A Bézier-Based Approach to Unstructured Moving Meshes

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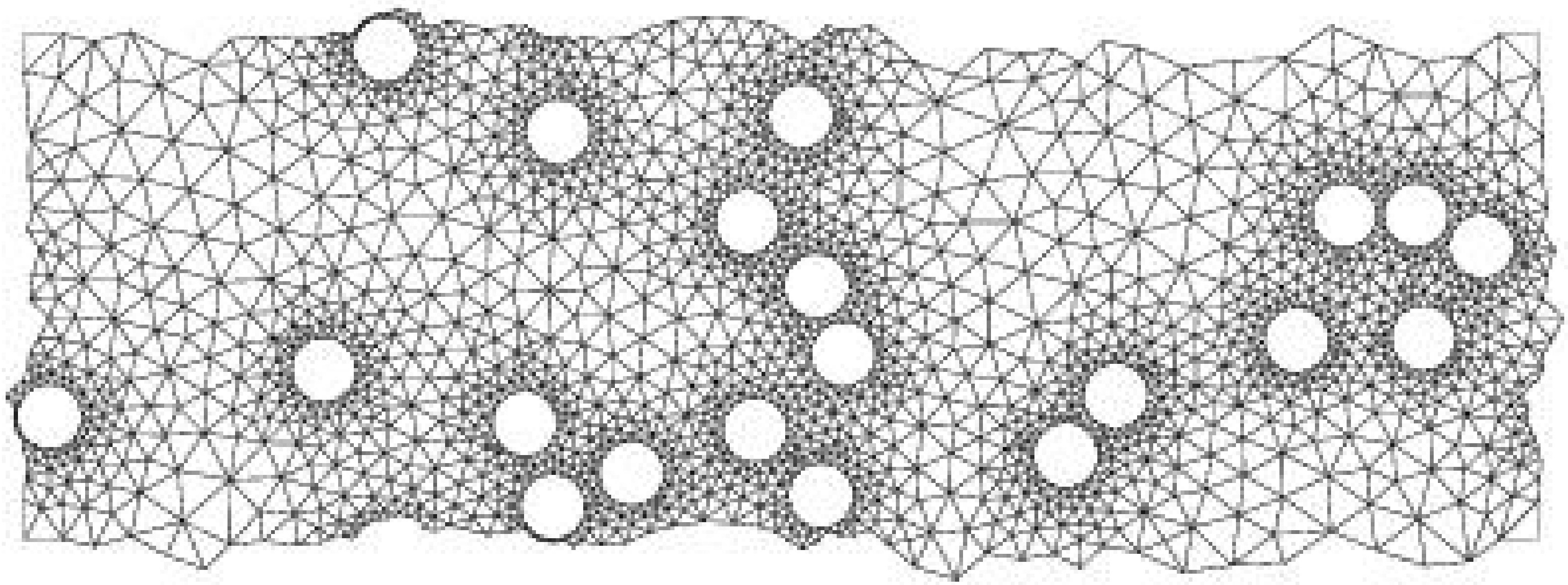
CS 294-1: Meshing and Triangulation

May 7th, 2008

Motivation

Why moving meshes?



Simulation of fluid particle flows by ***Bertrand Maury***
www.math.u-psud.fr/~maury

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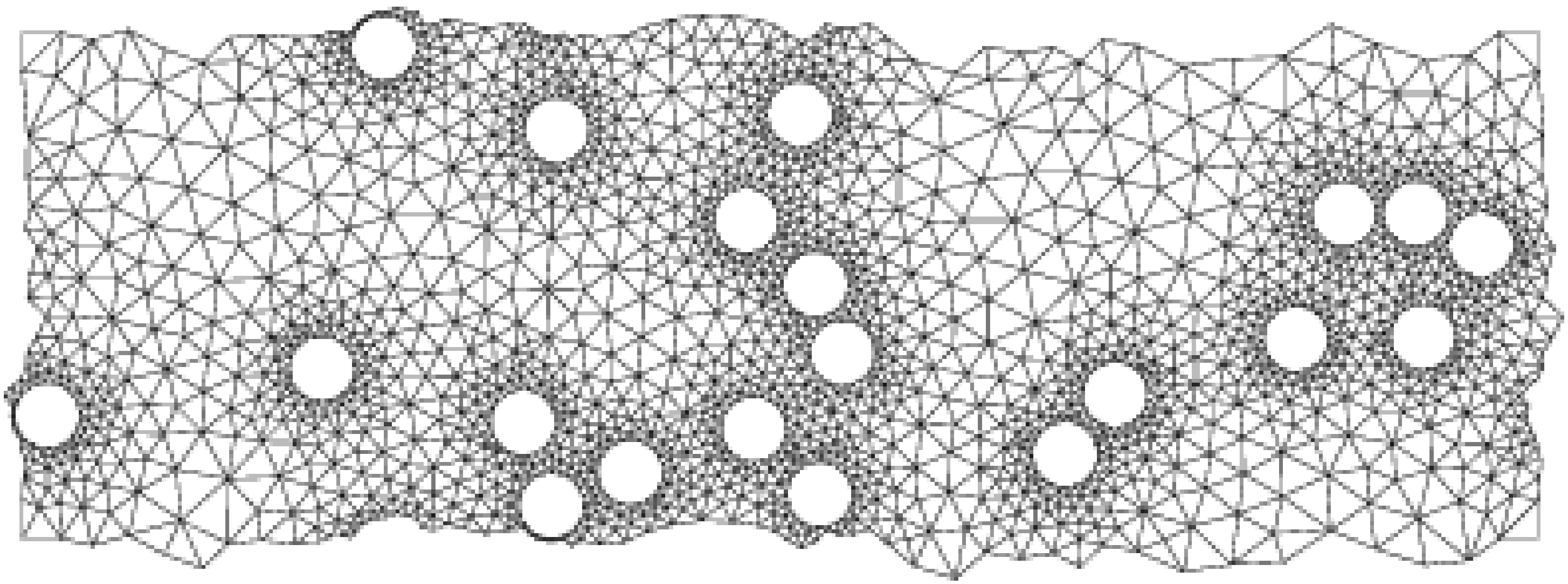

