, w(p) = W(u,v)\}$

$$D(u,v) > c \Rightarrow W(u,v) \geq 1 \qquad \text{(B)}$$

Retiming for Minimum Clock Cycle



W – register path weight matrix, min # of registers on all paths between u and v
D – path delay matrix, max delay among all paths between u and v with W(u,v) registers

W

$$\begin{array}{c|cccc} & V0 & V1 & V2 & V3 \\ \hline V0 & 0 & 2 & 2 & 2 \\ V1 & 0 & 0 & 0 & 0 \\ V2 & 0 & 2 & 0 & 0 \\ V3 & 0 & 2 & 2 & 0 \end{array}$$

D

$$\begin{array}{cccc|c} V0 & V1 & V2 & V3 & \\ \hline 0 & 3 & 6 & 13 & V0 \\ 13 & 3 & 6 & 13 & V1 \\ 10 & 13 & 3 & 10 & V2 \\ 7 & 10 & 13 & 7 & V3 \end{array}$$

$C \leq \alpha \Leftrightarrow \forall p$, if $d(p) > \alpha$ then $w(p) \geq 1$
i.e. for the clock cycle to be less than α there must be a latch in the path

Conditions for Retiming

- Assume that we are asked to check if a retiming exists for a clock cycle α
- Legal retiming: $w_r(e) \geq 0$ for all e. Hence
  $w_r(e) = w(e) + r(v) - r(u) \geq 0$ or
  $r(u) - r(v) \leq w(e)$
- For all paths p: $u \rightarrow v$ such that $d(p) \geq \alpha$, we require $w_r(p) \geq 1$
  - Thus

$$\begin{aligned} 1 \leq w_r(p) &= \sum_{i=0}^{k-1} w_r(e_i) \\ &= \sum_{i=0}^{k-1} [w(e_i) + r(v_{i+1}) - r(v_i)] \\ &= w(p) + r(v_k) - r(v_0) \\ &= w(p) + r(v) - r(u) \end{aligned}$$

Or take the least w(p) (tightest constraint) $r(u)-r(v) \leq W(u,v)-1$

I.e. there are many paths *p*, choose the *p* that gives the tightest constraint

Note: this is independent of the path from u to v, so we just need to apply it to u, v such that $D(u,v) > \alpha$

Solving the Constraints

- All constraints in difference of 2 variable form
- How to solve?

Correlator: α = 7

W

	V0	V1	V2	V3
V0	0	2	2	2
V1	0	0	0	0
V2	0	2	0	0
V3	0	2	2	0

D

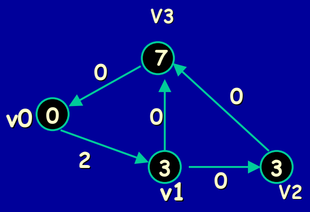
	V0	V1	V2	V3
V0	0	3	6	13
V1	13	3	6	13
V2	10	13	3	10
V3	7	10	13	7

Legal: $r(u)-r(v) \leq w(e)$

$$\boxed{\begin{aligned} r(v_0)-r(v_1) &\leq 2 \\ r(v_1)-r(v_2) &\leq 0 \\ r(v_1)-r(v_3) &\leq 0 \\ r(v_2)-r(v_3) &\leq 0 \\ r(v_3)-r(v_0) &\leq 0 \end{aligned}}$$

D>7: $r(u)-r(v) \leq W(u,v)-1$

$$\boxed{\begin{aligned} r(v_0)-r(v_3) &\leq 1 \\ r(v_1)-r(v_0) &\leq -1 \\ r(v_1)-r(v_3) &\leq -1 \\ r(v_2)-r(v_0) &\leq -1 \\ r(v_2)-r(v_1) &\leq 1 \\ r(v_2)-r(v_3) &\leq -1 \\ r(v_3)-r(v_1) &\leq 1 \\ r(v_3)-r(v_2) &\leq 1 \end{aligned}}$$



Solving the Constraints

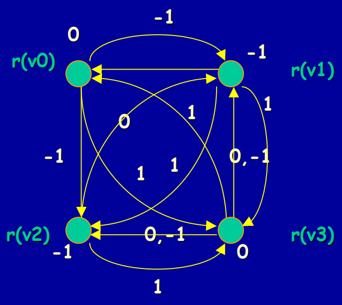
- Do shortest path on constraint graph
  - Bellman Ford Algorithm, O(|V|³)
- A solution exists if and only if there exists no negative weighted cycle.

