Scaling Laws for Cooperative Node Localization in NLOS Wireless Networks

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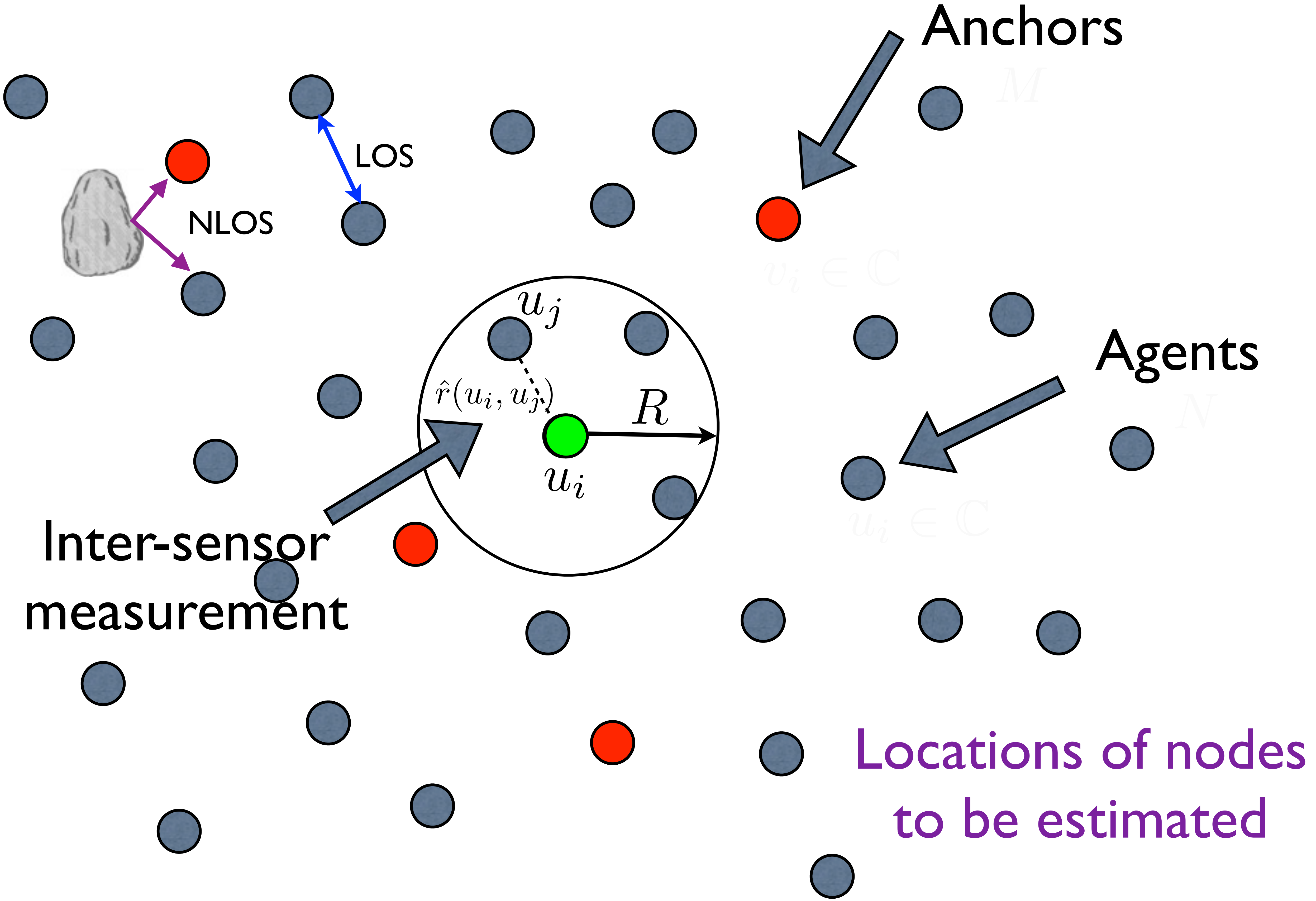
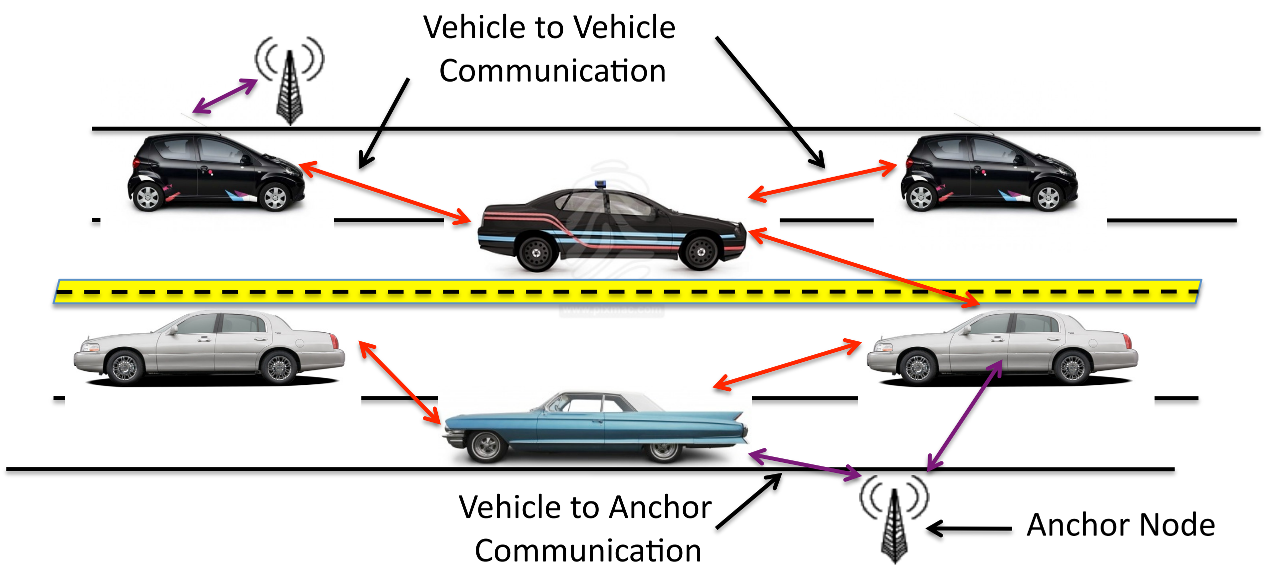
Problem Setup