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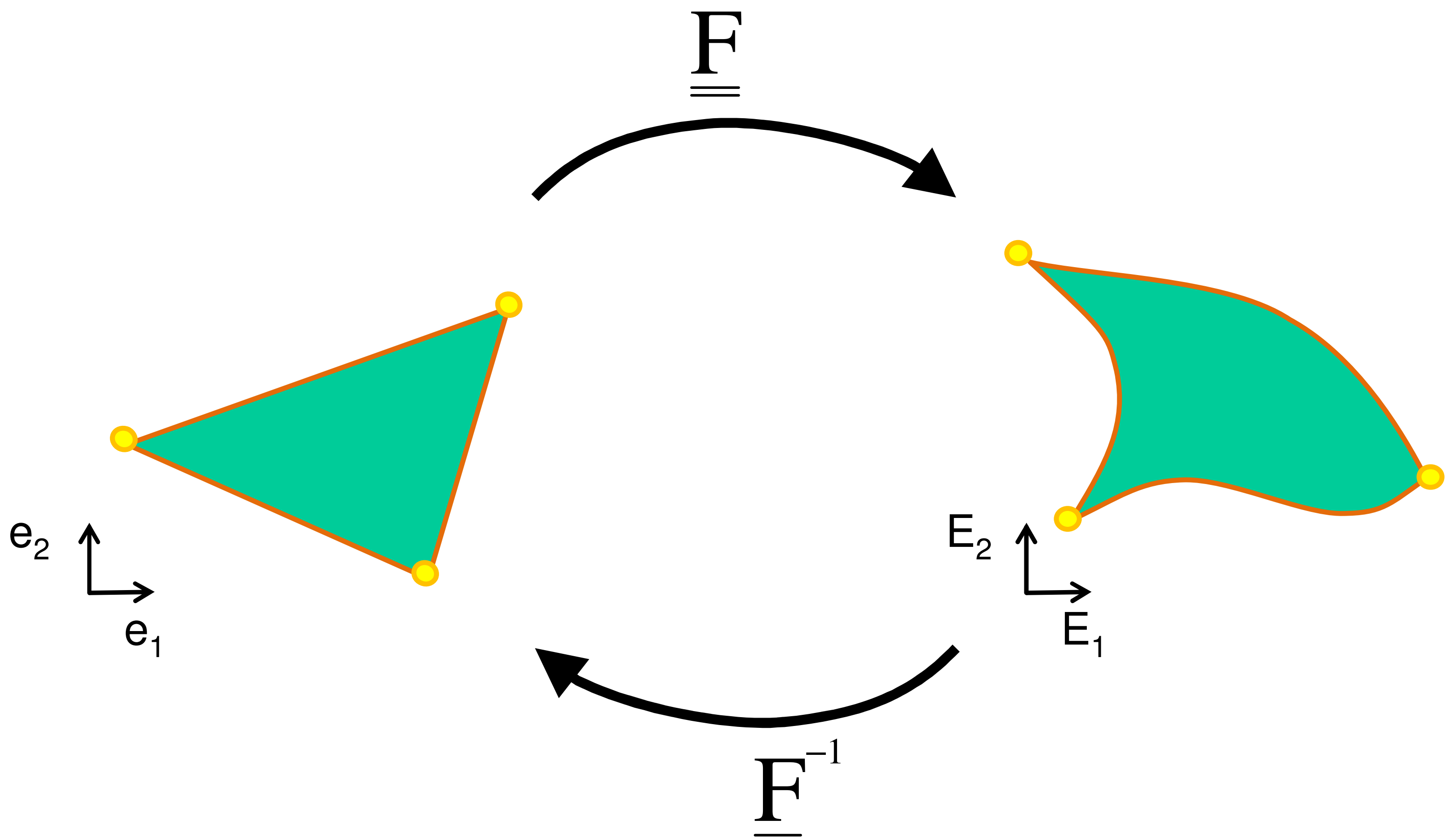
Why curved elements?



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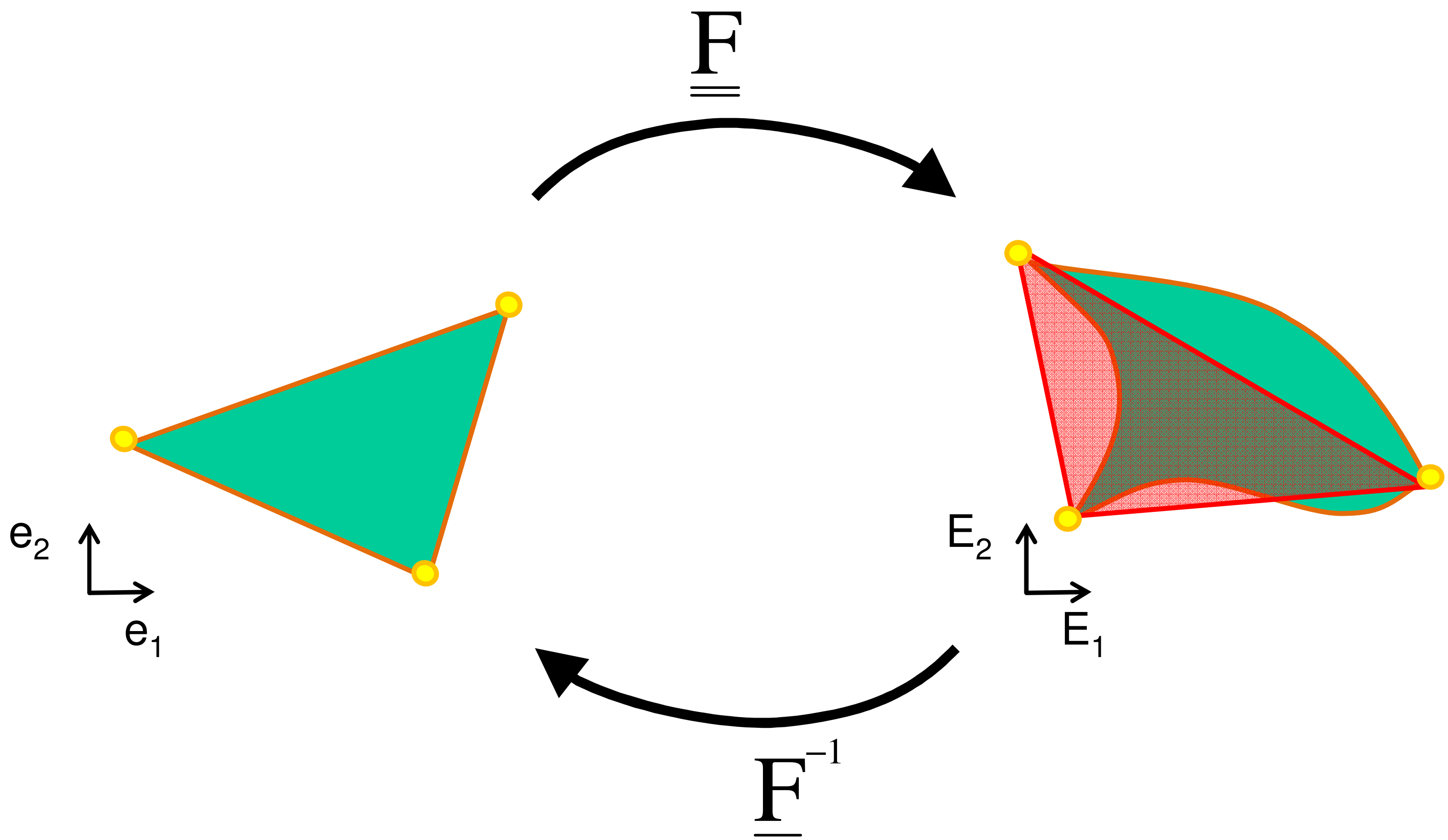
Why curved elements?