Legal: r(u)-r(v)≤w(e)

$$r(v_0)-r(v_1)\le 2$$
$$r(v_1)-r(v_2)\le 0$$
$$r(v_1)-r(v_3)\le 0$$
$$r(v_2)-r(v_3)\le 0$$
$$r(v_3)-r(v_0)\le 0$$

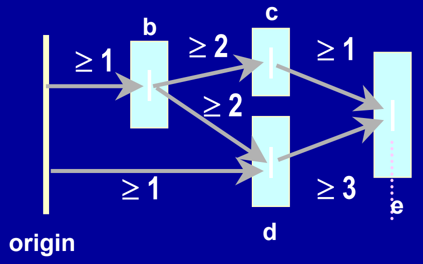
D>7:
r(u)-r(v)≤W(u,v)-1

$$r(v_0)-r(v_3)\le 1$$
$$r(v_1)-r(v_0)\le -1$$
$$r(v_1)-r(v_3)\le -1$$
$$r(v_2)-r(v_0)\le -1$$
$$r(v_2)-r(v_1)\le 1$$
$$r(v_2)-r(v_3)\le -1$$
$$r(v_3)-r(v_1)\le 1$$
$$r(v_3)-r(v_2)\le 1$$



A solution is $r(v_0) = r(v_3) = 0$, $r(v_1) = r(v_2) = -1$

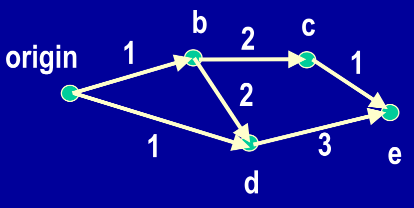
Representing Constraints



b ≥ origin + 1 c ≥ b + 2

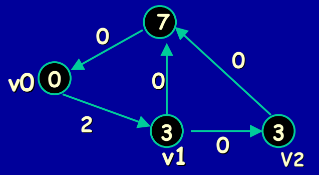
d ≥ origin + 1 d ≥ b + 2

e ≥ c + 1 e ≥ d + 3



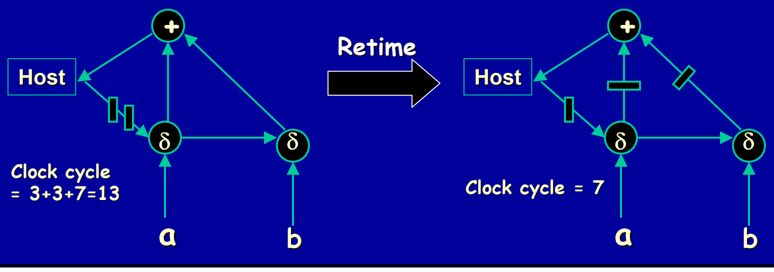
Retiming

To find the minimum cycle time, do a binary search among the entries of the D matrix 0(|V|³log|V|)



$$W = \begin{array}{c|cccc} & V0 & V1 & V2 & V3 \\ \hline V0 & 0 & 2 & 2 & 2 \\ V1 & 0 & 0 & 0 & 0 \\ V2 & 0 & 2 & 0 & 0 \\ V3 & 0 & 2 & 2 & 0 \end{array} \qquad D = \begin{array}{cccc|c} V0 & V1 & V2 & V3 & \\ \hline 0 & 3 & 6 & 13 & V0 \\ 13 & 3 & 6 & 13 & V1 \\ 10 & 13 & 3 & 10 & V2 \\ 7 & 10 & 13 & 7 & V3 \end{array}$$

