

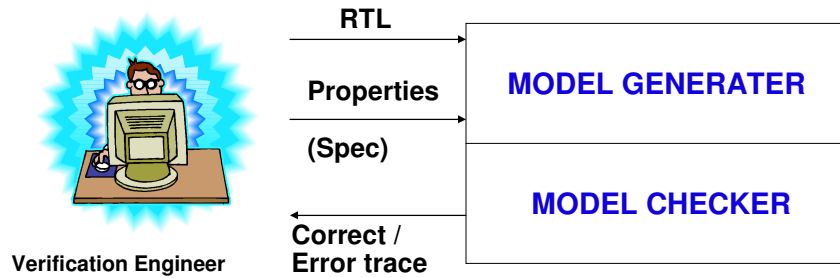
Model Checking

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(with thanks to Kenneth McMillan)

1

Formal Verification as practiced today



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2

Today's Lecture

What you know: How to formally specify properties using temporal logic

Today:

- Given a FSM description and a temporal logic property, how do we automatically check if that property holds?
 - Model checking
- Survey of some other formal verification topics
- What's next in verification?

Recap: Terminology and Temporal Logic

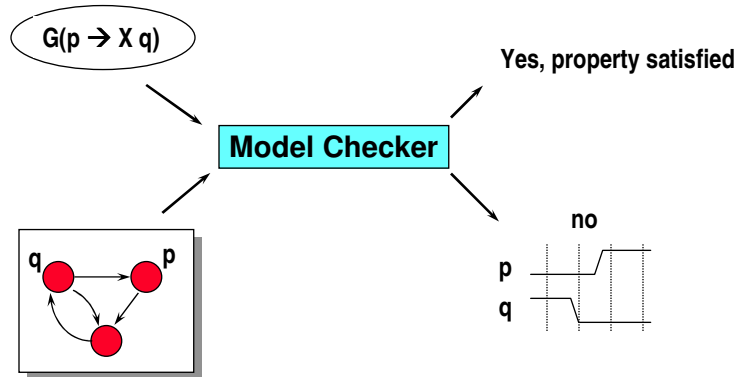
Behavior / Execution / Trace / Run / Path

A property corresponds to a “set of behaviors”

Operators to express properties over time:

- | | |
|----------|-------------------------------|
| G | “globally” |
| F | “eventually”/ “in the future” |
| X | “in the next state” |
| U | “until” |

Model Checking



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5

Brief History of Finite-State Model Checking

- 1977: Pnueli introduces use of (linear) temporal logic for program verification [1996 Turing Award]
- 1981: Model checking introduced by Clarke & Emerson and Quielle & Sifakis
 - But capacity limited by "state explosion"
- 1986: Bryant publishes paper on BDDs
- 1987: McMillan comes up with idea for "Symbolic Model Checking" (using BDDs)
 - First step towards tackling state explosion
- 1987-1999: Flurry of activity on model checking with BDDs, lots of progress using: abstraction, compositional reasoning, ...
 - More techniques to tackle state explosion
- 1999: Clarke et al. introduce "Bounded Model Checking" using SAT
 - Exploits advantages of SAT over BDDs
- 1999-date: More advances based on both BDDs and SAT, industrial use increases especially for corner-case and control logic debugging

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6

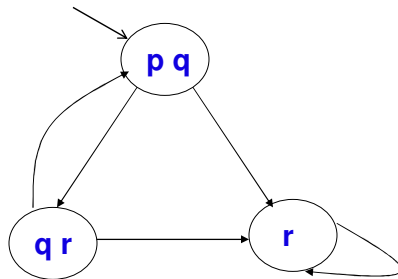
Outline

- Recap of Computation Tree Logic and why it is useful for designing verification algorithms
- Model Checking with BDDs
- Bounded Model Checking with SAT

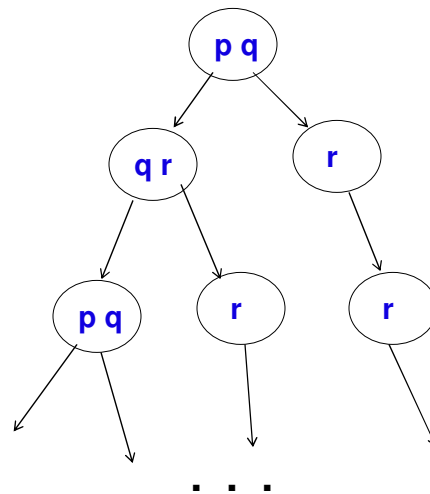
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Labelled State Transition Graph



“Kripke structure”



Infinite Computation Tree

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8

Temporal Logic

Linear Temporal Logic (LTL)

- Properties expressed over a single time-line

Computation Tree Logic (CTL, CTL*)

- Properties expressed over a tree of all possible executions
- CTL* gives more expressiveness than LTL
- CTL is a subset of CTL* that is easier to verify than arbitrary CTL*

Computation Tree Logic (CTL*)

Introduce two new operators called “Path quantifiers”

- **A p** : Property p holds along all computation paths
- **E p** : Property p holds along at least one path
- **Example:**
“From any state, it is possible to get to the reset state ”

A G (E F reset)

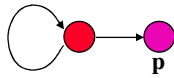
- **CTL:** Every F, G, X, U must be preceded by either an A or a E
 - E.g., Can't write A (FG p)
- LTL is just like having an “A” on the outside

Why CTL?