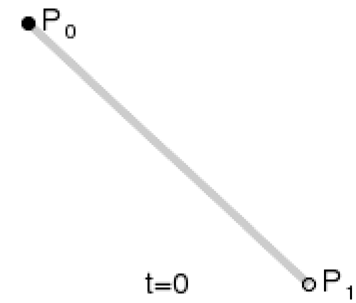


Outline

1. Bézier-based mesh
2. Mesh modification operation and improvement methods
3. Quality measurement of curved elements
4. Simulation results

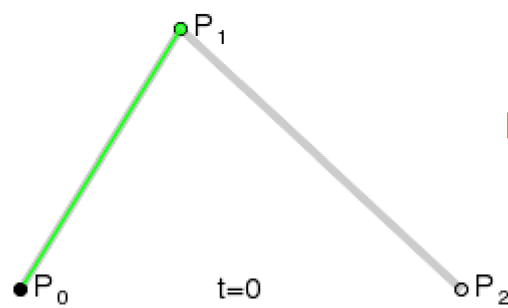
Bézier-based Mesh

Bézier curve



n=1

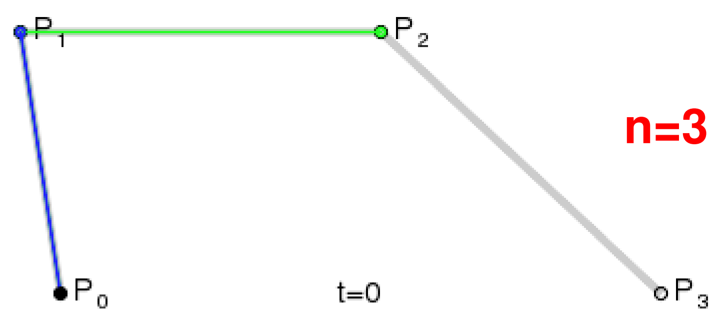
$$B(t) = B_{P_0P_1\ldots P_n}(t)$$
$$= (1-t)\,B_{P_0P_1\ldots P_{n-1}}(t) + t\,B_{P_1P_2\ldots P_n}(t)$$



n=2

With base cases:

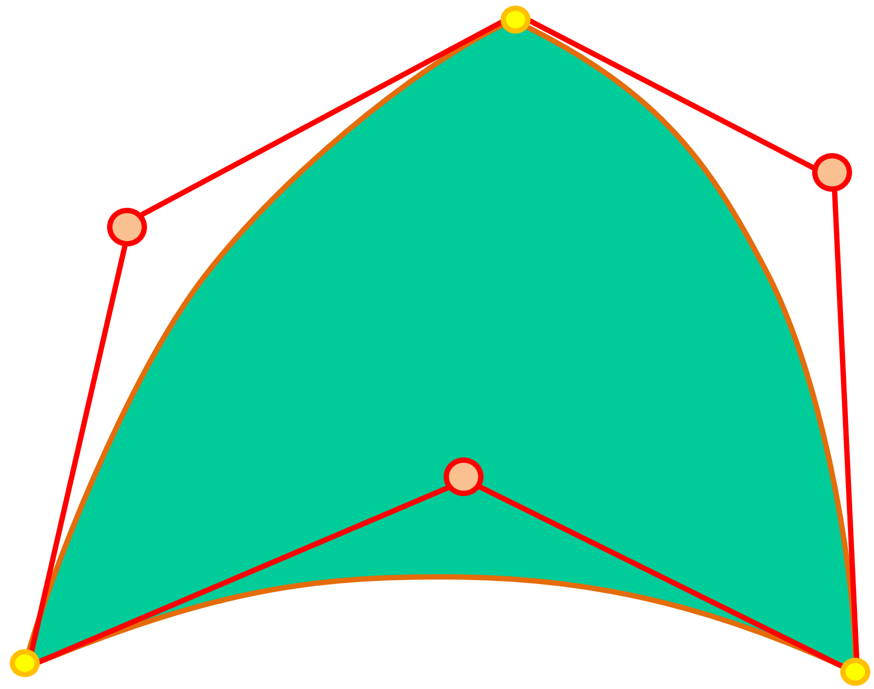
$$B_{P_0P_0}(t) = P_0$$
$$B_{P_1P_1}(t) = P_1$$



n=3

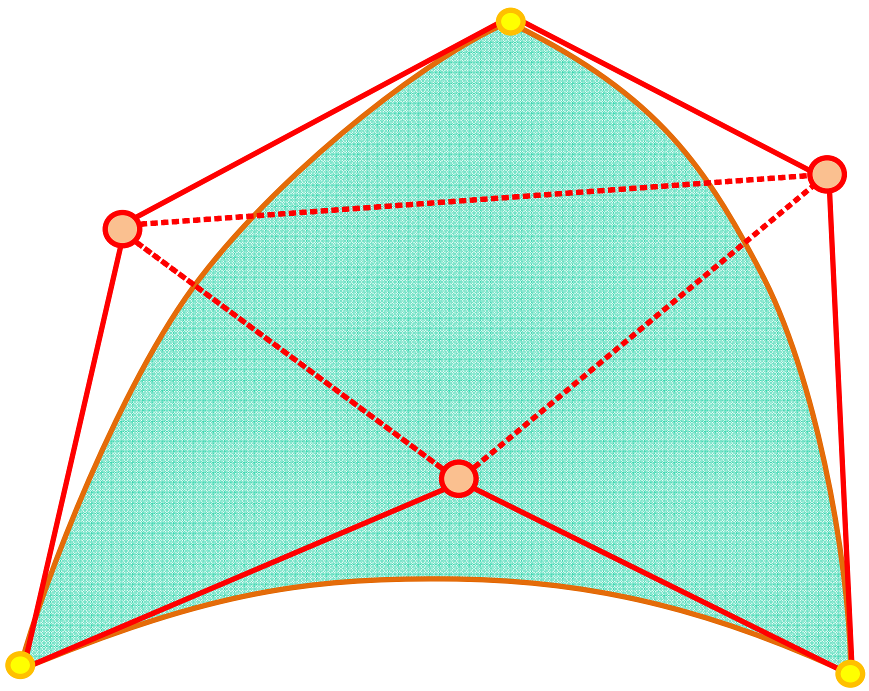
http://en.wikipedia.org/wiki/B%C3%A9zier_curve