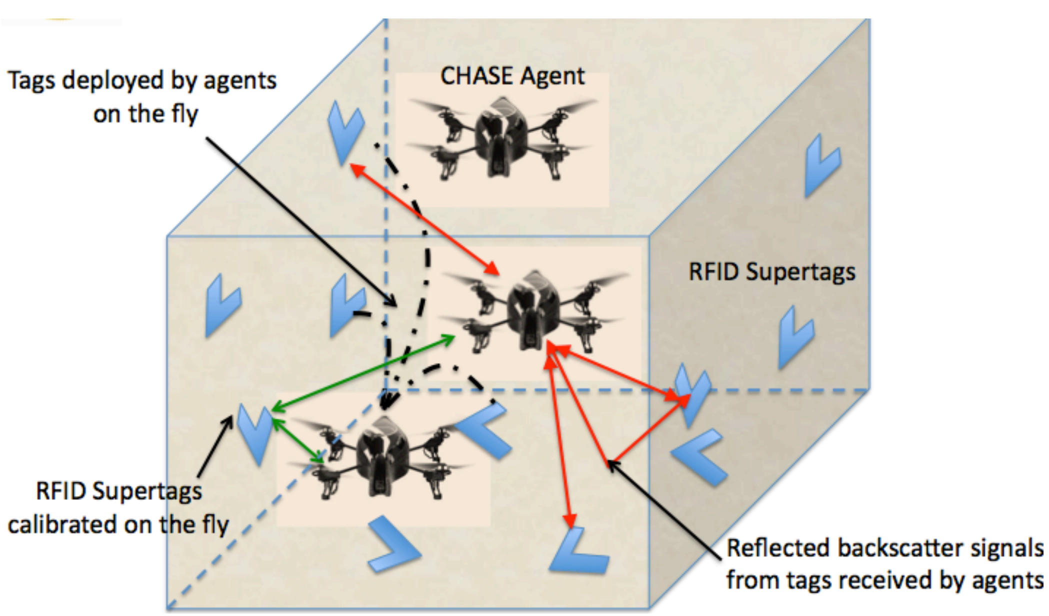


Applications

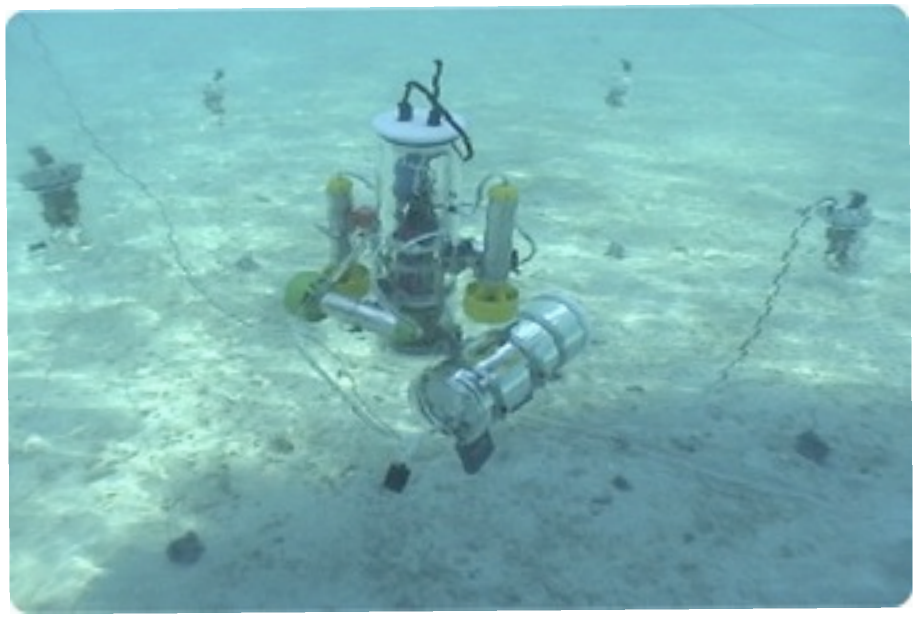
Sensor Networks



Intelligent Transportation Systems



Robotics



Underwater Sensor Networks

Measurement Model

$$\hat{r}(u_i, u_j) = r(u_i, u_j) + n_{ij}$$

Signal

$||u_i - u_j||$ Time of Arrival

$\frac{1}{||u_i - u_j||^\gamma}$ Received Signal Strength (RSSI)

$\tan^{-1} \frac{Im(u_i - u_j)}{Re(u_i - u_j)}$ Angle of Arrival

Noise

$$\alpha p_{LOS}(.) + (1 - \alpha) p_{NLOS}(.)$$

α Fraction of LOS signals

$p_{LOS}(.)$ Gaussian, Uniform etc

$p_{NLOS}(.)$ Exponential, Uniform etc

Problem

M - Anchors, N- Agents, α LOS Fraction

$$\mathbb{E}\left[\sum_{i=1}^{N} ||u_i - \hat{u}_i||^2 \;\middle|\; \underline{\hat{r}}\right] = f(M, N, \alpha)$$

$$f(M, N, \alpha) = ?$$

How does the error behave as a function of the number of nodes and fraction of LOS readings?