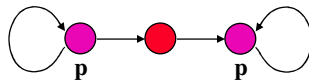
- Verifying LTL properties turns out to be computationally harder than CTL
- Exponential in the size of the LTL expression
 - linear for CTL
- For both, verification is linear in the size of the state graph

CTL as a way to approximate LTL

- $AG\ EF\ p$ is weaker than $GF\ p$ Good for finding bugs...



- $AF\ AG\ p$ is stronger than $FG\ p$



Good for verifying correctness...

CTL Model Checking

So, we've decided to do CTL model checking.

What are the algorithms?

Recap: Reachability Analysis

Given:

1. A Boolean formula corresponding to initial states R_0
2. δ

To find: All states reachable from R_0 in 1, 2, 3, ...
transitions (clock ticks)

Strategy: Denote set of states reachable from R_0 in k (or
less) clock ticks as R_k

$$R_{k+1}(s^+) = R_k(s^+) + \exists s \{ R_k(s) . \delta(s, s^+) \}$$

Backwards Reachability Analysis

Given:

1. A Boolean formula corresponding to error states E_0
2. δ

To find: All states that can reach E_0 in 1, 2, 3, ... transitions (clock ticks)

Strategy: Denote set of states reachable from E_0 in k (or less) clock ticks as E_k

$$E_{k+1}(s) = E_k(s) + \exists s^+ \{ E_k(s^+) \cdot \delta(s, s^+) \}$$

Verification of $G p$

Corresponding CTL formula is AGp

- Remember that p is a function of s
- **Forward Reachability Analysis:**
 - Check if any $R_k(s) \cdot p'(s)$ is true for any s
- **Backward Reachability Analysis:**
 - Set $E_0 = p'$
 - Check if $E_k(s) \cdot R_0(s)$ is true for any s

Model Checking Arbitrary CTL

Need only consider the following types of CTL properties:

- $E X p$
- $E G p$
- $E (p U q)$

Why? \leftarrow all others are expressible using above

- $A G p = ?$
- $A G (p \rightarrow (A F q)) = ?$

Model Checking CTL Properties

We define a general recursive procedure called “Check” to do this

Definition of Check:

- Input: A CTL property Π (and implicitly, δ)
 - Output: A Boolean formula B representing the set of states satisfying Π
- If $B(s) \cdot R_0(s) \neq 0$, then Π is true (in the initial state)

The “Check” procedure

Cases:

- If Π is a Boolean formula, then $\text{Check}(\Pi) = \Pi$
- Else:
 - $\Pi = \text{EX } p$, then $\text{Check}(\Pi) = \text{CheckEX}(\text{Check}(p))$
 - $\Pi = \text{E}(p \text{ U } q)$, then
 $\text{Check}(\Pi) = \text{CheckEU}(\text{Check}(p), \text{Check}(q))$
 - $\Pi = \text{E G } p$, then $\text{Check}(\Pi) = \text{CheckEG}(\text{Check}(p))$
- Note: What are the arguments to CheckEX, CheckEU, CheckEG? CTL properties or Boolean formulas?

CheckEX

CheckEX(p) returns a set of states such that p is true in their next states

How to write this?

CheckEU

CheckEU(p, q) returns a set of states, each of which is such that

- Either q is true in that state
- Or p is true in that state and you can get from it to a state in which p U q is true

Seems like circular reasoning!

But it works out: using a recursive computation like in reachability analysis

- We compute a series of approximations leading to the right answer

CheckEU

CheckEU(p, q) returns a set of states, each of which is such that

- Either q is true in that state
- Or p is true in that state and you can get from it to a state in which p U q is true

Let Z_0 be our initial approximation to the answer to CheckEU(p, q)

$$Z_k(s) = \{ q(s) + [p(s) \cdot \exists s^+ \{ \delta(s, s^+) \cdot Z_{k-1}(s^+) \}] \}$$

What's a good choice for Z_0 ? Why will this terminate?