Bézier element

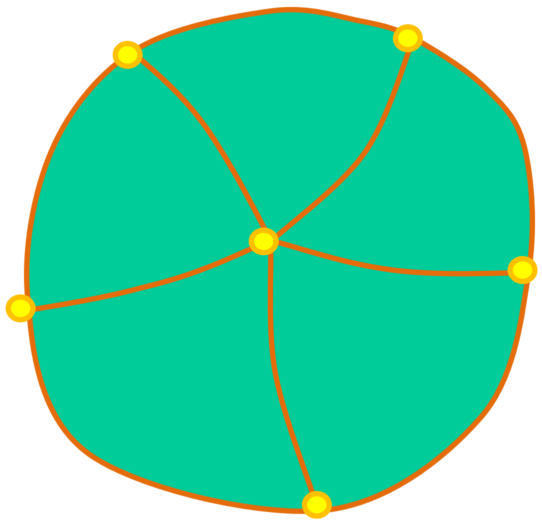


Bézier element

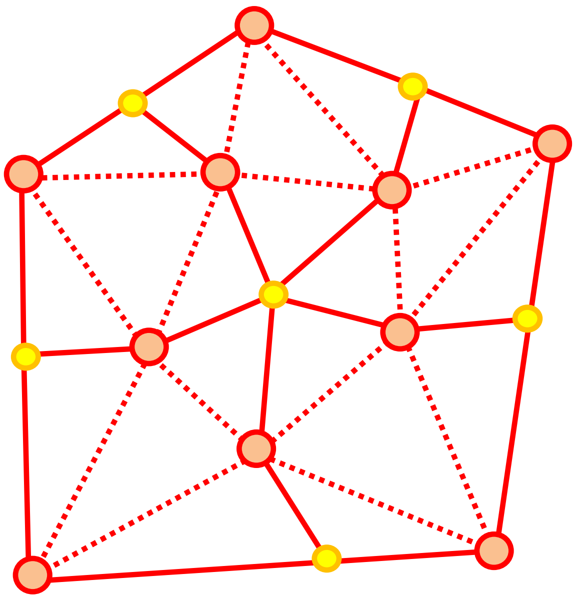
with control triangles



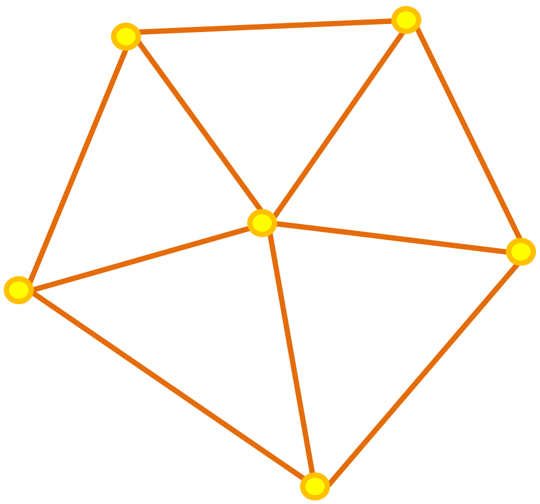
Bézier mesh



curved mesh

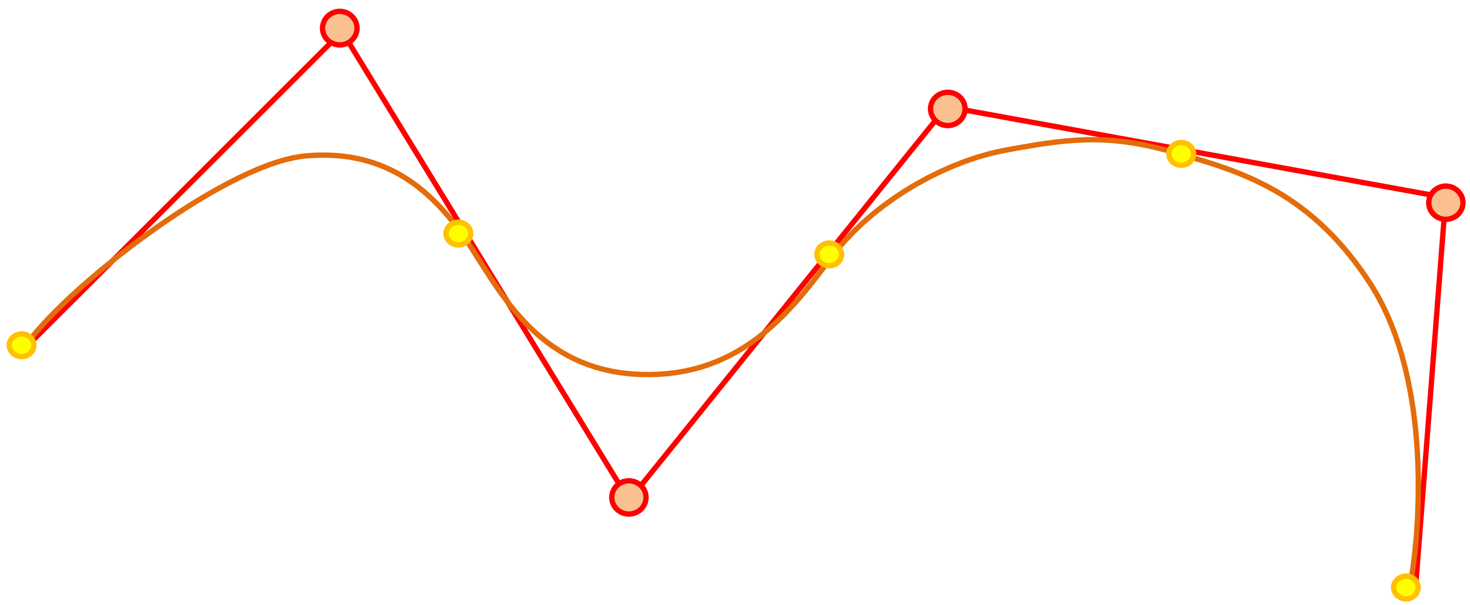


control mesh



logical mesh

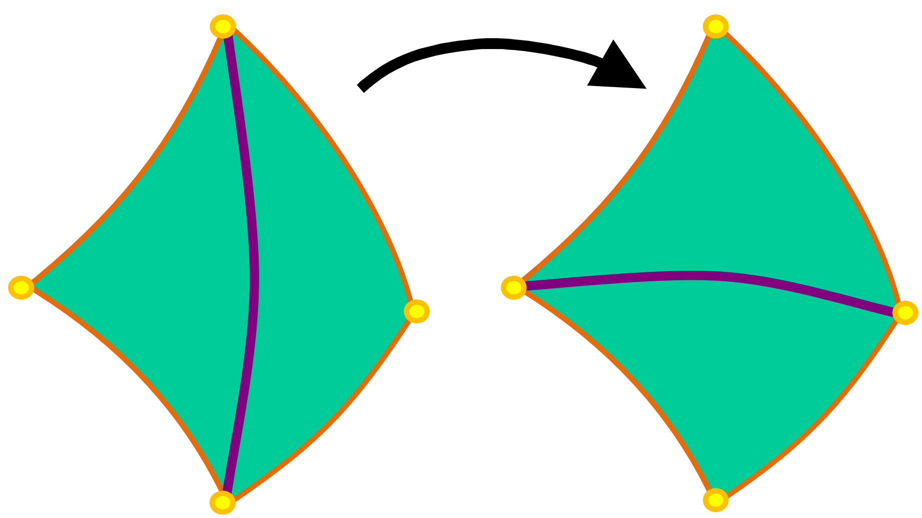
B-splines on mesh boundary



Quadratic B-spline with its control polygon