Analyze the *Cramer Rao Lower Bound*

Cramer Rao Lower Bound

CRLB - lower bound - best possible unbiased estimator

$\underline{\mathbf{u}}$ Vector of node locations

$\hat{\underline{\mathbf{u}}}$ Vector of estimated node locations

$\hat{\underline{r}}$ Vector of pairwise measurements

Cramer Rao theorem states

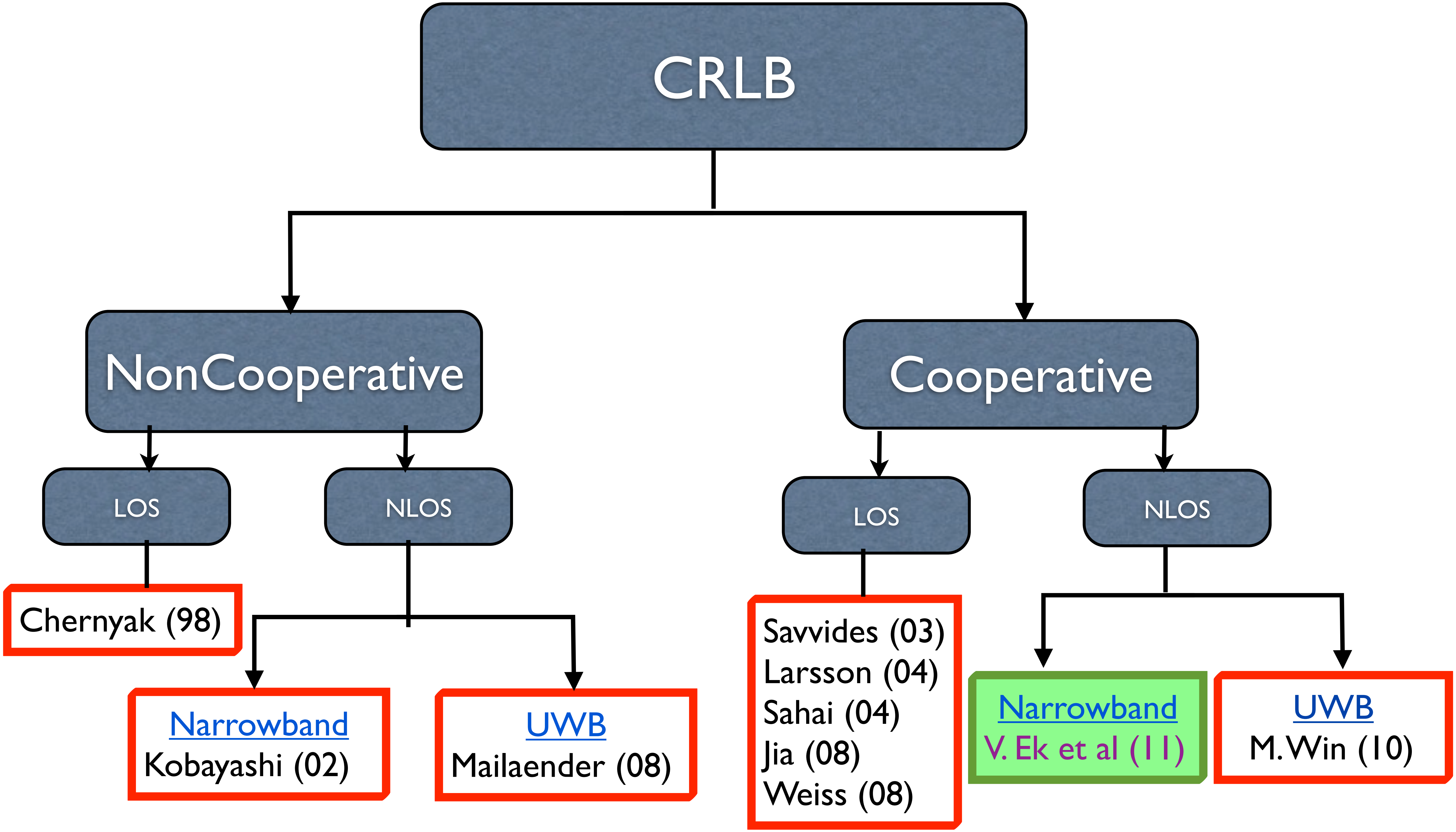
Design guidelines on number of anchors to be deployed etc..

$$\mathbb{E}[(\underline{\mathbf{u}} - \hat{\underline{\mathbf{u}}})(\underline{\mathbf{u}} - \hat{\underline{\mathbf{u}}})*] \succeq \mathbf{F}^{-1}$$

$\mathbf{F}$ Fisher Information Matrix

$$F_{ij} \triangleq \mathbb{E}\left\{\frac{\partial p(\hat{\underline{r}}|\underline{u})}{\partial u_i}\frac{\partial p(\hat{\underline{r}}|\underline{u})}{\partial u_j}\right\}$$

Literature



CRLB
NonCooperative
Cooperative
LOS
NLOS
LOS
NLOS
Chernyak (98)
Narrowband
Kobayashi (02)
UWB
Mailaender (08)
Savvides (03)
Larsson (04)
Sahai (04)
Jia (08)
Weiss (08)
Narrowband
V. Ek et al (11)
UWB
M.Win (10)

Prior work and our results

Prior work

- Focus mostly on LOS case. For NLOS, assume prior knowledge of which measurements are NLOS.
- CRLB expression dependent on node locations - not good for design guidelines.

Our work

- Analysis of the behavior as a function of the fraction of NLOS readings - no prior knowledge assumed.
- CRLB expression independent of node locations - good for design guidelines.

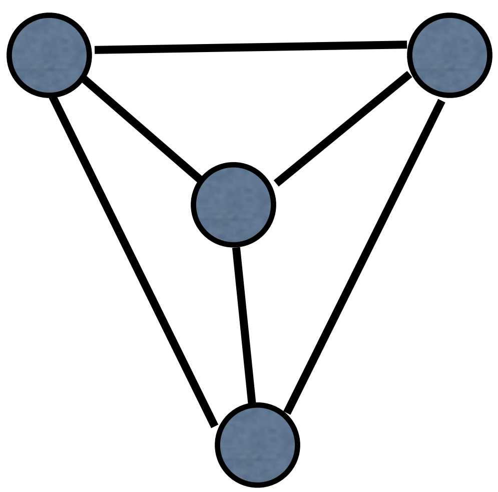
Results

Localization errors

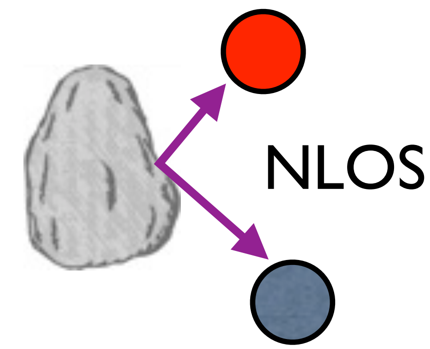
Network geometry

Bad

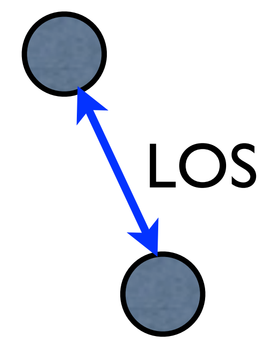
Good



Noise



Separation principle?

