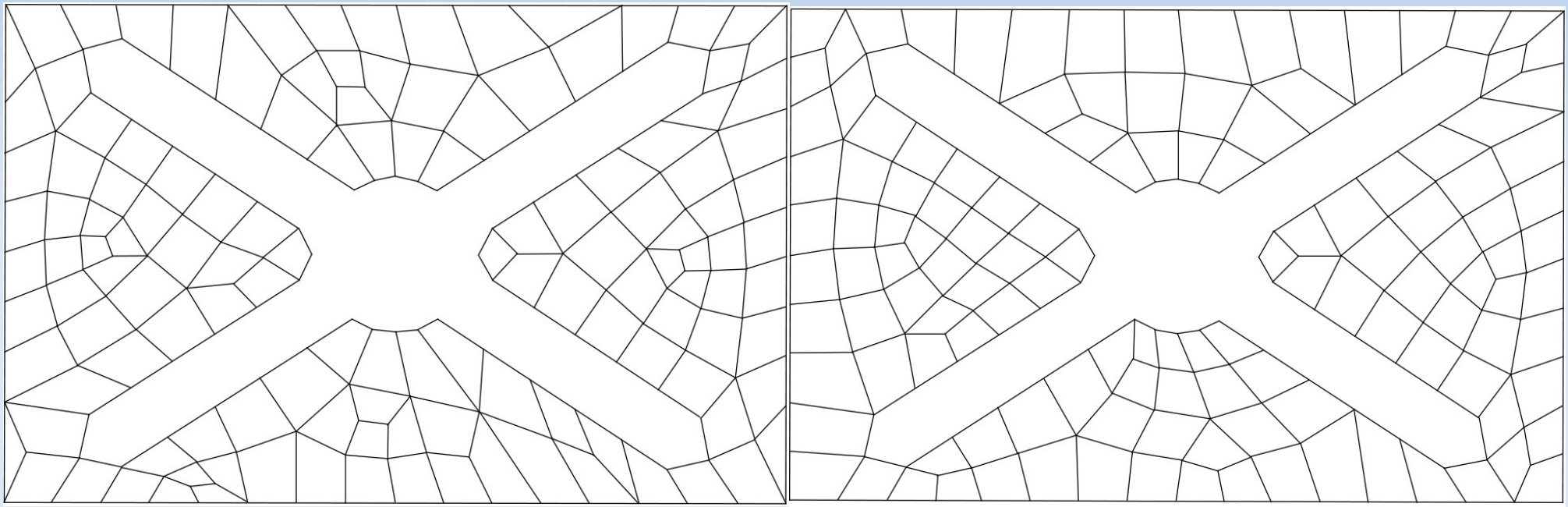
CleanUp: Improving Quadrilateral Finite Element Meshes

Paul Kinney



Presentation by

James F. Hamlin

April 23, 2008

Philosophy

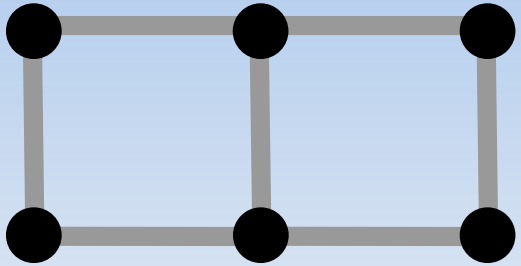
Quadrilateral Mesh Improvement

Permanent nodes and edges cannot be changed.

May modify connectivity and insert/remove interior nodes as required.

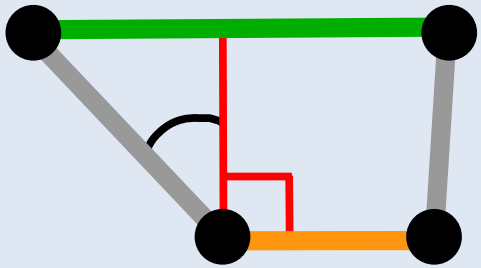
Mesh Improvement Goals

A square is the ideal element.



Thus:

- Minimize “skew”: |angle - 90°|
- Minimize aspect ratio: $\frac{\text{||longest edge||}}{\text{||shortest edge||}}$



Also:

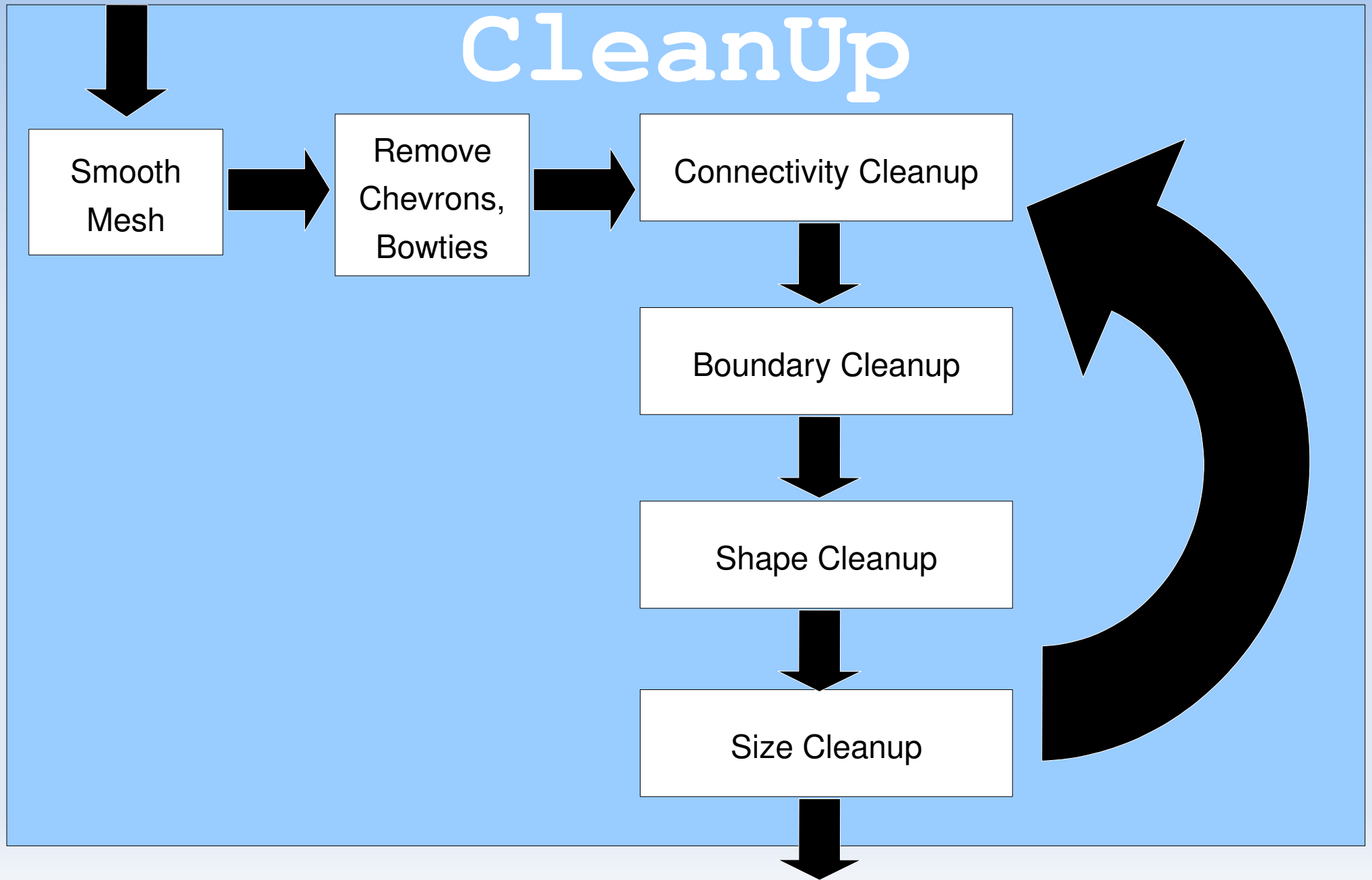
- Keep bad elements away from the boundary.