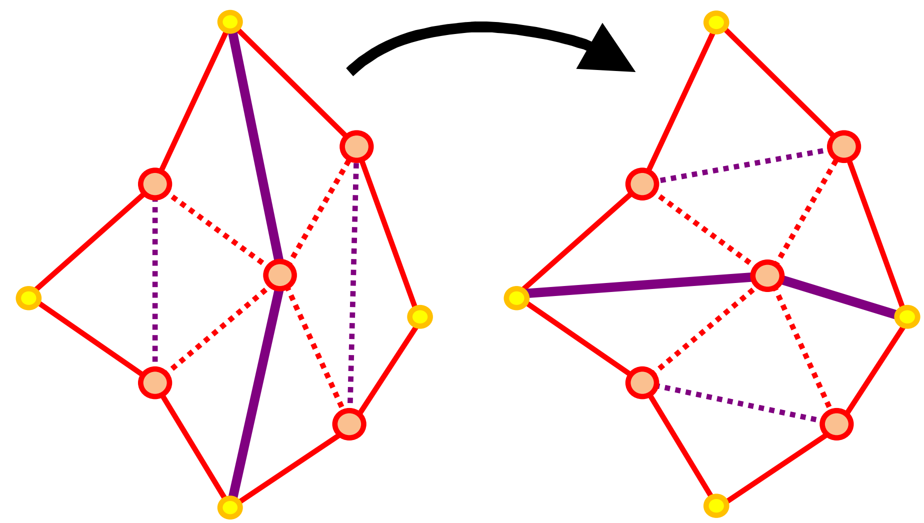
Mesh Modification Operation and Improvement Methods

Edge flipping



curved mesh

single edge flip

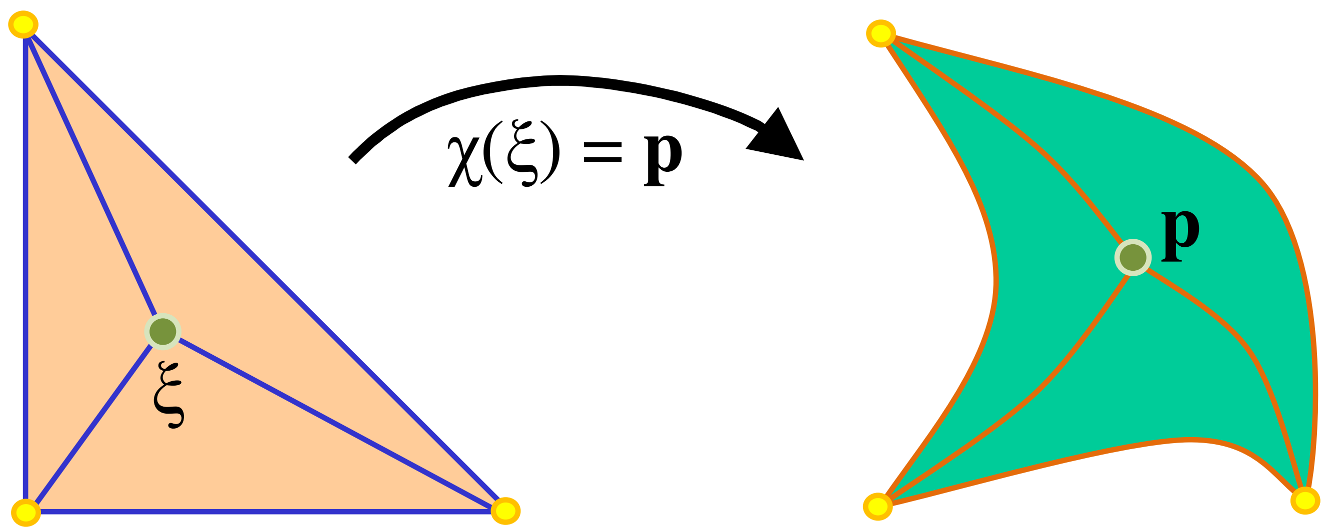


control mesh

four edges in flipping involved

Vertex insertion

Utilizing isoparametric concept

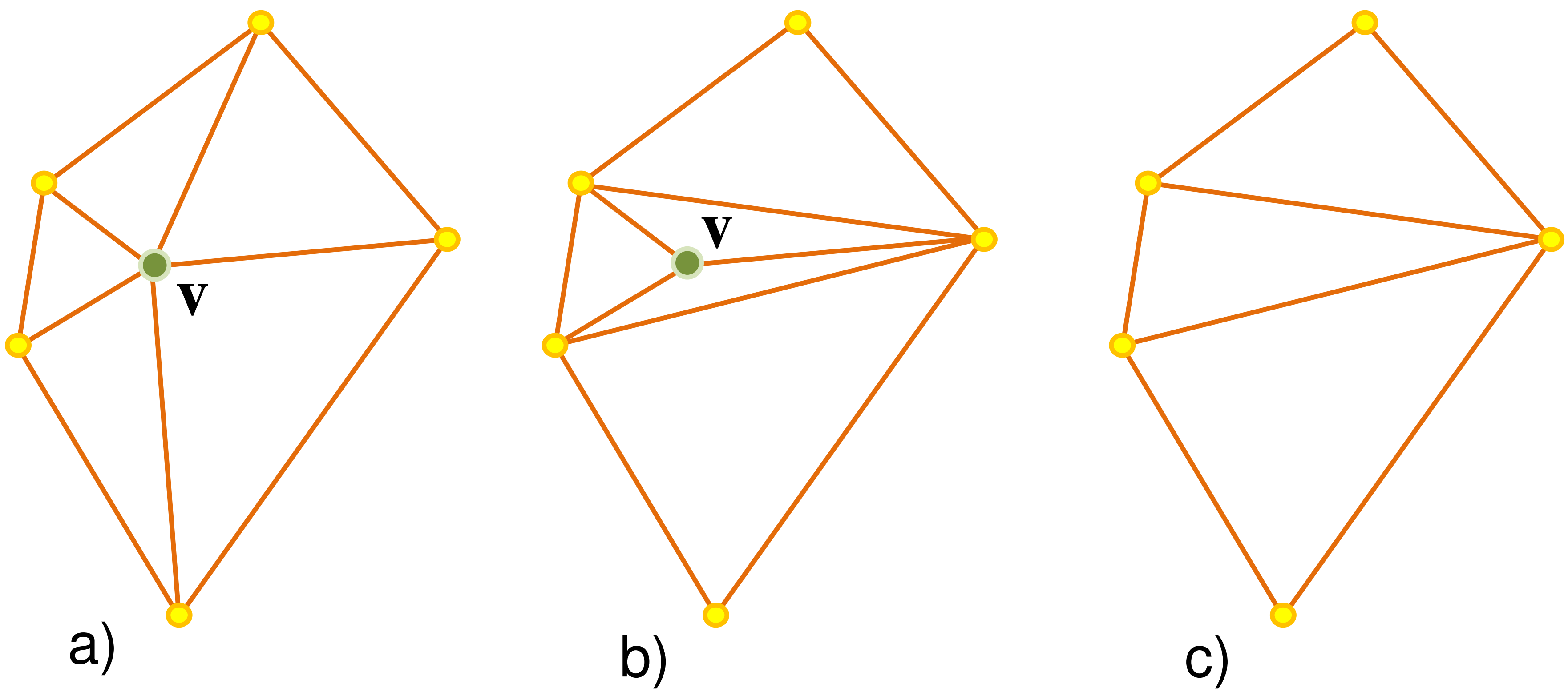


reference element
(unit right triangle)