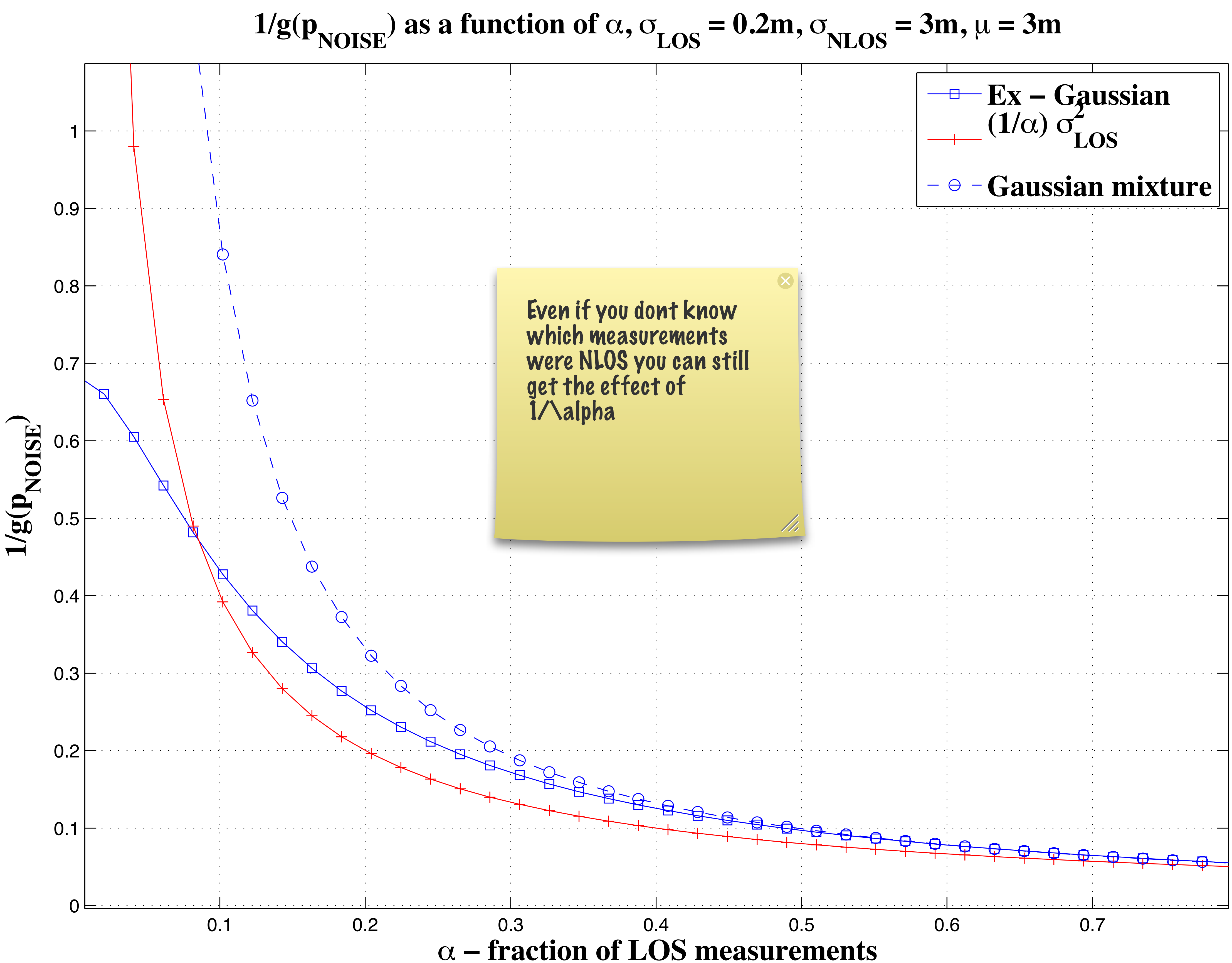


Theorem:

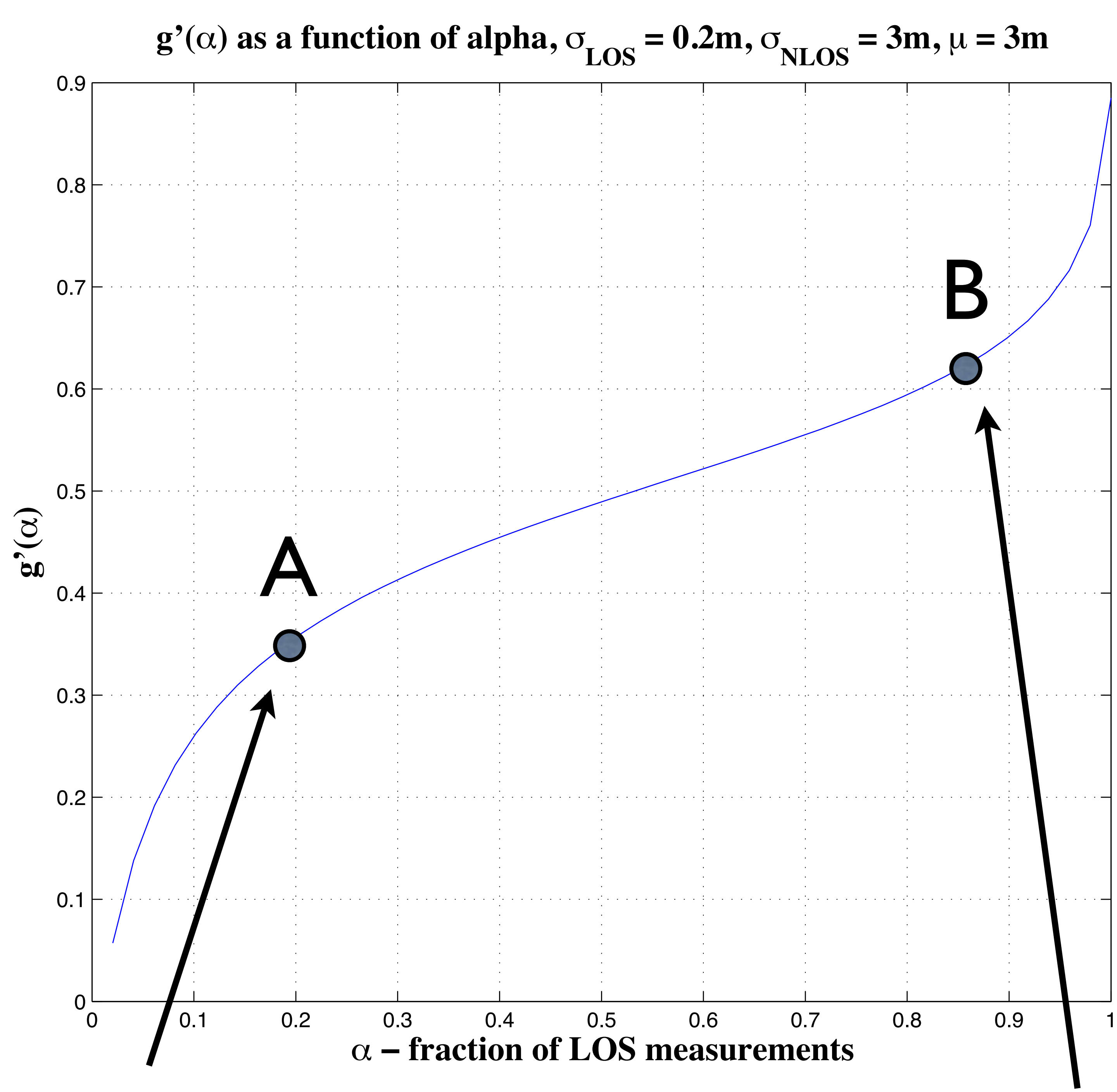
Fisher Information Matrix F can be written as