Kinney's CleanUp

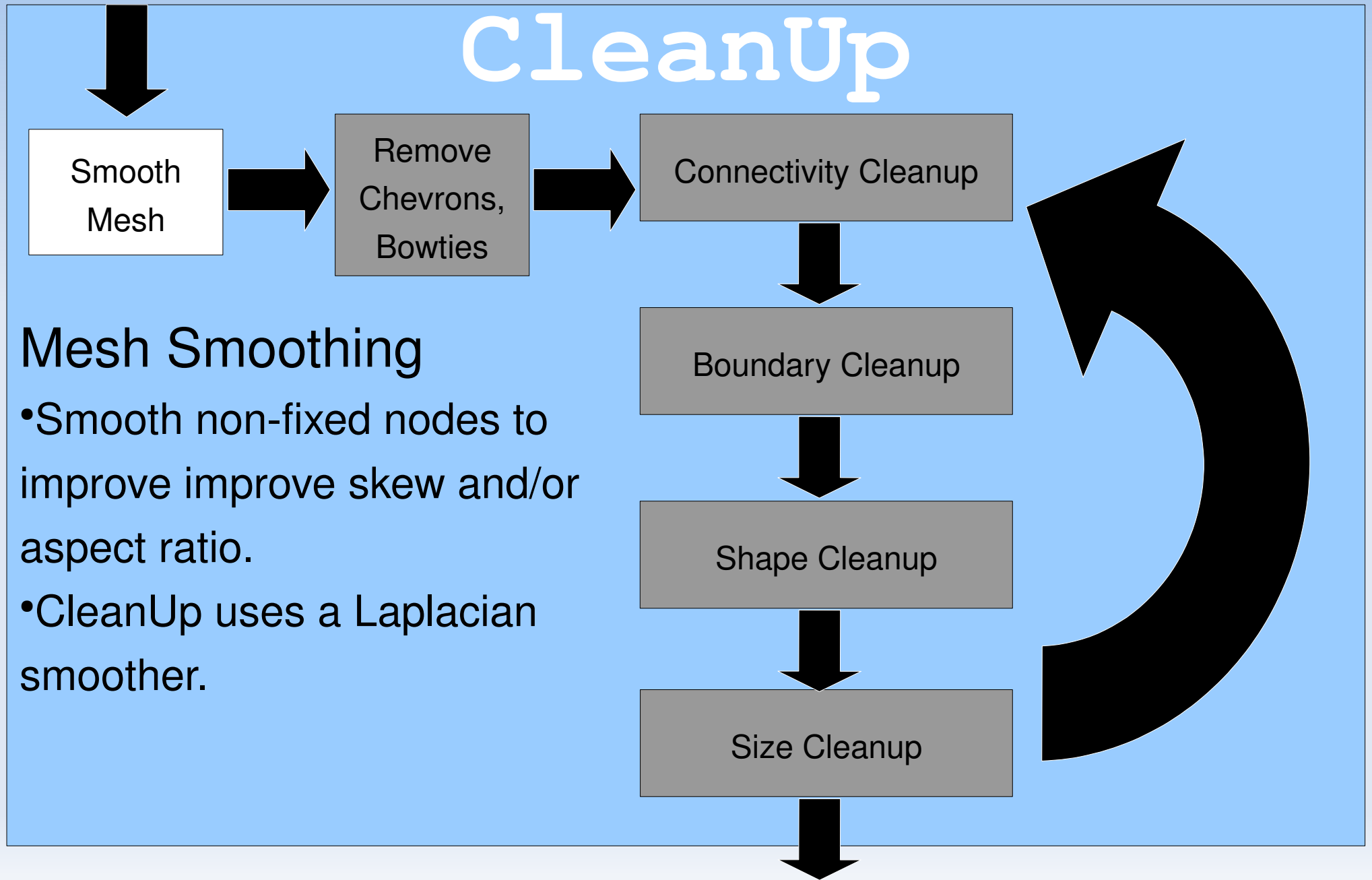
Largely heuristic

- Find bad mesh configurations
- Map to improved configurations
- Apply improvements *ad nauseum*

Algorithm Framework



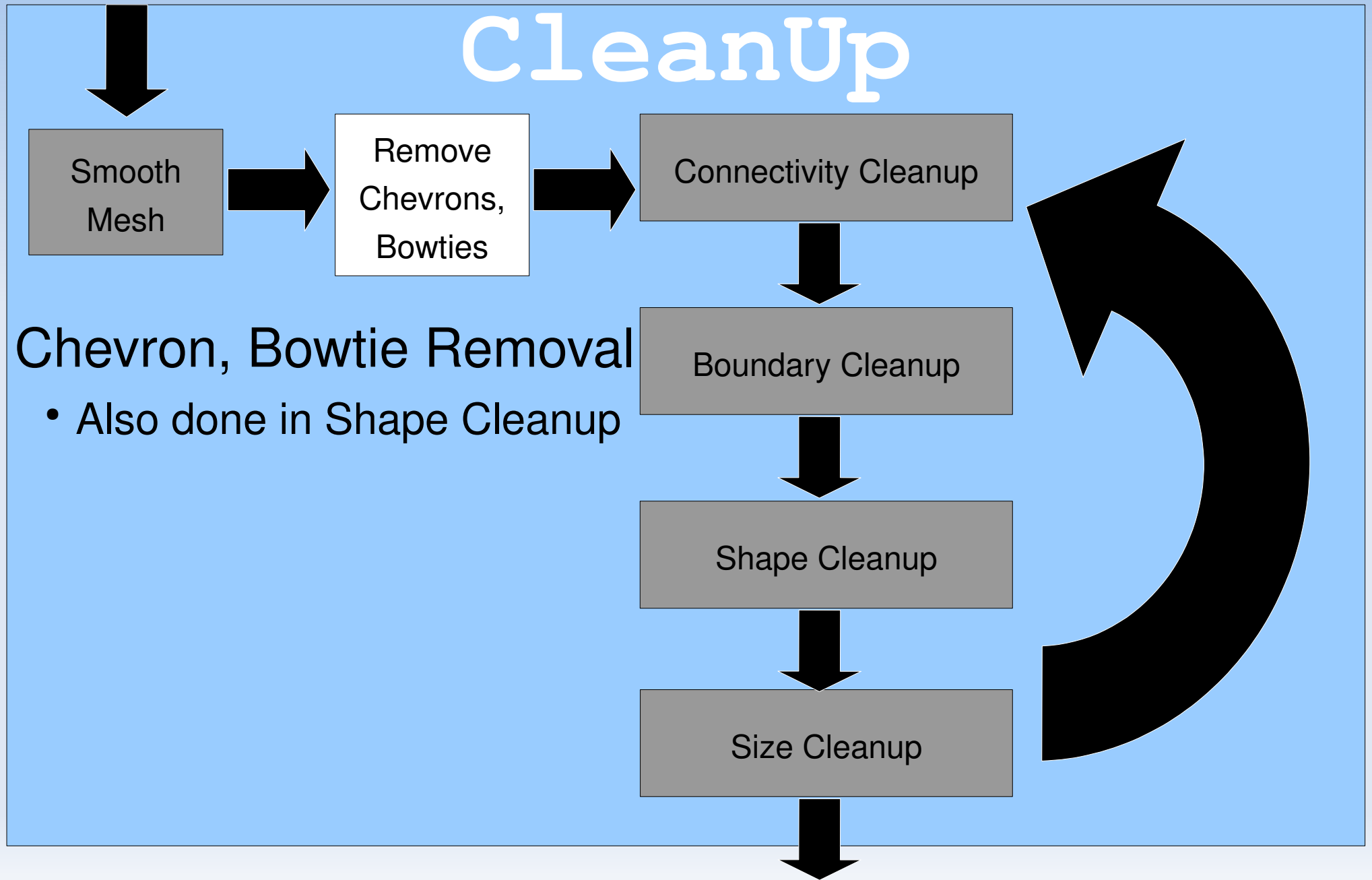
Algorithm Framework



Mesh Smoothing

- Smooth non-fixed nodes to improve improve skew and/or aspect ratio.
- CleanUp uses a Laplacian smoother.