Retimed correlator:

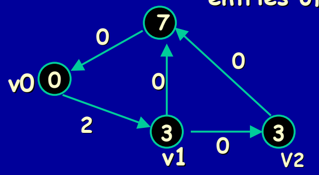


Clock cycle = 3+3+7=13

Clock cycle = 7

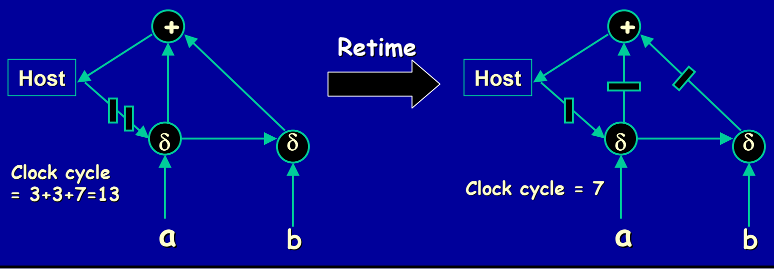
Retiming

To find the minimum cycle time, do a binary search among the entries of the D matrix 0(|V|³log|V|)



$$W = \begin{array}{c|cccc} & V0 & V1 & V2 & V3 \\ \hline V0 & 0 & 2 & 2 & 2 \\ V1 & 0 & 0 & 0 & 0 \\ V2 & 0 & 2 & 0 & 0 \\ V3 & 0 & 2 & 2 & 0 \end{array} \qquad D = \begin{array}{cccc|c} V0 & V1 & V2 & V3 & \\ \hline 0 & 3 & 6 & 13 & V0 \\ 13 & 3 & 6 & 13 & V1 \\ 10 & 13 & 3 & 10 & V2 \\ 7 & 10 & 13 & 7 & V3 \end{array}$$

Retimed correlator:



