A Bézier-Based Approach to Unstructured Moving Meshes

Cardoze D., Cunha A., Miller G., Phillips T., and Walkington N.

Hagen Wille
CS 294-1: Meshing and Triangulation

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Motivation

Why moving meshes?

Simulation of fluid particle flows by Bertrand Maury
www.math.u-psud.fr/~maury
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Why curved elements?

\[ F \]
Motivation

Why curved elements?

\[ F \]

\[ F^{-1} \]
Outline

1. Bézier-based mesh
2. Mesh modification operation and improvement methods
3. Quality measurement of curved elements
4. Simulation results
Bézier-based Mesh
Bézier curve

\[ B(t) = B_{P_0P_1...P_n}(t) = (1-t)B_{P_0P_1...P_{n-1}}(t) + tB_{P_1P_2...P_n}(t) \]

With base cases:
\[ B_{P_0P_0}(t) = P_0 \]
\[ B_{P_1P_1}(t) = P_1 \]

http://en.wikipedia.org/wiki/B%C3%A9zier_curve
Bézier element
Bézier element with control triangles
Bézier mesh

curved mesh  control mesh  logical mesh
B-splines on mesh boundary

Quadratic B-spline with its control polygon
Mesh Modification Operation and Improvement Methods
Edge flipping

curved mesh
single edge flip

curvature mesh
four edges in flipping involved
Vertex insertion
Utilizing isoparametric concept

\[ \chi(\xi) = p \]

reference element
(unit right triangle)

Bézier element
Vertex removal
operating in logical mesh

a) b) c)
Mesh refinement
operating in logical mesh

Enforce *Delaunay* property with edge flip

Adapt *Ruppert’s* algorithm for refining curved elements which are too large or have a „bad“ logical triangle (poor aspect ratio)
Mesh refinement

Changes of *Ruppert’s* algorithm for curved elements

- boundary encroachment
- diametral circle
- lens
- point location
Mesh Coarsening

Utilizing function-based coarsening paradigm of *Talmor et al* (Lecture 21)

Adaptation necessary for boundaries that must be maintained – *Douglas-Peucker* algorithm

Incremental removal of vertices to preserve Delaunay property
Edge smoothing

curved mesh

control mesh

a) b) c) d)
Quality Management of Curved Elements
Metric for Bézier elements
measuring the 'curvature'\

\[ \int_k \left| \frac{J}{A_k} \right| \]

- \( k \) … element being considered
- \( A_k \) … area of curved element
- \( J \) … Jacobian of the geometric mapping \( \chi(\xi) \)
Optimizing control point position

\[ \max \min_{x \in K, i \in M} \{q_i(x)\} \]

- \( M \) … index set of triangles incident to control point
- \( x \) … control point location
- \( q_i \) … quality value of triangle \( i \) in \( M \)
Local maxima of \( \min_{i \in M} \{q_i(x)\} \)

\[
q_i(x) = A \\
q_i(x) = \frac{4\sqrt{3}A}{l_1^2 + l_2^2 + l_3^2}
\]
Simulation results
Simulation cycle

Generate initial mesh;
repeat{
    compute velocity field with FEA;
    push mesh forward;
    improve mesh:
        1. Enforce Delaunay property
        2. Refine
        3. Coarsen
        4. Smooth
}
Single (blood) cell in tube
Cells with different viscosity
Cell pushed through orifice
Cell pushed through orifice
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