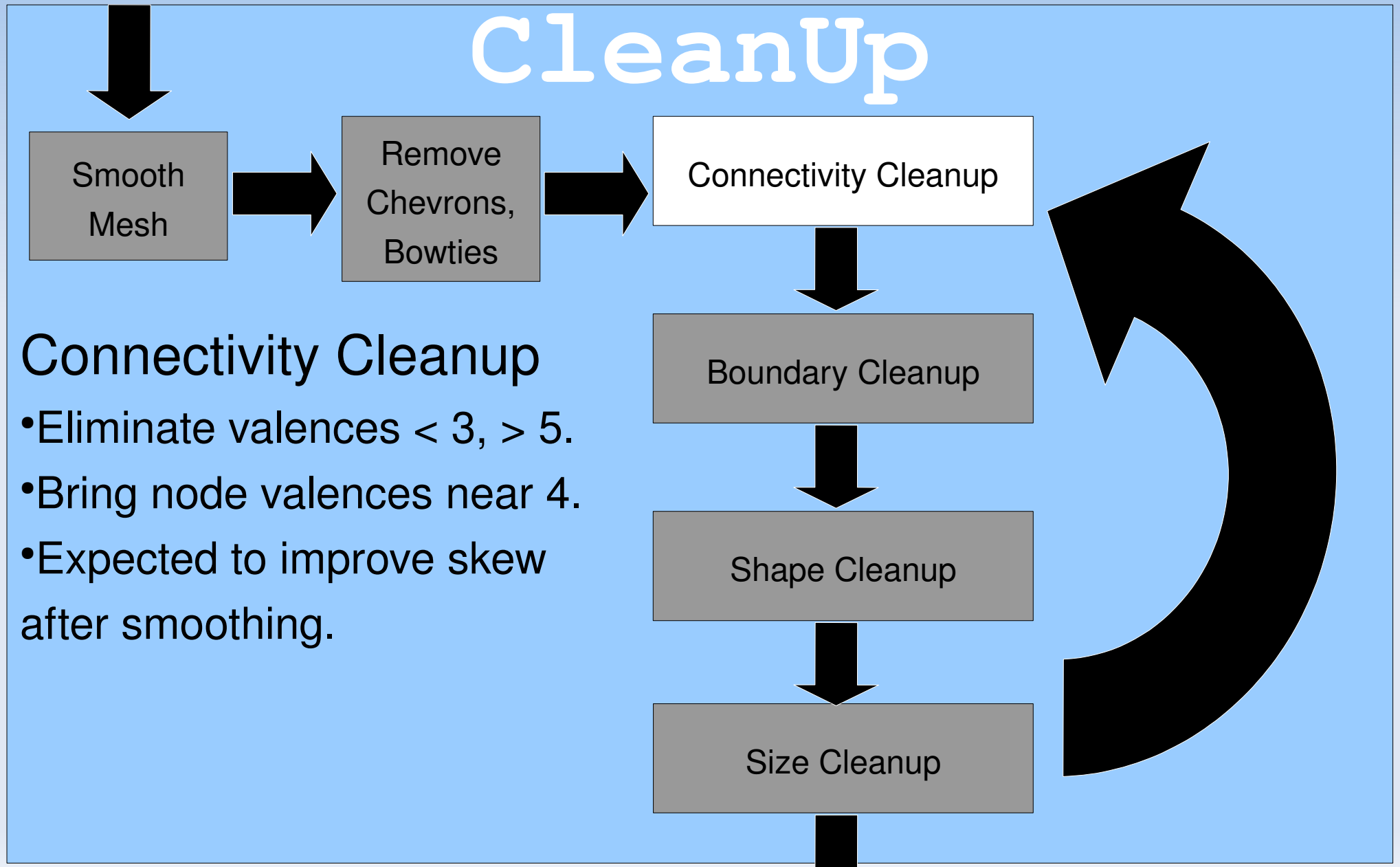
Algorithm Framework



Chevron, Bowtie Removal

- Also done in Shape Cleanup

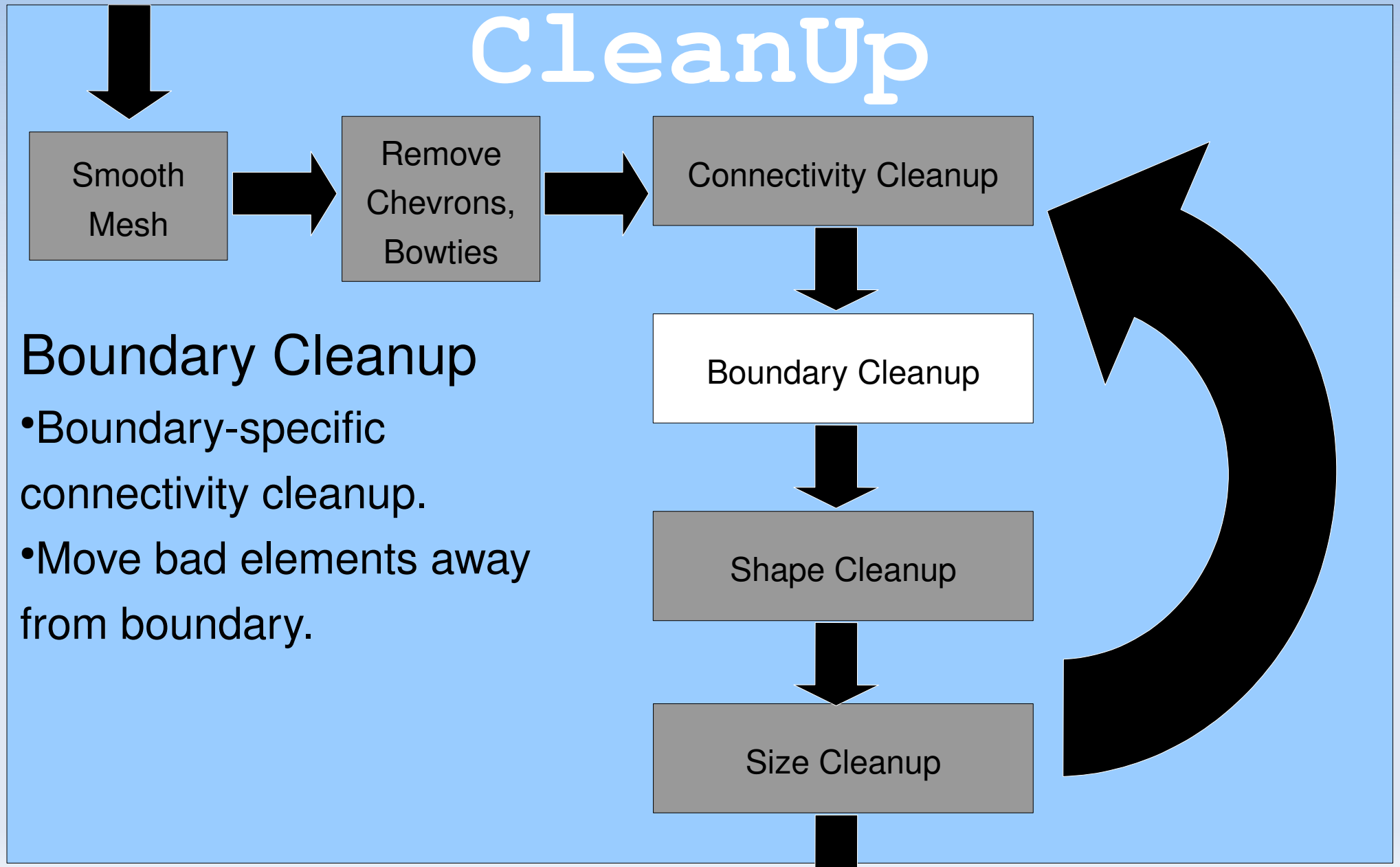
Algorithm Framework



Connectivity Cleanup

- Eliminate valences < 3, > 5.
- Bring node valences near 4.
- Expected to improve skew after smoothing.

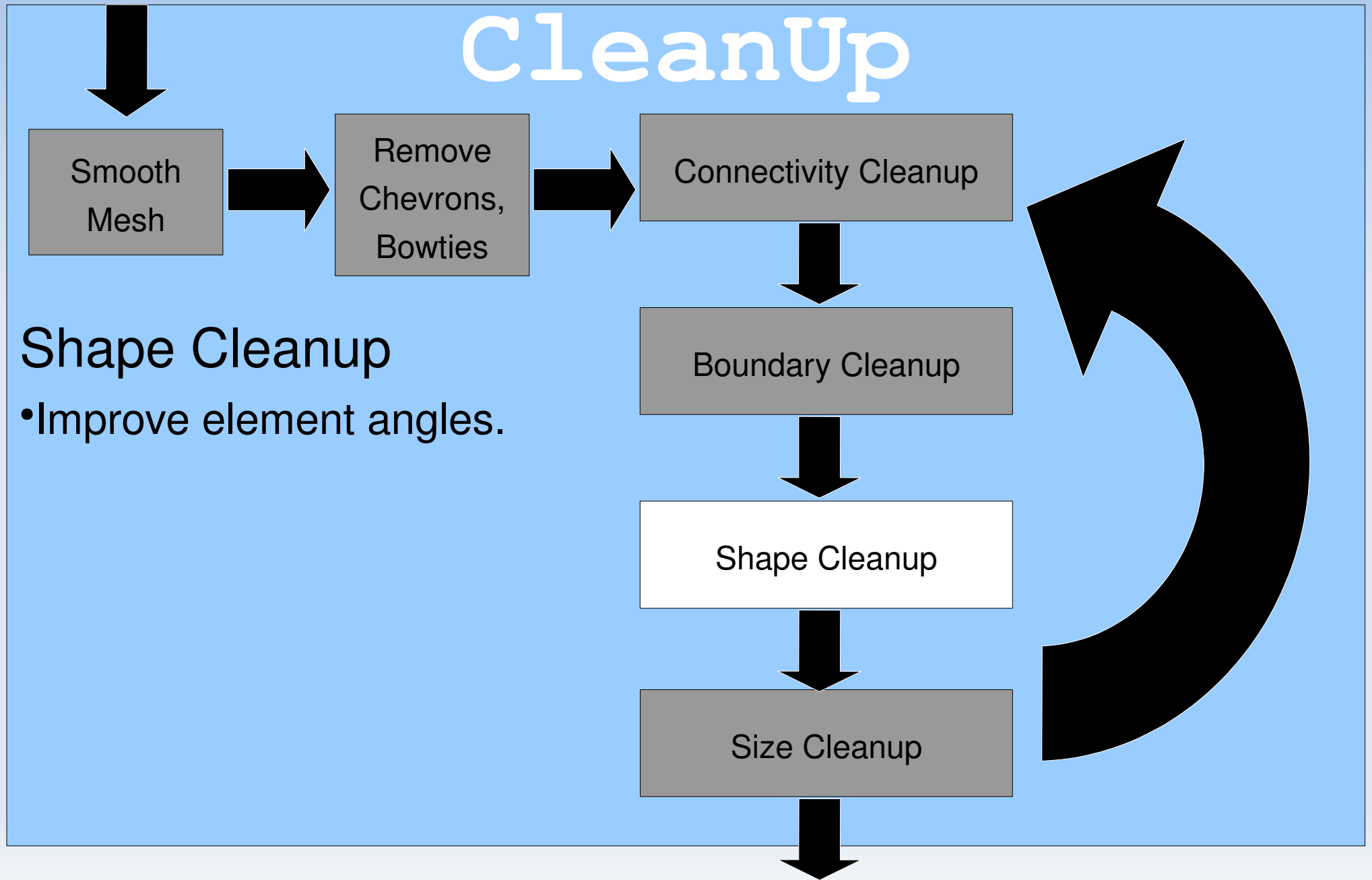
Algorithm Framework



Boundary Cleanup

- Boundary-specific connectivity cleanup.
- Move bad elements away from boundary.