Bézier element

Vertex removal

operating in logical mesh



Mesh refinement

operating in logical mesh

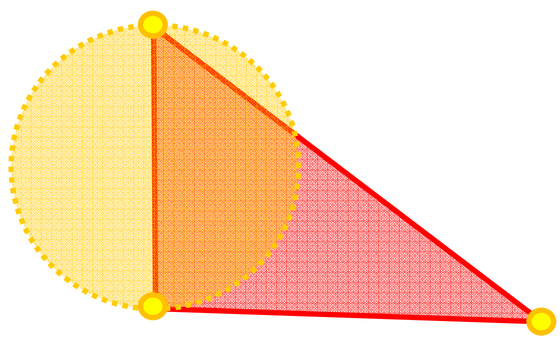
Enforce *Delaunay* property with edge flip

Adapt *Ruppert's* algorithm for refining curved elements which are too large or have a „bad“ logical triangle (poor aspect ratio)

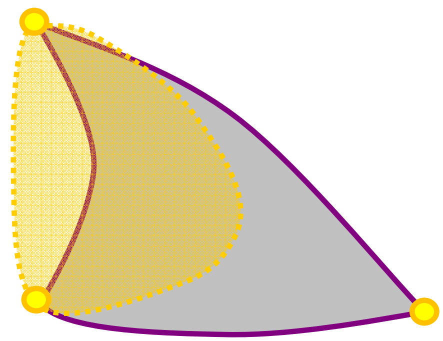
Mesh refinement

Changes of *Ruppert's* algorithm for curved elements

boundary encroachment

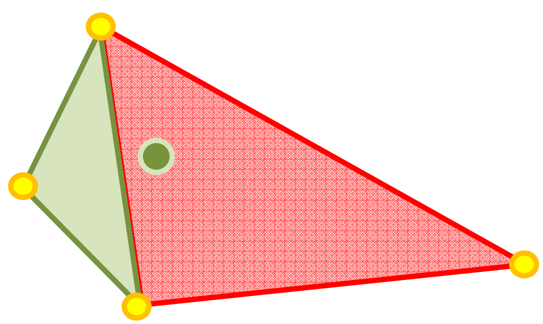


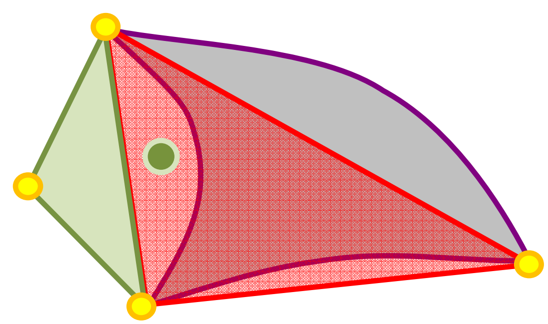
diametral circle



lens

point location





Mesh Coarsening

Utilizing function-based coarsening paradigm of *Talmor et al* (Lecture 21)

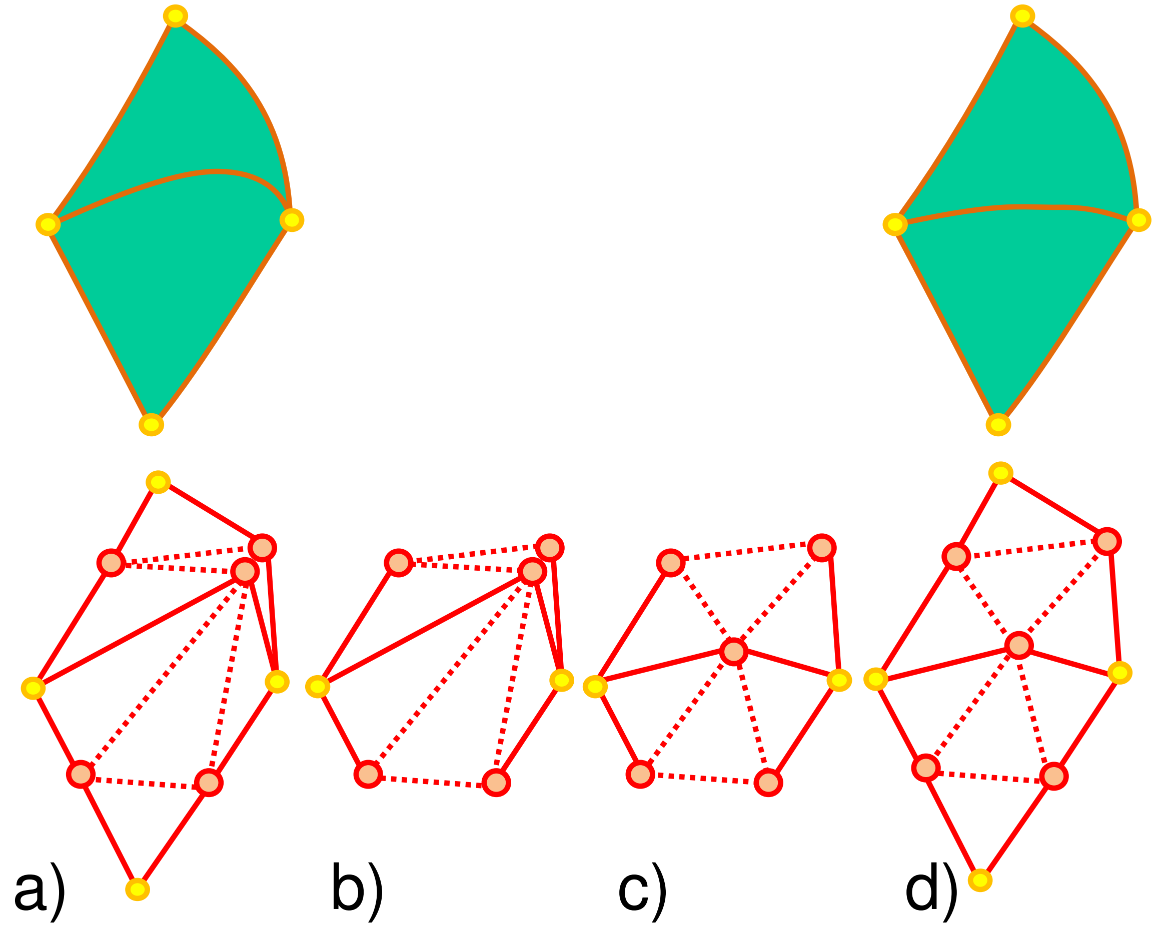
Adaptation necessary for boundaries that must be maintained – *Douglas-Peucker* algorithm