$$F = g(p_{NOISE})F_G$$

Effect of noise



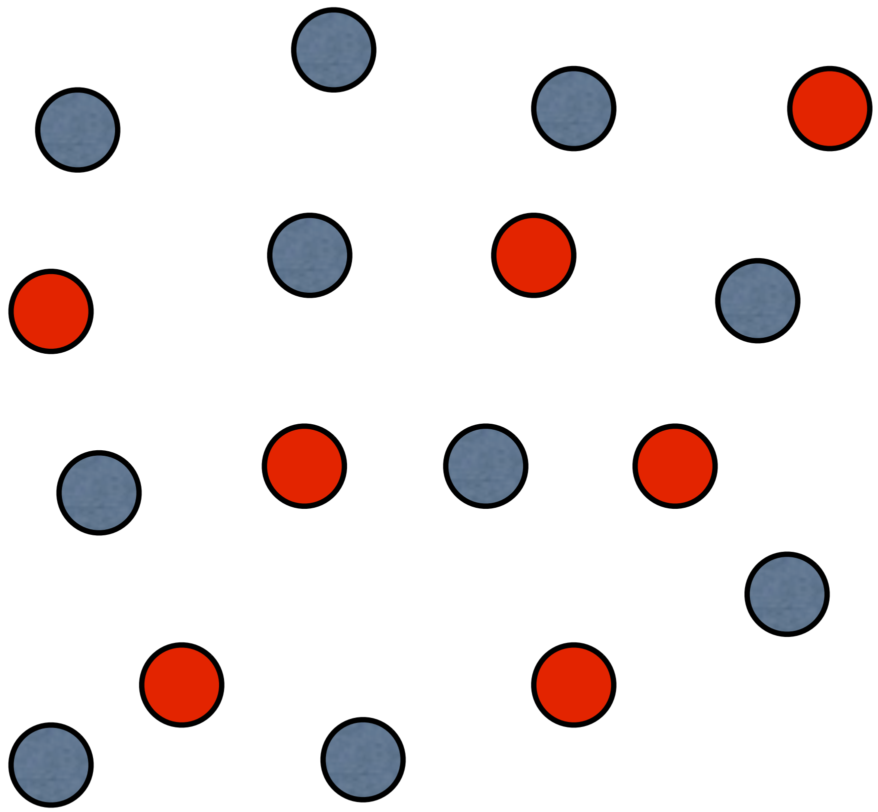
Effect of noise

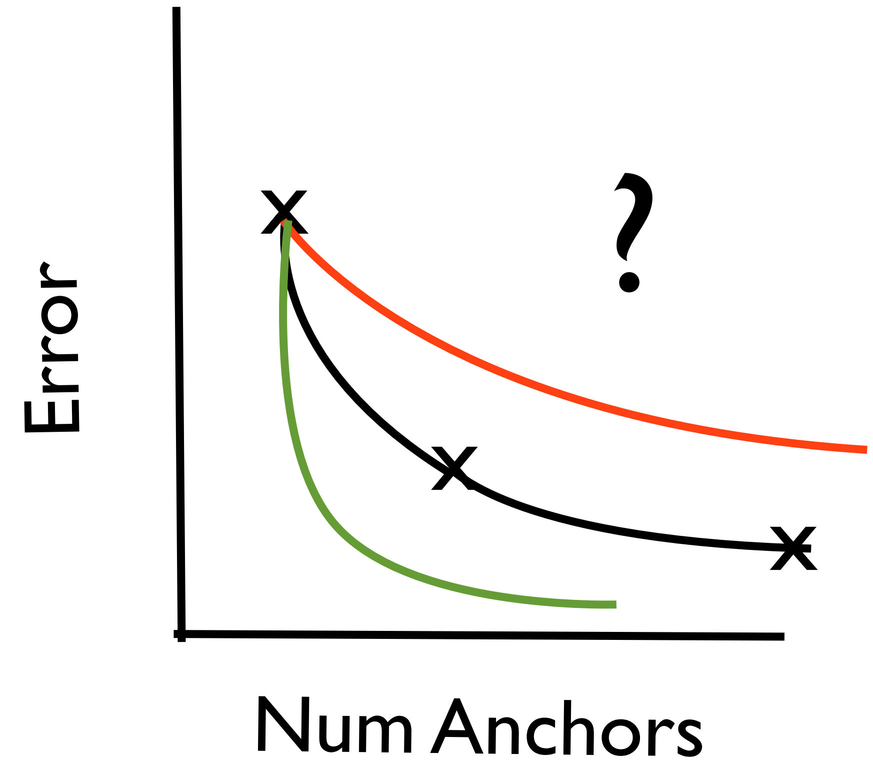


At lower values of α small fraction of LOS readings - significant gains

At higher values of α small fraction of NLOS readings - significant degradation

Effect of anchors





Measure of "precision"