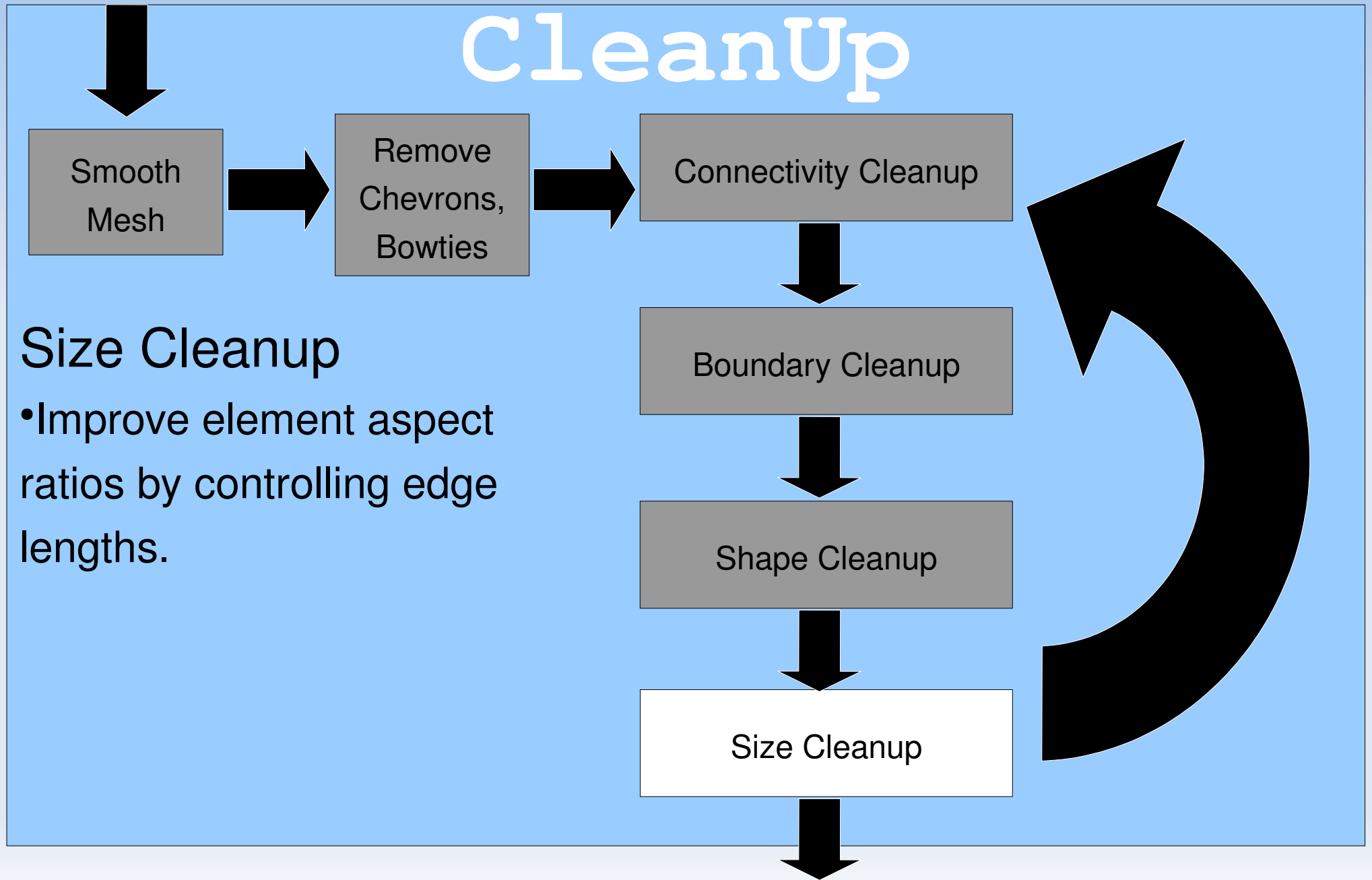
Algorithm Framework



Shape Cleanup

- Improve element angles.

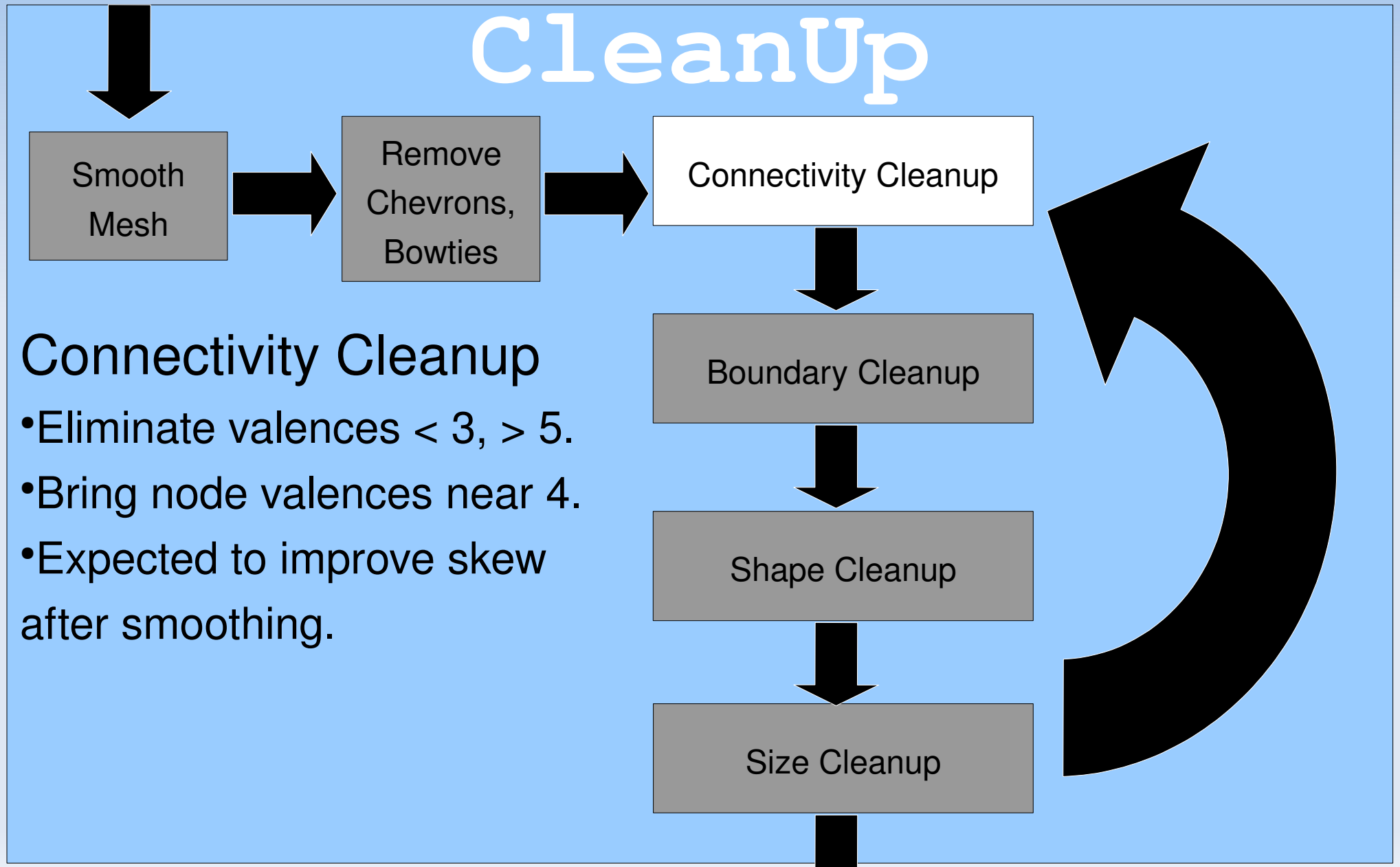
Algorithm Framework



Size Cleanup

- Improve element aspect ratios by controlling edge lengths.

Algorithm Framework



Connectivity Cleanup

- Eliminate valences < 3, > 5.
- Bring node valences near 4.
- Expected to improve skew after smoothing.

Node Valence

A node's **valence** is the number of edges that adjoin it.

A node is **regular** if its valence is 4. Otherwise, it is **irregular**.

Valence and avg. angle:

