Discussion: Memory & Computation Considerations

**Methods: Lifting Formulation [1]**

\[ y[i] = w * x = \sum_{i=0}^{N-1} w[i] x[i] \]

Let \( w = Bh \), where \( B \in \mathbb{R}^{N \times N} \) and \( x = Cm \), where \( C \in \mathbb{R}^{N \times K} \).

\[ y = \sum_{i=0}^{N-1} (C m_i) * w = \sum_{i=0}^{N-1} m_i (c_i) w \]

\[ = \text{circ}(c_1) B \text{ circ}(c_2) B \ldots \text{circ}(c_L) B \]

Let \( A = [\text{circ}(c_1) B \text{ circ}(c_2) B \ldots \text{circ}(c_L) B] \) and let \( X = hm^T \) which is a rank-1 matrix and a reordering of all the elements-wise multiples of \( m \) and \( h \).

\[ y = A(X) \]

**Methods: Ways to solve the problem**

- Non-Convex Formulations
  - Problem Formulation
  - Optimization Methods
  - Extended Approaches

- Semidefinite Program
  - Problem Formulation
  - Optimization Methods
  - Extended Approaches

- Nuclear Norm Minimization
  - Problem Formulation
  - Optimization Methods
  - Extended Approaches

**Theoretic Guarantees**

- Alternating Methods [3]
  - If \( A \) satisfies RIP conditions then the alternating minimization procedure has geometric convergence, where each iteration involves solving two least-square problems.
  - Jain et al. also outlines method for initialization.

- SDP and Nuclear Norm Minimization [2]
  - Recht et al. shows equivalence between solving nuclear norm minimization problems and the previously written SDP.
  - In Recht et al., it is shown that if \( A \) fulfills a RIP condition then \( X \) can be recovered exactly solving a nuclear norm minimization problem.

- Burer-Monteiro Heuristic [2,8,9]
  - Largely experimental results, but authors conclude that method is not strongly affected by inherent non-convexity.

- Discussion of when local minima provide global minima of SDP formulation.

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- For applications in imaging, high pixel or voxel count could make \( N \) and \( K \) very large and I would suggest using the original non-convex method solved by either alternating method or method of multipliers as suggested by Burer et al. [7] (also the recommendation for large scale low rank matrix completion).

- In addition, using the non-convex formulation or the Burer-Monteiro heuristic implicitly enforce that your solution is rank-1.

- Useful for enforcing additional constraints (i.e. sparsity on \( u \) or \( v \)).

- For large \( N \) and \( K \), SDP and nuclear norm minimization forms are memory inefficient as they must store a \( N \) by \( K \) matrix.

- Nuclear norm minimization is computationally limited because computing an SVD for large matrices is expensive.

Conclusion

- References
  5. Sturm et al. Using SeDuMi 1.02, a MATLAB Algorithm for solving Semidefinite Programs (1999)