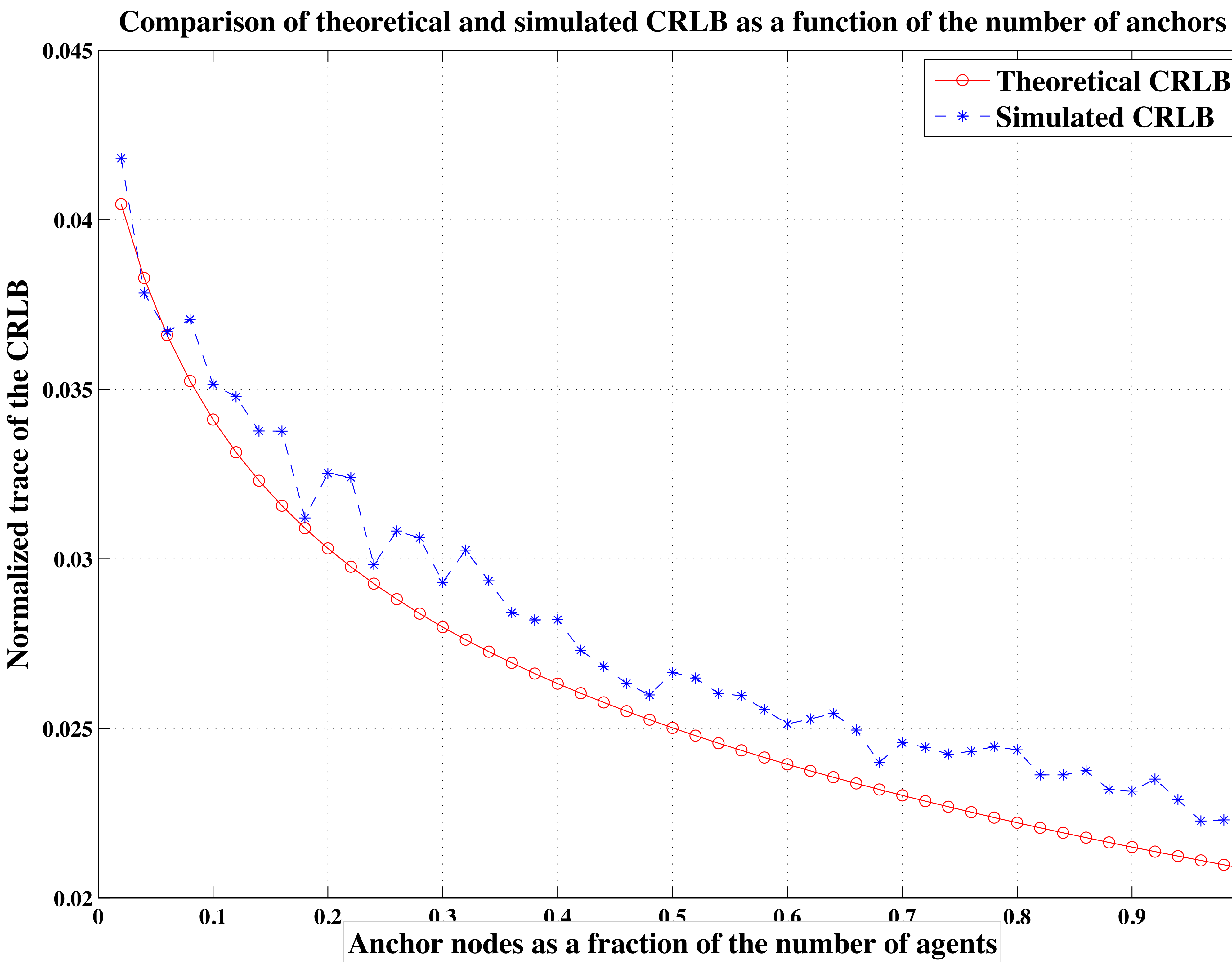
Additional "precision" due to M' anchors

Theorem:

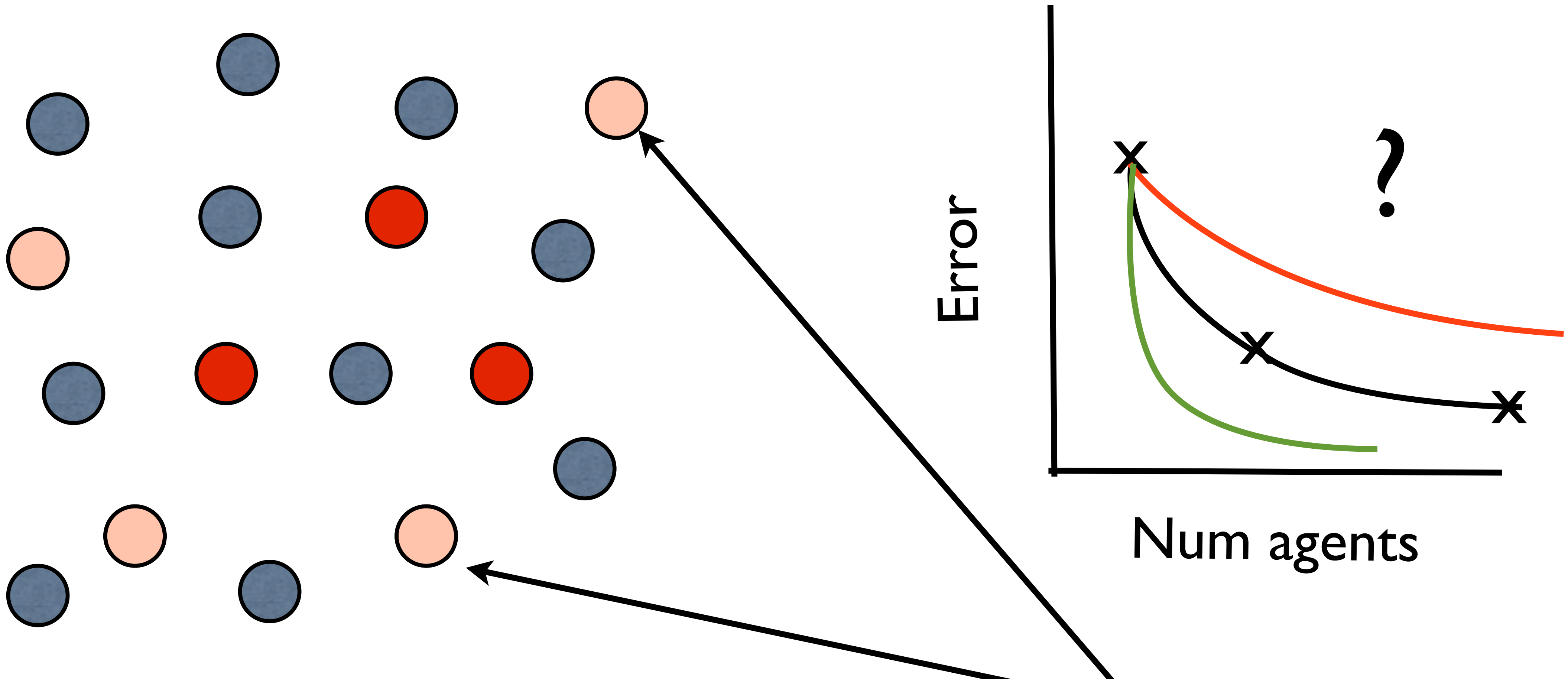
$$\mathrm{Trace}(F'^{-1}) = \frac{1}{g(p_{NOISE})} \sum_{i=1}^{2} \frac{1}{\lambda_i + \frac{\rho M'}{2\sigma^2}} \quad w.h.p$$

$$\mathbb{E}\left[\sum_{i=1}^{N} ||u_i - \hat{u}_i||^2 \;\middle|\; \underline{\hat{r}} \right]$$

(for distance measurements)


Comparison of theoretical and simulated CRLB as a function of the number of anchors
Theoretical CRLB
Simulated CRLB
Normalized trace of the CRLB
Anchor nodes as a fraction of the number of agents
0.045
0.04
0.035
0.03
0.025
0.02
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