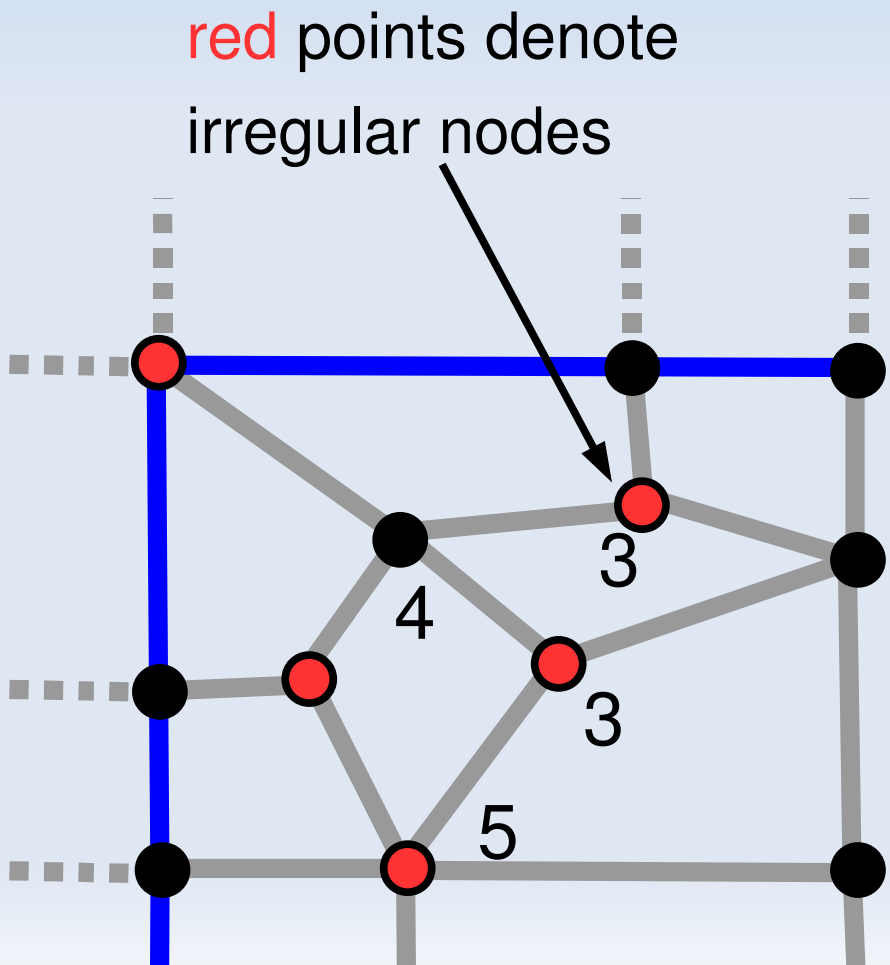
2: 180°

3: 120°

4: 90°

5: 72°

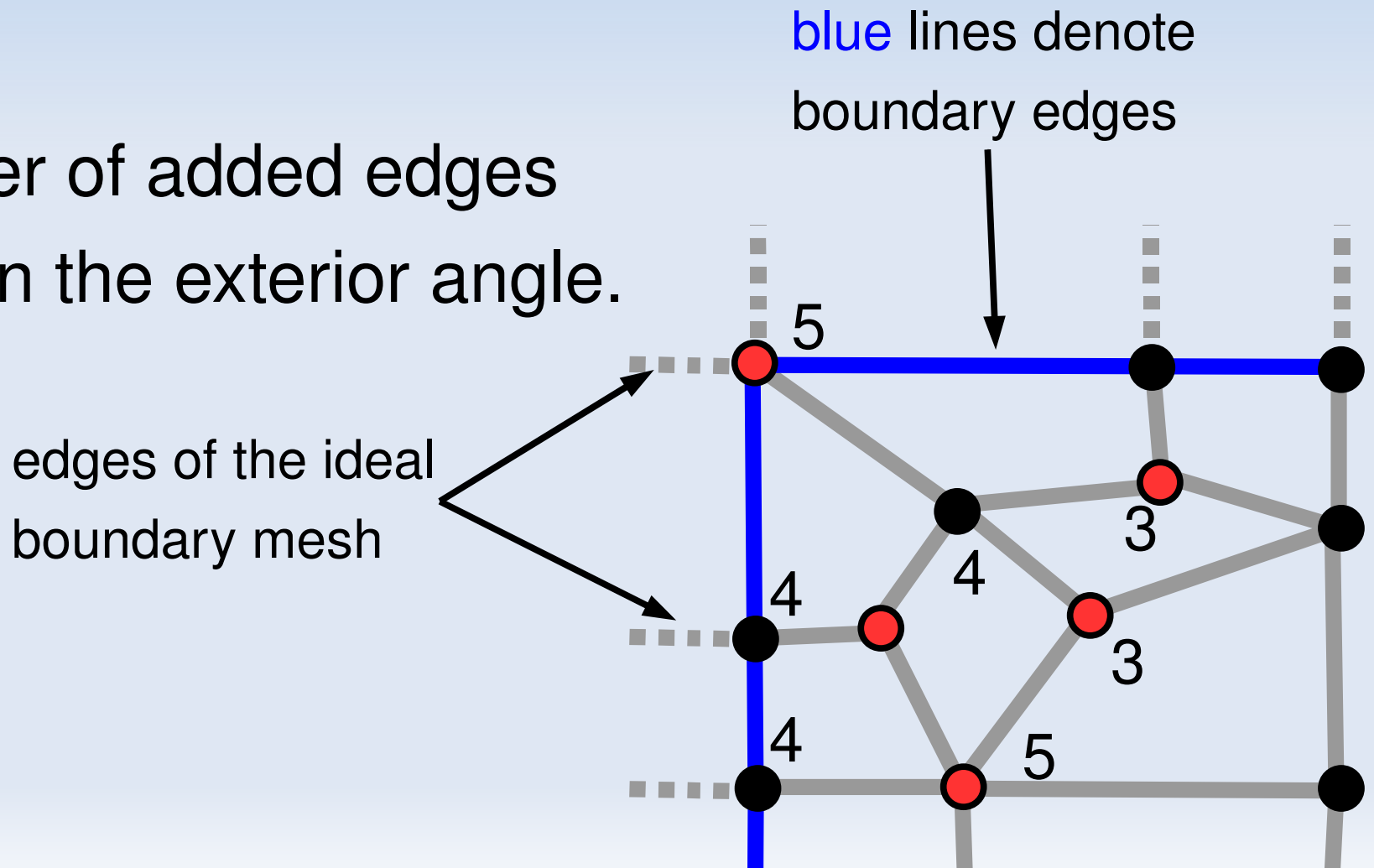
6: 60°



Boundary Node Valence

Imagine that an ideal mesh extends beyond the boundary.

The number of added edges depends on the exterior angle.



Valence Patterns

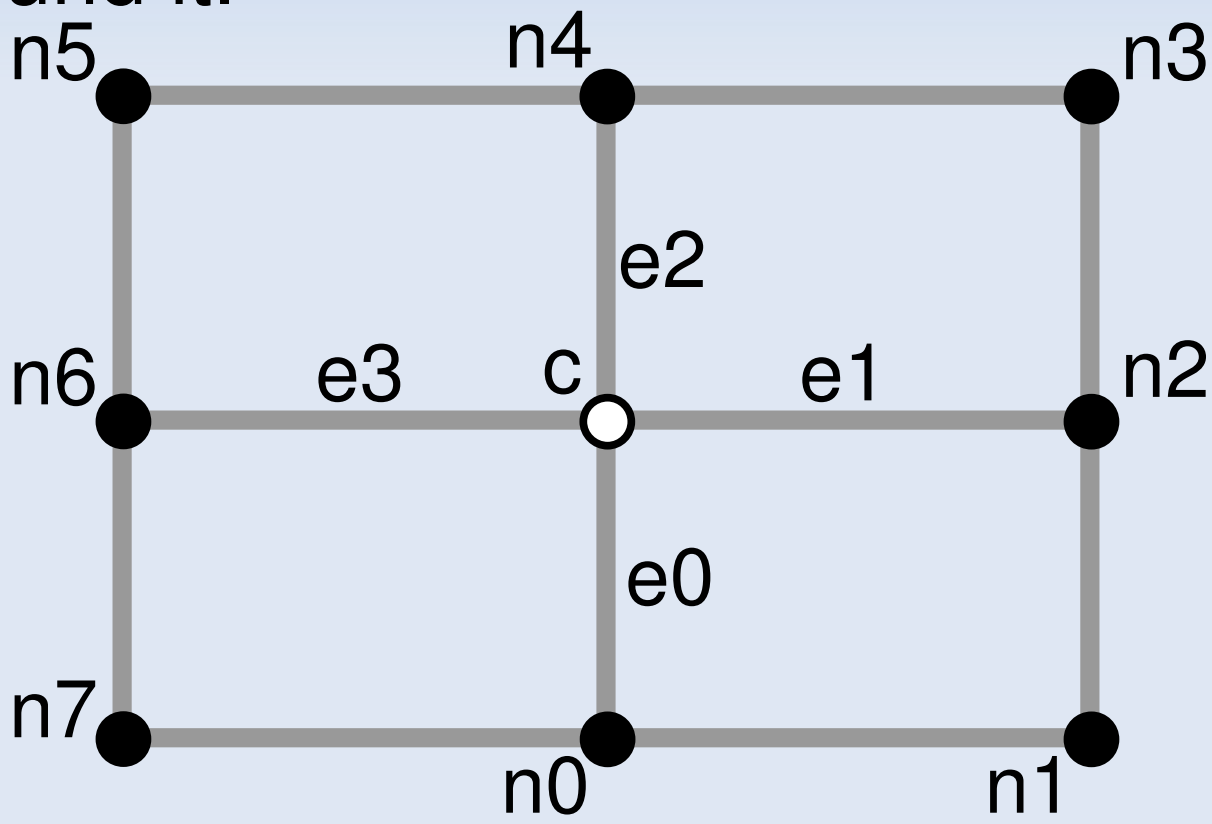
For each interior node c with valence V, we consider the 2V nodes in its link n0, ..., n(2V-1) oriented counterclockwise around it.

n0..n(2V-1)

c's neighboring nodes

e0..e(V-1)

c's neighboring edges



Valence Patterns

Match the valences of c and its neighbors to a known pattern.