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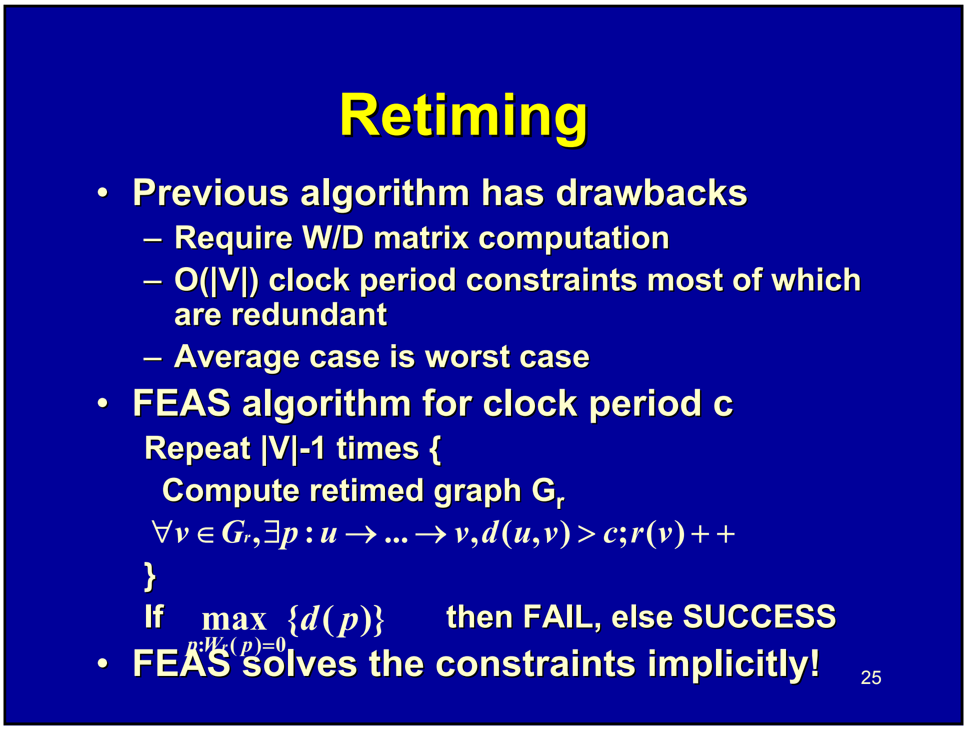
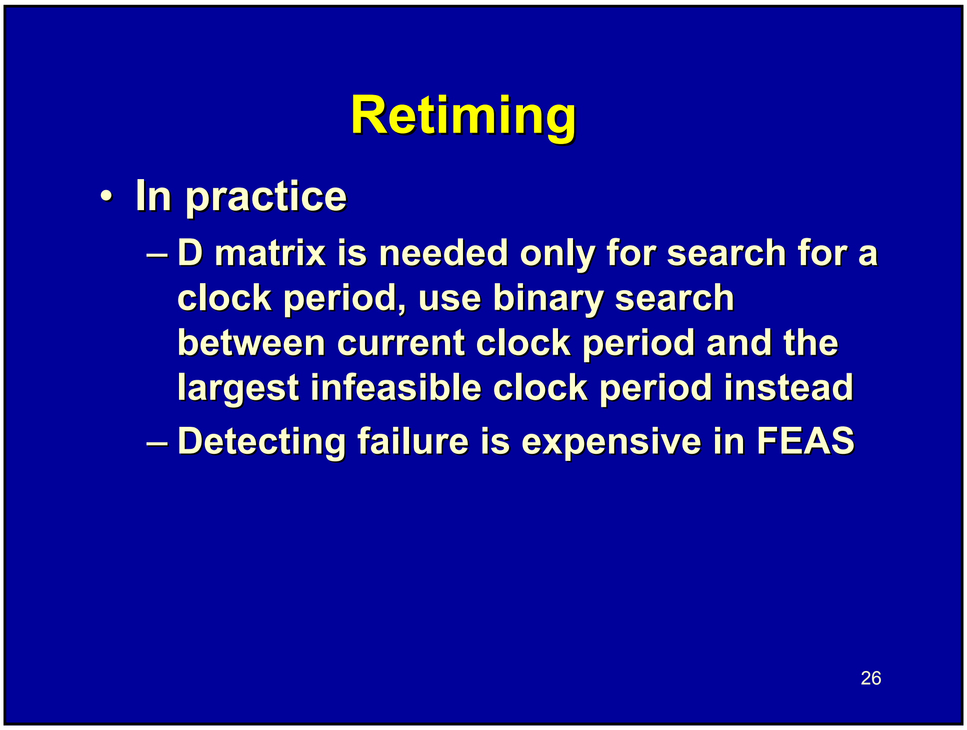


Retiming

- Previous algorithm has drawbacks
  - Require W/D matrix computation
  - O(|V|) clock period constraints most of which are redundant
  - Average case is worst case
- FEAS algorithm for clock period c

```
Repeat |V|-1 times {
  Compute retimed graph $G_r$
  $\forall v \in G_r, \exists p : u \to \dots \to v, d(u,v) > c; r(v)++$
}
If $\max_{p:W_r(p)=0} \{d(p)\}$ then FAIL, else SUCCESS
```

- FEAS solves the constraints implicitly!

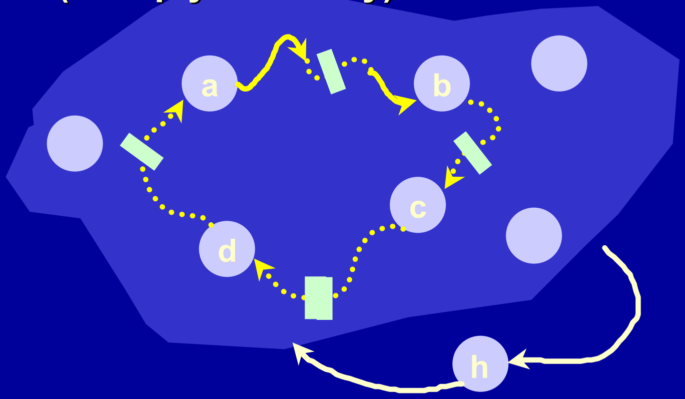


Retiming

- In practice
  - D matrix is needed only for search for a clock period, use binary search between current clock period and the largest infeasible clock period instead
  - Detecting failure is expensive in FEAS