Incremental removal of vertices to preserve Delaunay property

Edge smoothing

curved mesh

control mesh



Quality Management of Curved Elements

Metric for Bézier elements

measuring the ‚curvature‘

$$\int_k \frac{|J|}{A_k}$$

k … element being considered

A_k … area of curved element

J … Jacobian of the geometric mapping $\chi(\xi)$

Optimizing control point position

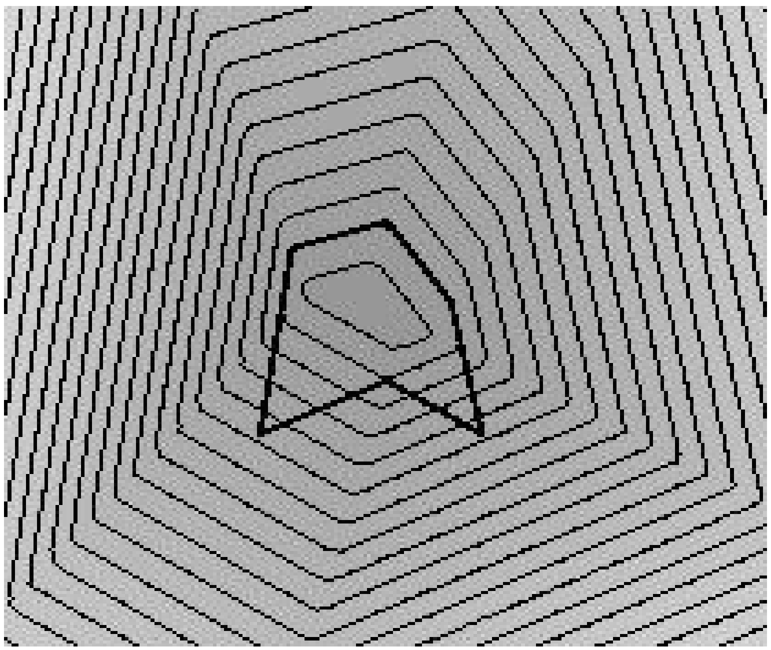
$$\max_{x \in K} \min_{i \in M} \{q_i(x)\}$$

M … index set of triangles incident to control point

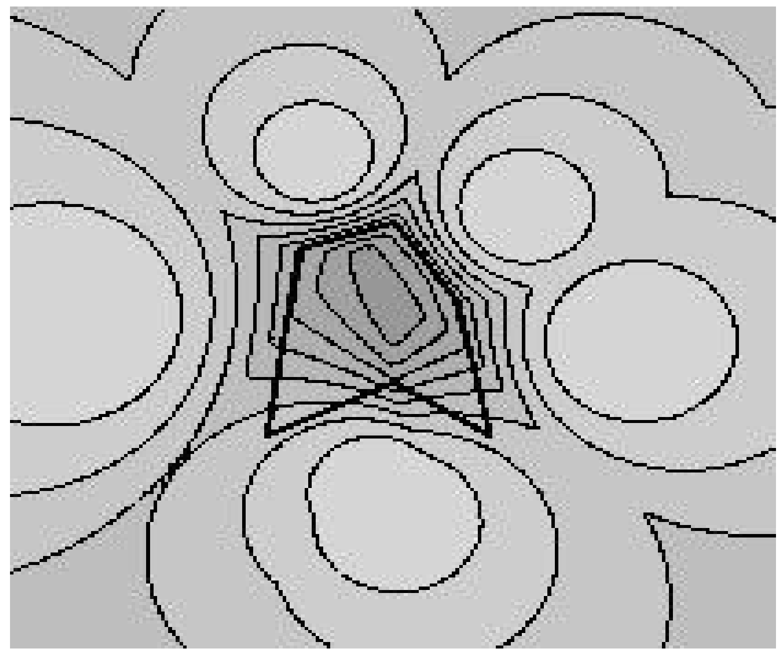
x … control point location

q_i … quality value of triangle i in M

Local maxima of $\min_{i \in M} \{q_i(x)\}$



$$q_i(x) = A$$



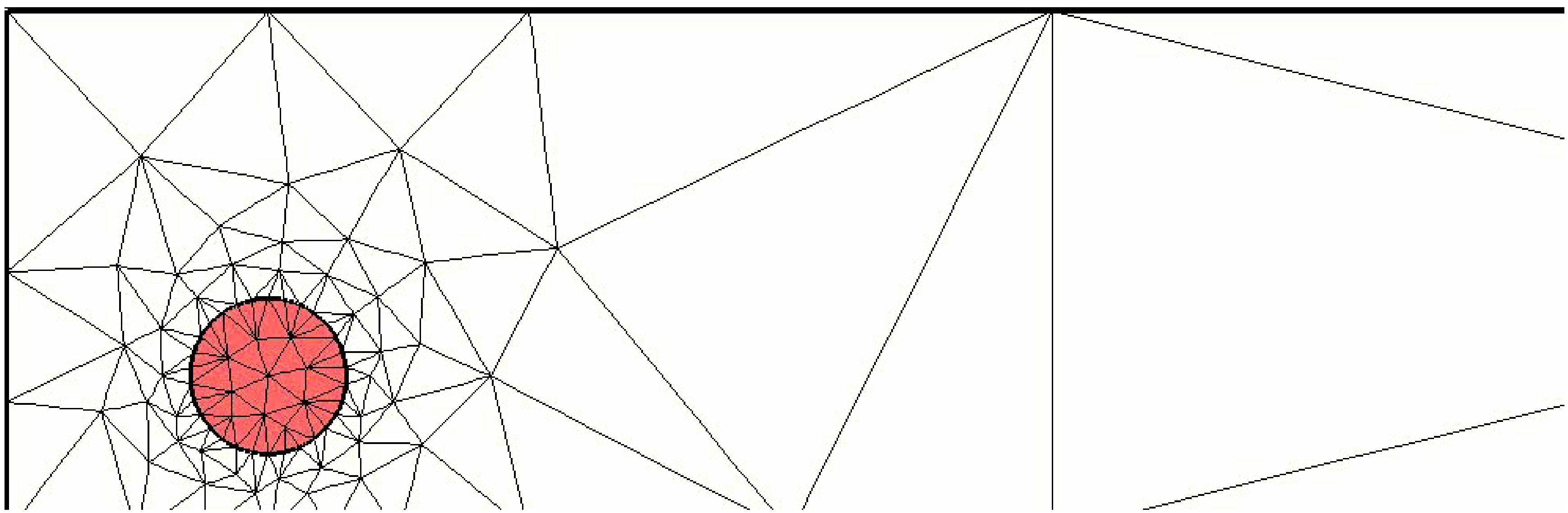
$$q_i(x) = \frac{4\sqrt{3}A}{l_1^2 + l_2^2 + l_3^2}$$