4 - 43545000

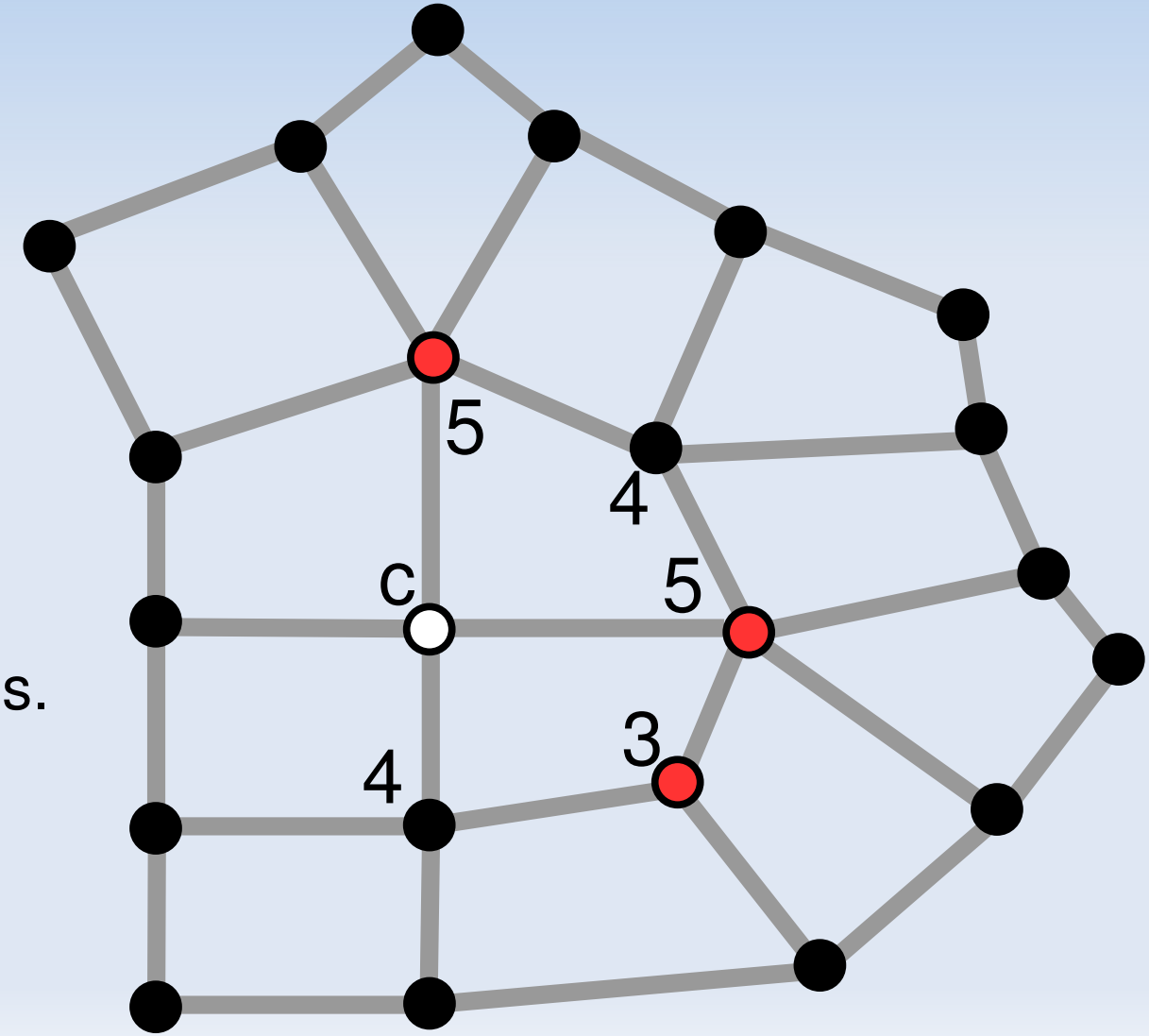
Center node valence

Neighbor node valences.

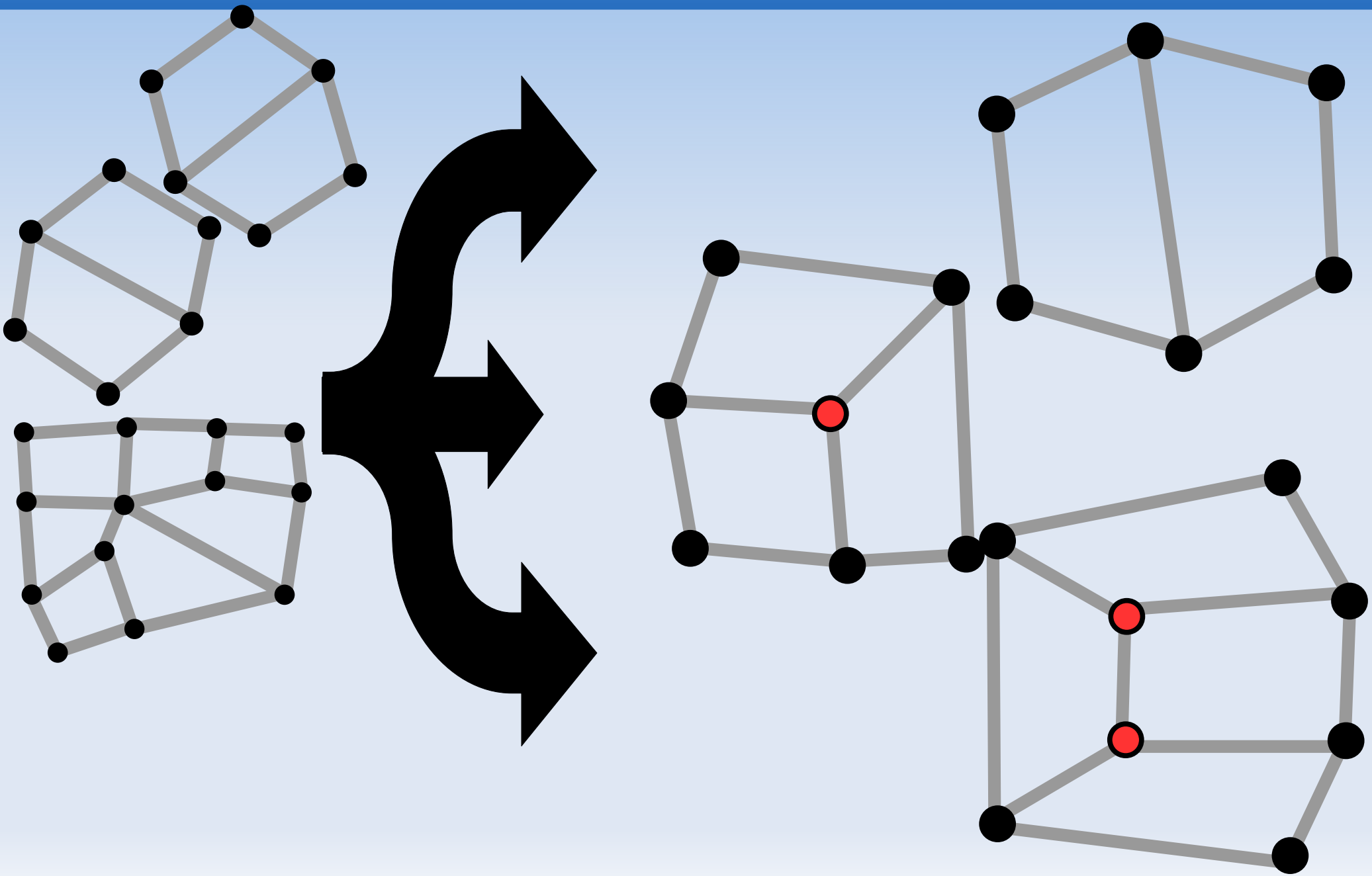
4+/-: 4 or more/less

5: 5 or more

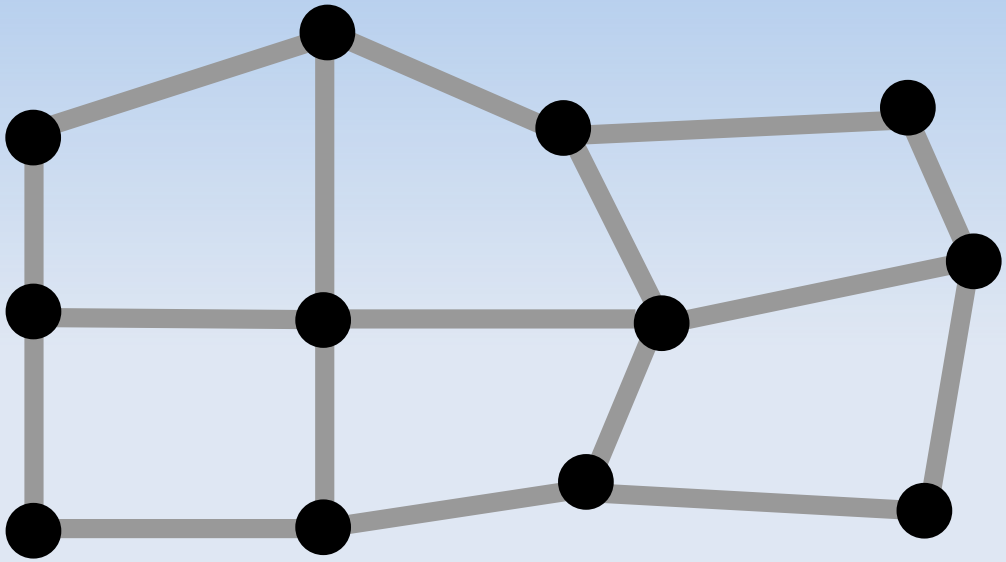
0: don't care.