Retiming - performance

- Predecessor heuristic - detect infeasibility (cheaply and early)



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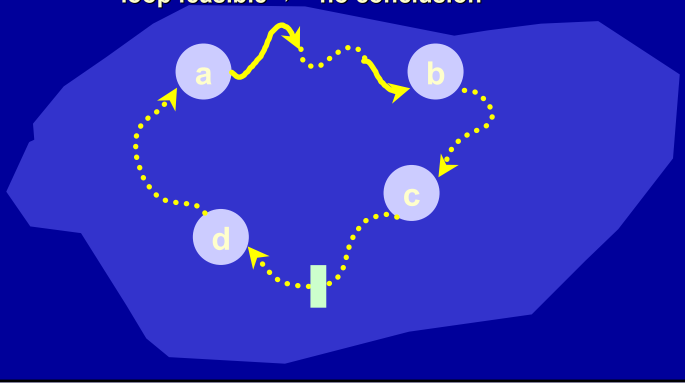
Retiming - performance

- Solve retiming for the loop
  - much smaller size than original graph
  - loop infeasible ⇨ no retiming at *c*
  - loop feasible ⇨ no conclusion



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Retiming For Minimum Area

Goal: minimize number of registers used

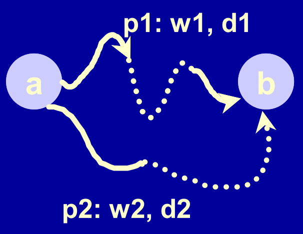
$$
\begin{aligned}
\min N_r &= \sum_{e\in E} w_r(e) \\
&= \sum_{e:u\to v} (w(e)+r(v)-r(u)) \\
&= \sum_{e\in E} w(e) + \sum_{e:u\to v} (r(v)-r(u)) \\
&= N + \sum_{u\to v} (r(v)-r(u)) \\
&= N + \sum_{v\in V} [r(v)(\#\, fanin(v) - \#\, fanout(v))] \\
&= N + \sum_{v\in V} a_v r(v)
\end{aligned}
$$

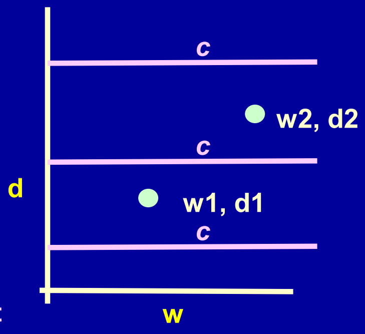
where a_v is a constant.

HW: Write dual of linear program

Retiming - performance

- Constrained min-area retiming
  - constraint generation





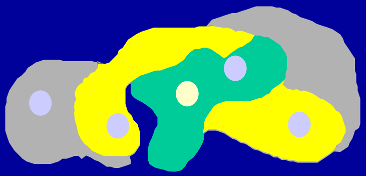
Effect: No constraint

Effect: No constraint

Effect: w_r (p1) $\geqslant 1$

Retiming - performance

- Mixed shortest path and longest path problem
  - Floyd-Warshall : too slow, too much memory
- Exploit sparsity of circuit graphs to explore gates within a *c* critical frontier

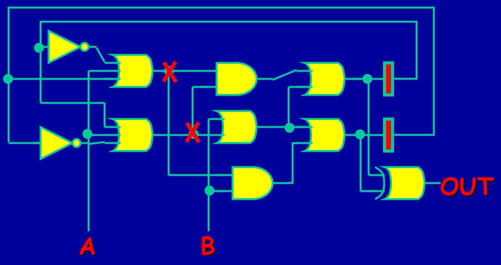


M : average #gates within *c* critical frontier
- average time : O(n M lg M)
- average memory: O(n)