Simulation results

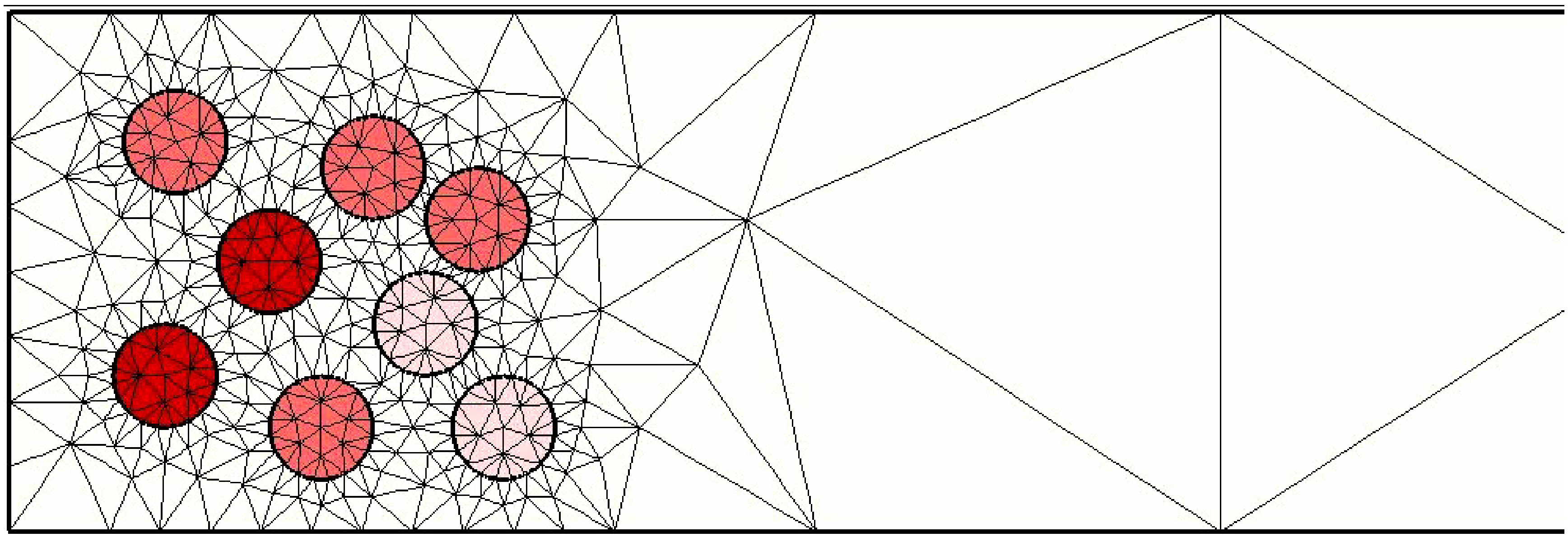
Simulation cycle

```
Generate initial mesh;
repeat{
    compute velocity field with FEA;
    push mesh forward;
    improve mesh:
        1.Enforce Delaunay property
        2.Refine
        3.Coarsen
        4.Smooth
}
```

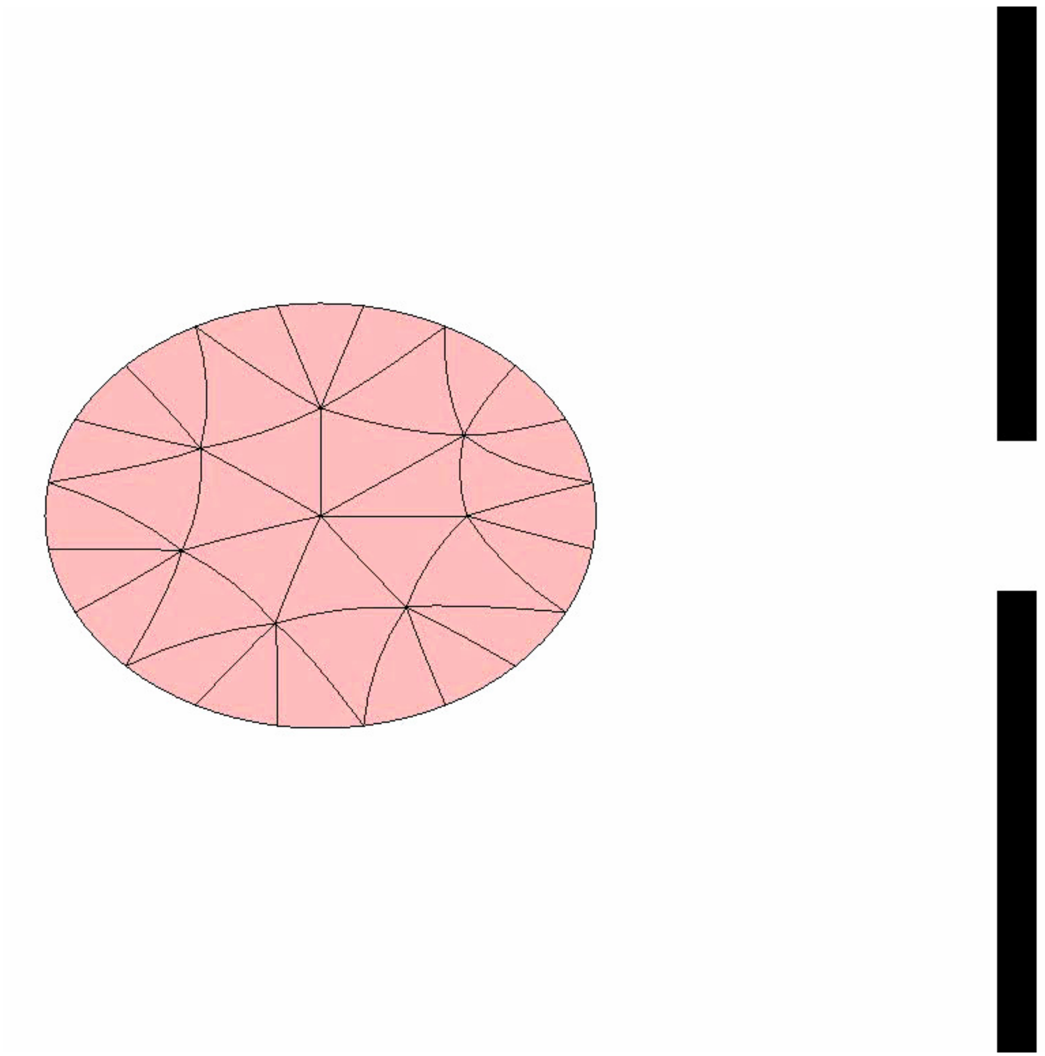
Single (blood) cell in tube



Cells with different viscosity



Cell pushed through orifice



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