Cleanup Operations



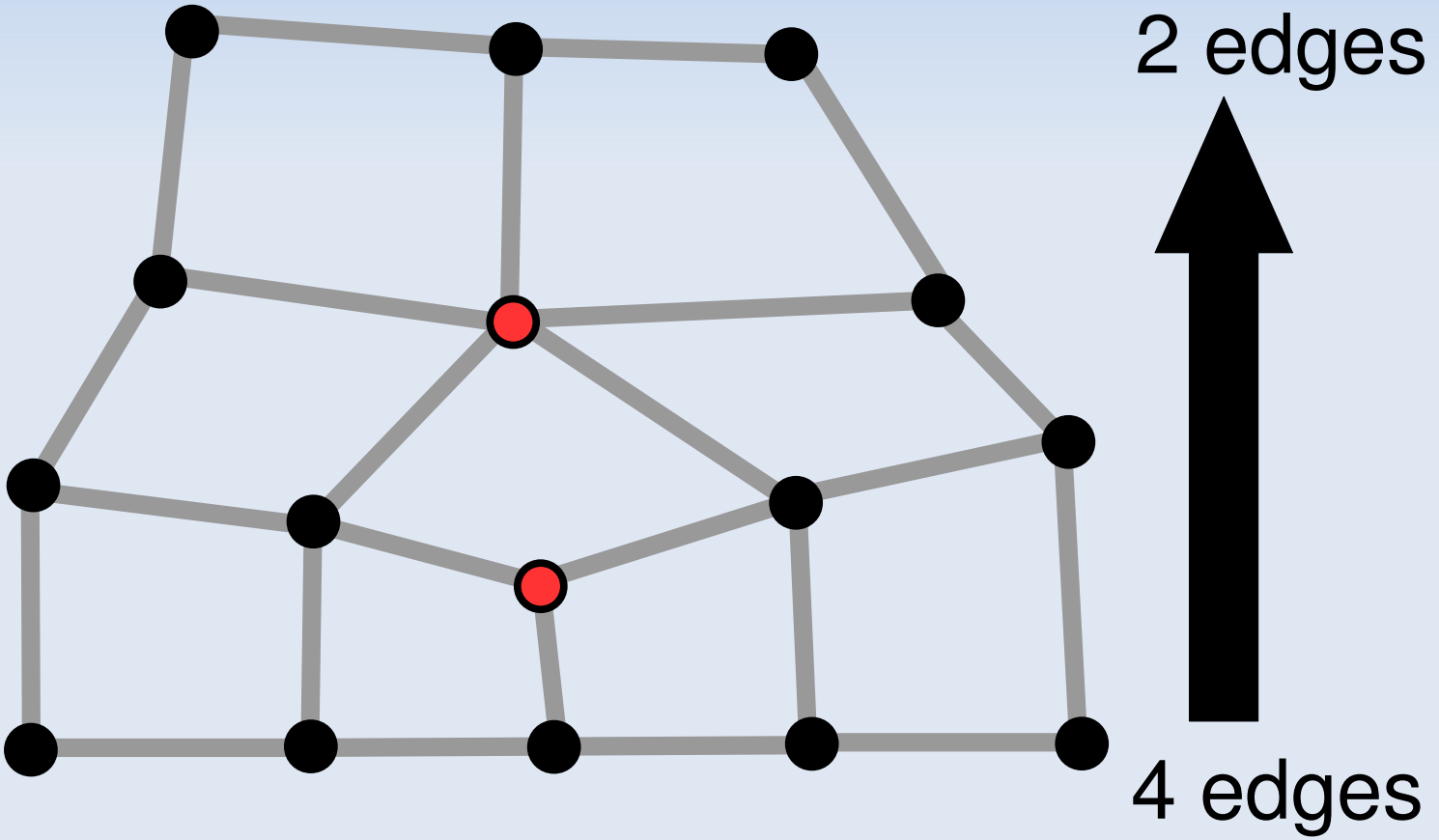
Standard Mesh Patterns



Opposite sides of mesh (top/bottom, left/right) have the same number of edges

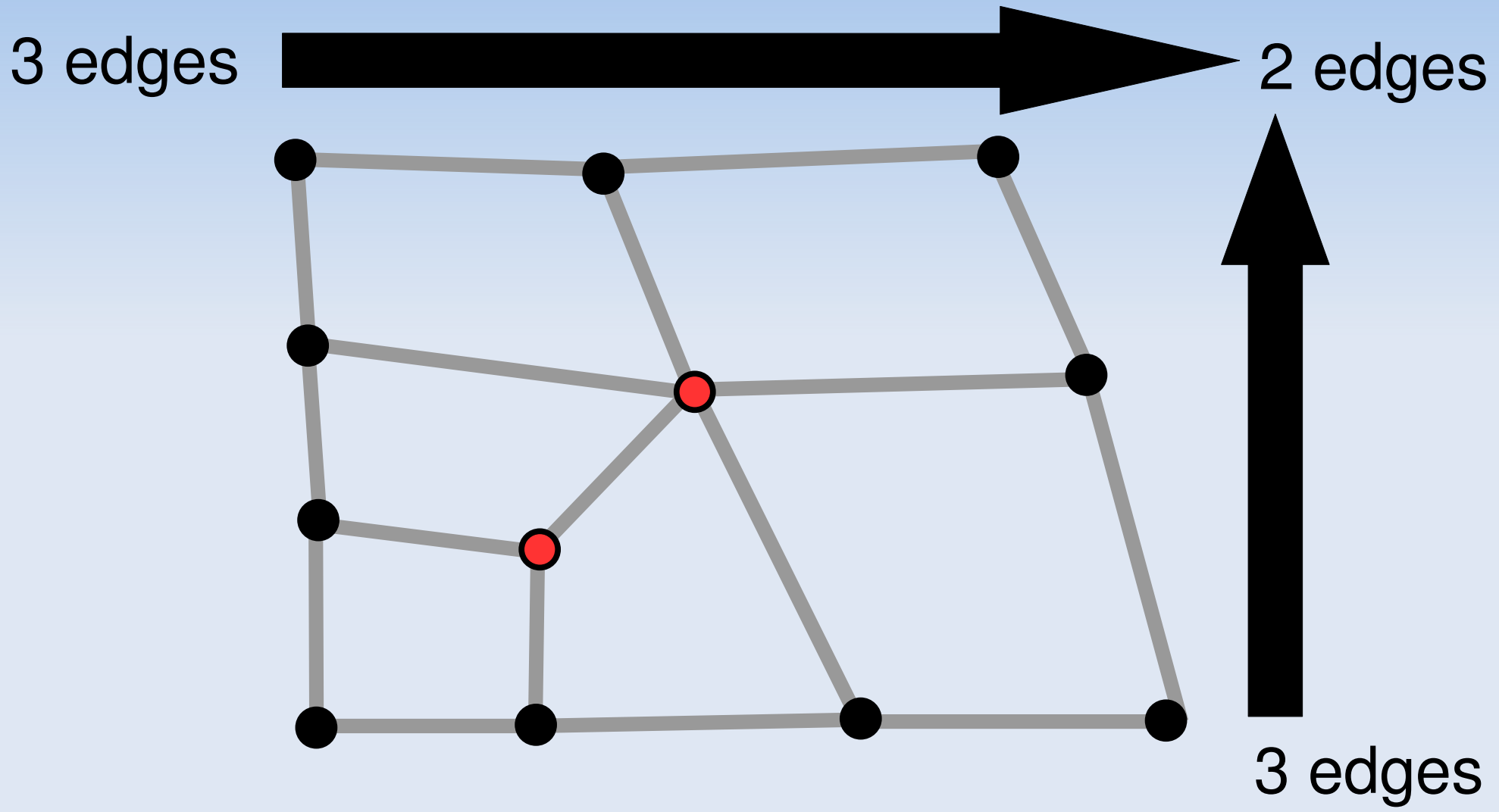
Standard Mesh Patterns

If we must have irregular nodes, it is because there is some transition across the mesh.



One pair of opposite sides of mesh differ by 2.

Standard Mesh Patterns



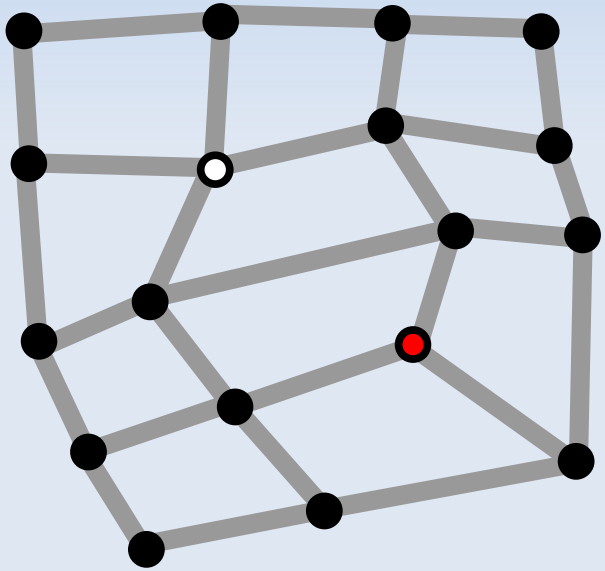
Both pairs of opposite sides of mesh differ by 1.

Connectivity Cleanup

Put together (by hand) a mapping:

Local situations in the mesh → Better local situations in the mesh

5 - 3443000000 →



...

...