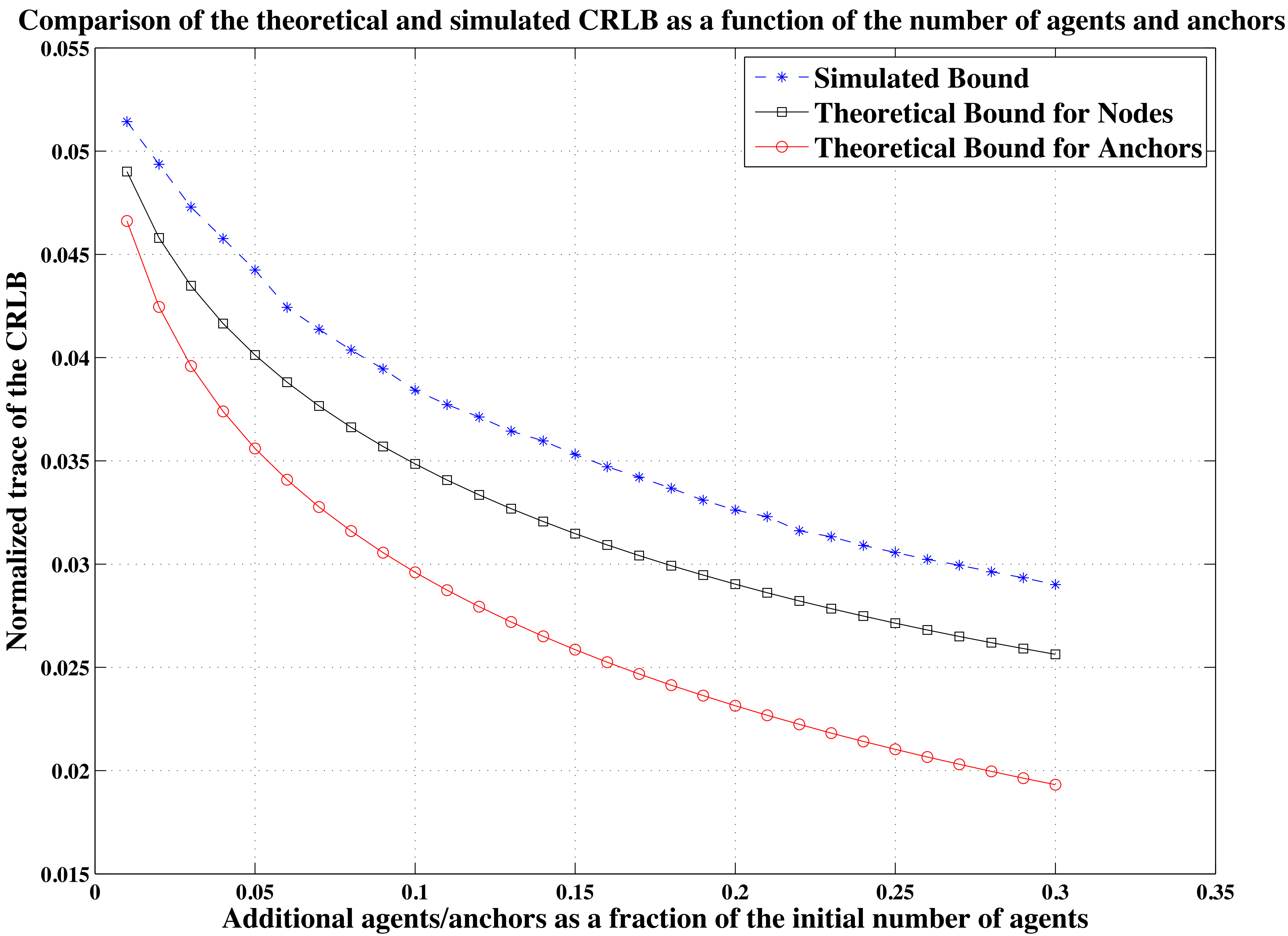
Effect of agents



"Virtual" anchors

Theorem:

$$Trace(F'^{-1}) = \frac{1}{g(p_{NOISE})} \sum_{i=1}^{2N} \frac{1}{\lambda_i + \frac{\rho N'}{2}}.$$



Comparison of the theoretical and simulated CRLB as a function of the number of agents and anchors
Simulated Bound
Theoretical Bound for Nodes
Theoretical Bound for Anchors
Normalized trace of the CRLB
Additional agents/anchors as a fraction of the initial number of agents
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Experimental results

Can we estimate the GPS accuracy in a city just based on GIS data?