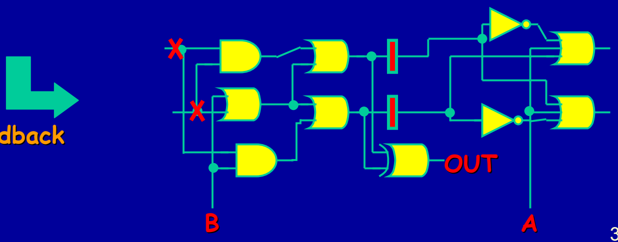
Retiming For Minimum Area

- In practice
  - We need W & D matrices to add clock period edges
    - Compute row of matrix at a time and avoid redundant edges
    - W is easy, D is a little harder
    - Take care to avoid adding redundant edges
  - Use minimum cost scaling to solve circulation problem
    - Numeric precision needs big integers!

Another Look at FSM Optimization

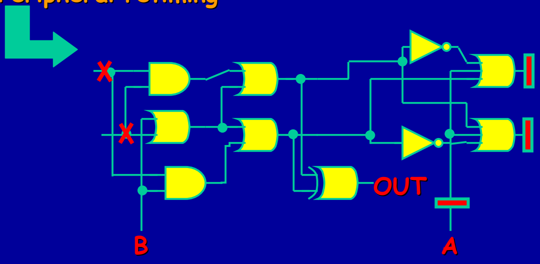


Break feedback

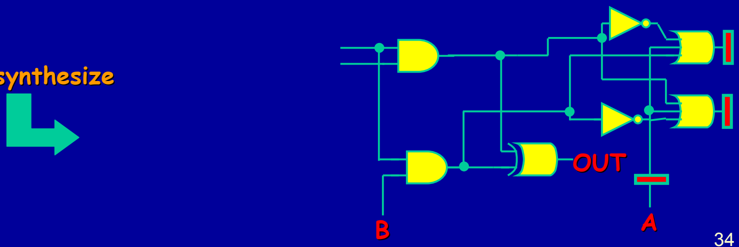


FSM Optimization

Peripheral retiming

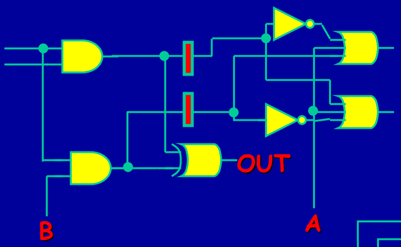


Resynthesize

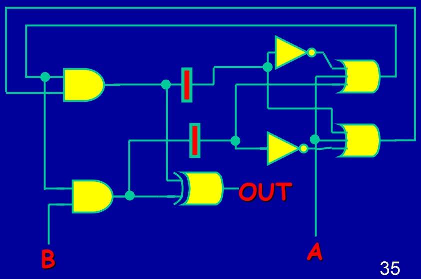


FSM Optimization

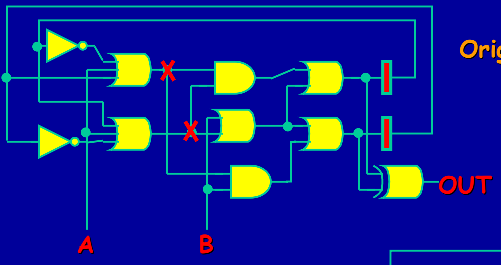
Retime



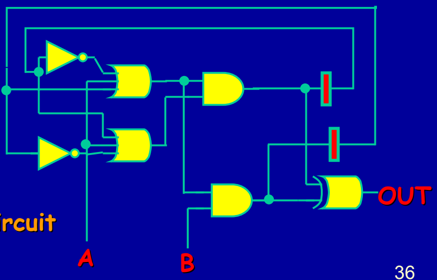
Reconnect



FSM Optimization



Original circuit



Resynthesized circuit

Retiming – initial states

- Circuits come in two flavors
  - Initial power-up then force set/reset lines
    - Retiming obeys delayed equivalence notion
  - Initial state loaded in
    - This is a problem