Summary

EGp computed similarly

Definition of Check:

- Input: A CTL property Π (and implicitly, δ)
- Output: A Boolean formula B representing the set of states satisfying Π

All Boolean formulas represented “symbolically” as BDDs

- “Symbolic Model Checking”

Bounded Model Checking [Biere, Clarke, Cimatti, Zhu99]

Given

- A finite state machine M (“transition system”)
- A property p

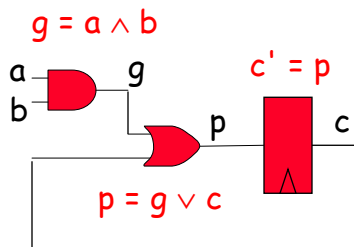
Determine

- Does M allow a counterexample to p of k transitions or fewer?

This problem can be translated to a SAT problem

Models

Transition system described by a set of constraints



Model:

$$C = \{ \begin{array}{l} g = a \wedge b, \\ p = g \vee c, \\ c' = p \end{array} \}$$

Each circuit element is a constraint
note: $a = a_t$ and $a' = a_{t+1}$

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25

Properties

We restrict our attention to safety properties.

Characterized by:

- Initial condition R_0
- Final condition E (representing "error" states)

A counterexample is a path from a state satisfying R_0 to state satisfying E , where every transition satisfies C .

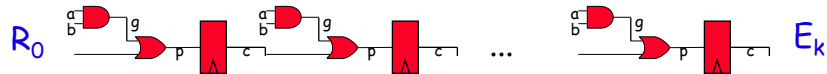
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26

Unfolding

Unfold the model k times:

$$U_k = C_0 \wedge C_1 \wedge \dots \wedge C_{k-1}$$



- Use SAT solver to check satisfiability of
$$R_0 \wedge U_k \wedge E_k$$
- A satisfying assignment is a counterexample of k steps

BMC applications

Debugging:

- Can find counterexamples using a SAT solver

Proving properties:

- Only possible if a bound on the length of the shortest counterexample is known.
 - I.e., we need a *diameter* bound. The diameter is the maximum length of the shortest path between any two states.
- Worst case is exponential. Obtaining better bounds is sometimes possible, but generally intractable.

New Developments in SAT-based MC

SAT-based bounded model checking has scaled to thousands of state bits and is very useful for debugging

- Can verify LTL properties too

Unbounded model checking is now also possible with SAT

But on some problems, BDD-based model checking is still better

Some Other Formal Verification Topics

Scaling up Model Checking

- **Abstraction: Keep only the relevant state variables**
- **Compositional Reasoning: Break a system up into modules, prove the property for the modules, combine the proofs**
- ...

Model Generation

- **Counterexample-guided Abstraction-Refinement**
- **Machine learning (especially for Environment model)**

Theorem proving is also used, sometimes combined with Model Checking

Some References for Further Study

- Model Checking, E. Clarke, O. Grumberg, D. Peled, MIT Press, 2000.
- Verification Tools for Finite-State Concurrent Systems, Clarke, Grumberg, Long (in prelim reading list)
- A. Biere, A. Cimatti, E. Clarke, O. Strichman, Y. Zhu. Bounded Model Checking. In *Advances in Computers*, vol. 58, Academic Press 2003.

Formal Verification in Industry

Some commercial tools in EDA: Synopsys Magellan, 0-In FV, Jasper JasperGold, Real Intent Verix, IBM RuleBase, ...

Theorem proving also used: e.g., Intel's Forte system, ACL2 prover at AMD

Software: Microsoft Static Driver Verifier (SDV), VeriSoft (Bell Labs), SPIN (Bell Labs, now NASA/JPL), ...

Industry view: Useful, but not the only tool

What's next in Verification?

- **Non-Boolean (infinite-state) Model Checking**
 - Software (why aren't FSMs enough to express these?)
 - Real-time systems
 - Hybrid systems
 - Verifying data-dependent properties
- **Computer Security**
- **Run-time Verification & Robustness**

Computer Security

How is verifying security different from other forms of verification?

- What's different about the properties?
- What's different about the system model?

An Example of a Security Problem

Assume cryptography works perfectly, can't be broken



It can still be possible to get unauthorized access to information!

- Encryption must be used carefully!

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35

Example: IBM 4758 Secure Co-processor

Used widely in the banking industry

Software Interaction



Common Cryptographic Architecture (CCA) Interface



MK is in-built, secret, unique to each chip

Key	Control Vector
K	$CV_K = \text{Read, Write}$
...	

Picture courtesy IBM

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36

The Problem

[discovered by M. Bond, et al. at Cambridge,UK]

Using perfectly legal CCA commands, it is possible to generate a control vector to do operations one is not allowed to do

- E.g., read and write account information

Has to be an “inside job” at one of the bank branches

Can be discovered by a form of Bounded Model Checking [Ganapathy et al., ICSE'05]

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37

The Vision



Design Engineer

Specification / Feedback



Correct-by-Construction
Design Compiler

VERIFIER

Robust Implementation

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38