Dilution of Precision

DoP - characterize location estimation error from GPS measurements

$$\mathrm{DoP}^2 = \mathrm{Trace}(F_g^{-1})$$

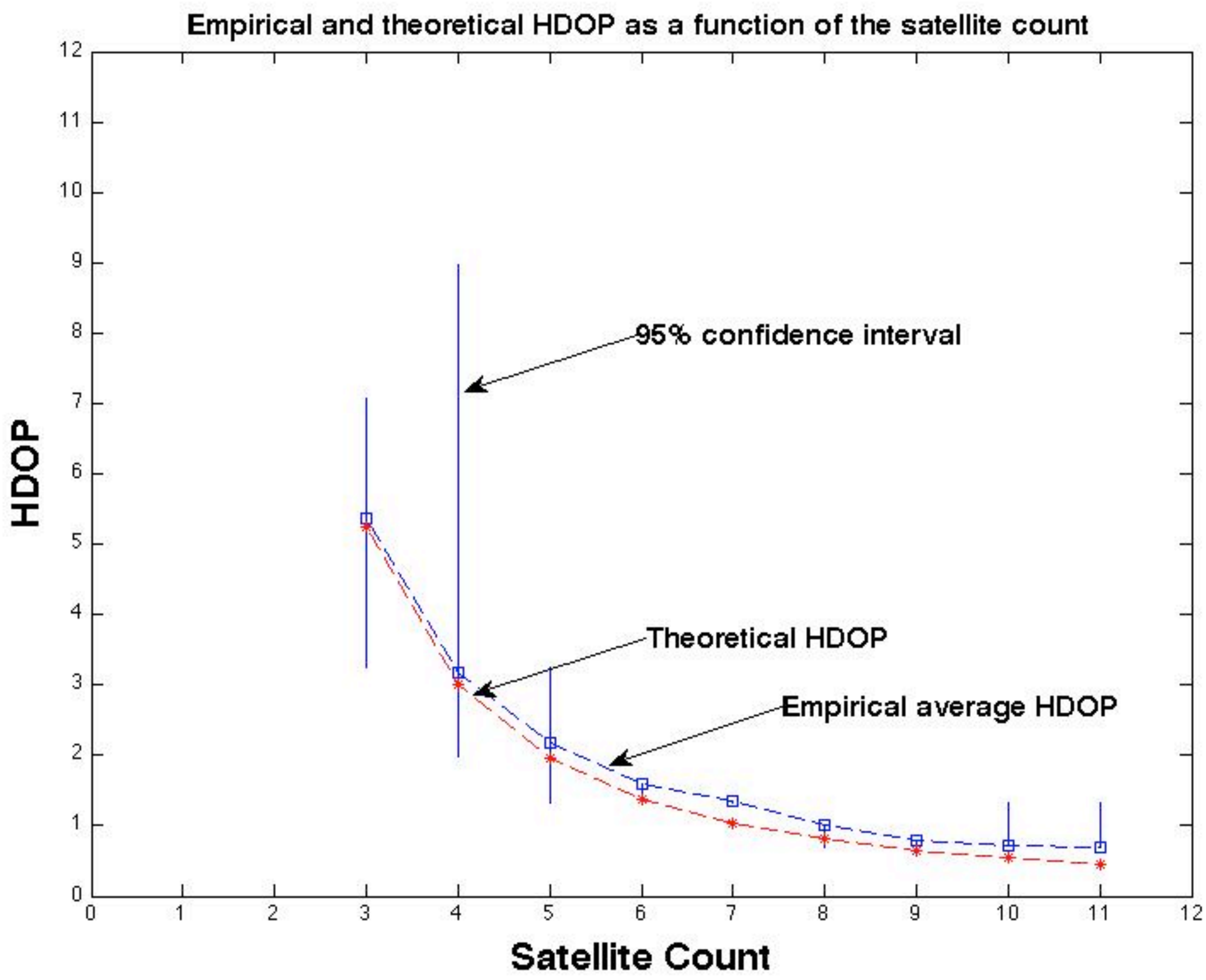
$$\mathrm{HDOP} = \frac{4}{s\left(1 - \frac{\sin(2\cos^{-1}(1-\frac{2s}{N}))}{2\cos^{-1}(1-\frac{2s}{N})}\right)}$$

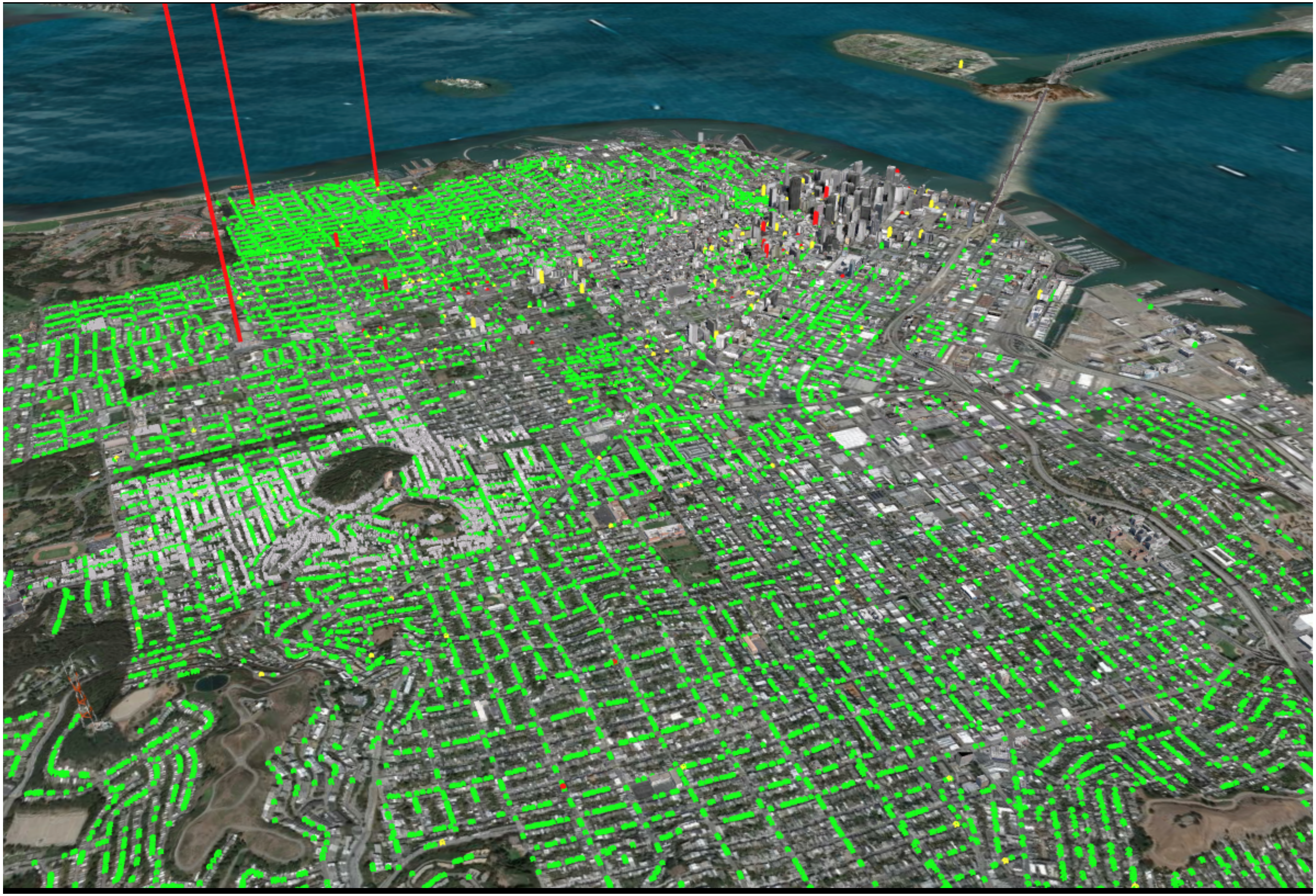
s - predicted number of satellites at a POI using building heights

Mask angle

Venkatesan Ekambaram, Christian Manasseh, Adam Goodliss, Raja Sengupta, Kannan Ramchandran, "A Systems Approach to Sizing of Co-operative High-Accuracy Location (C-HALO) services validated by experiments in San Francisco", ION GNSS 2011, Portland, Oregon.

Empirical and theoretical HDOP from data collected in SF





0.3 to 4% of San Francisco has a GPS visibility of less than 6 satellites

Accident Data

Conclusion

- Scaling of localization error in NLOS environments
  - number of anchors and agents.
  - fraction of LOS signals.
- Compact CRLB expression.
- Simulation and experimental results.

Thank you!