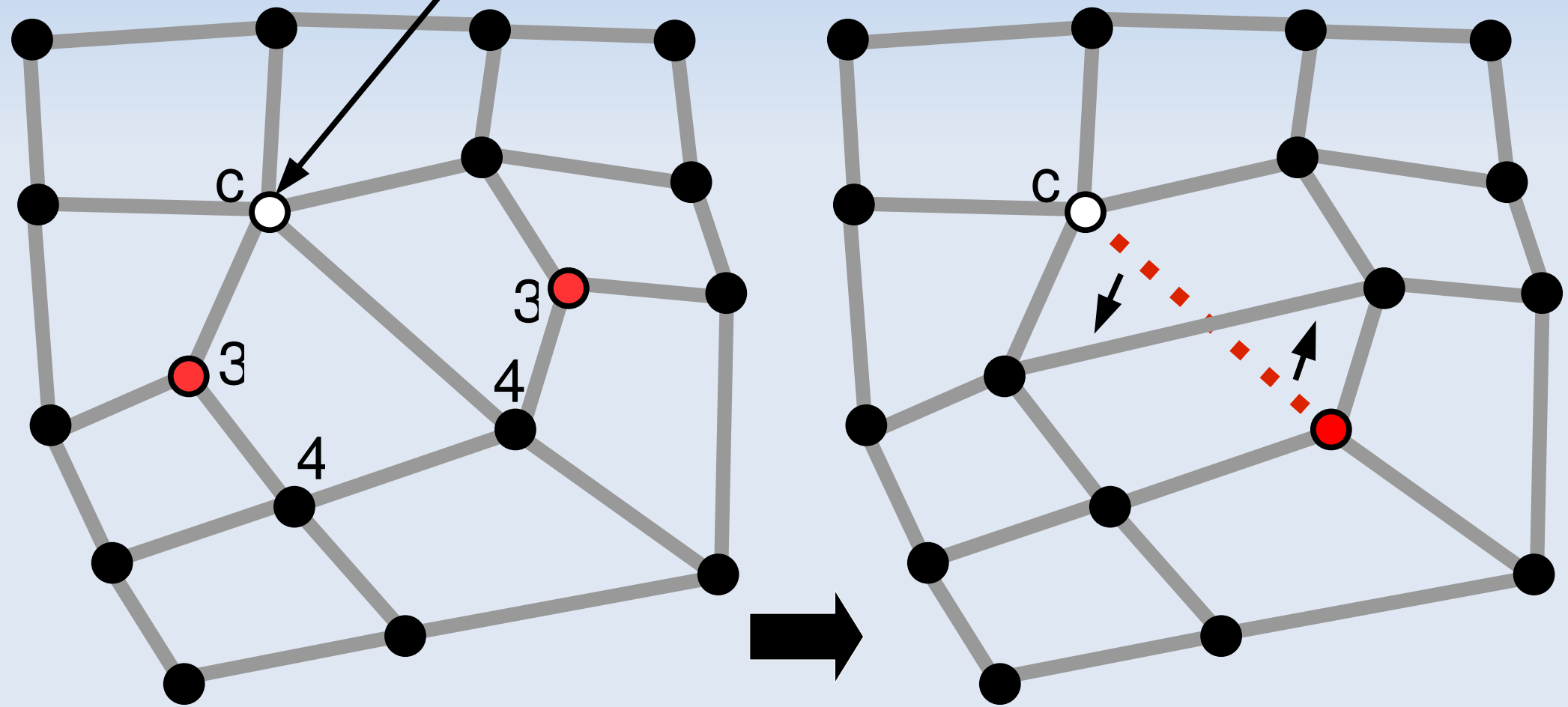
Connectivity Cleanup Operators

- CleanUp recognizes 64 different connectivity patterns, mapping to 27 different actions.

Here are a few...

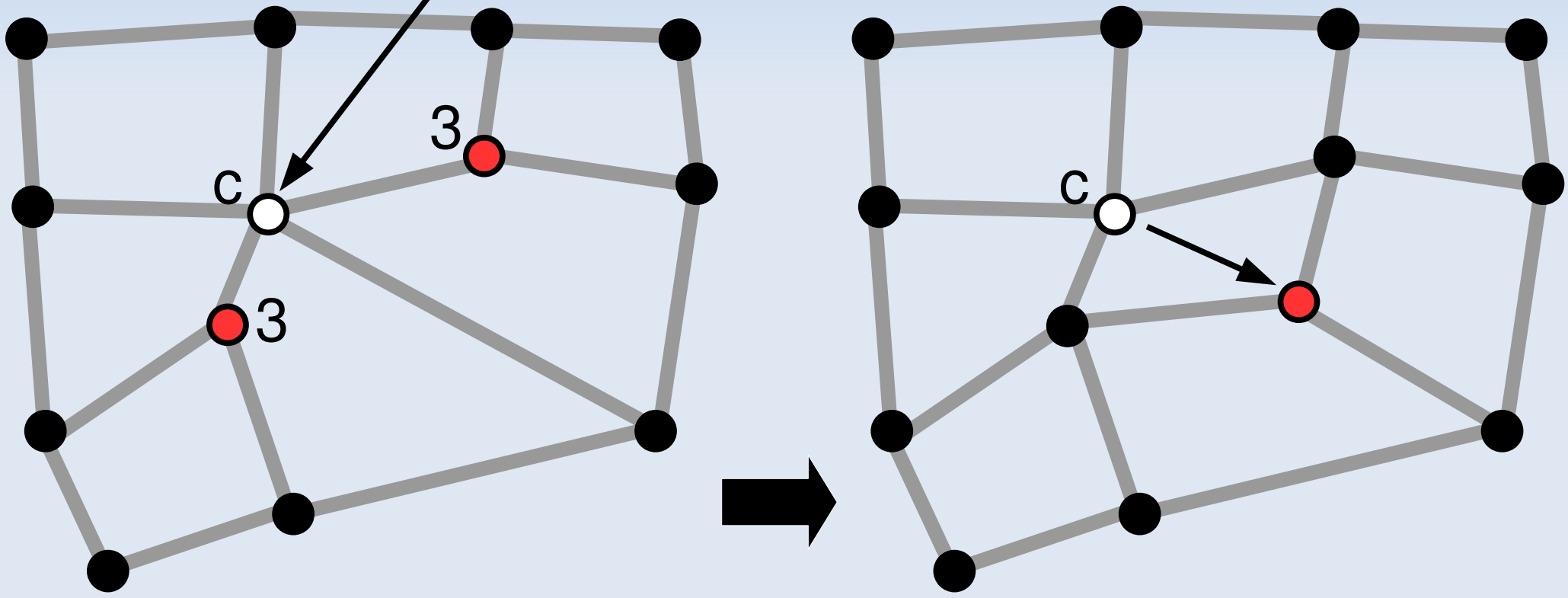
Switch Diagnoal

Case 5 - 3443000000

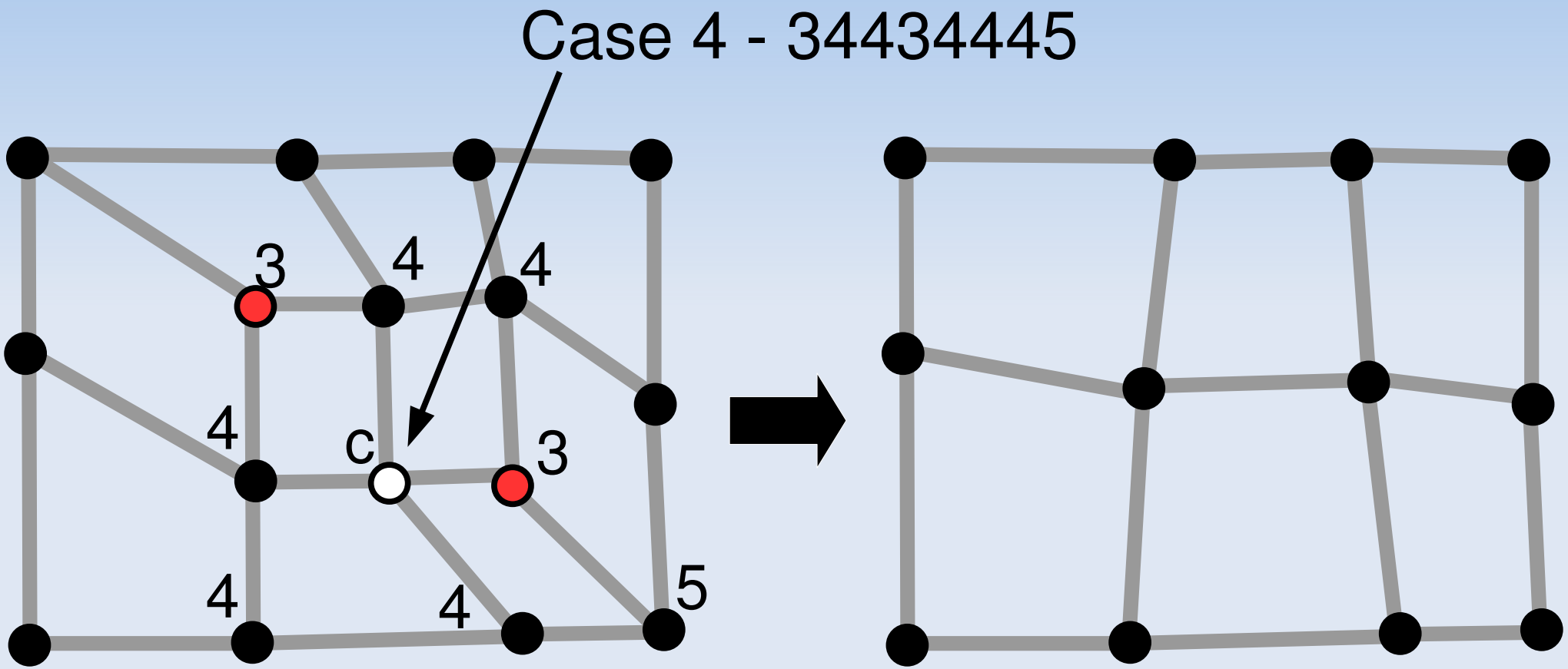


Element Open

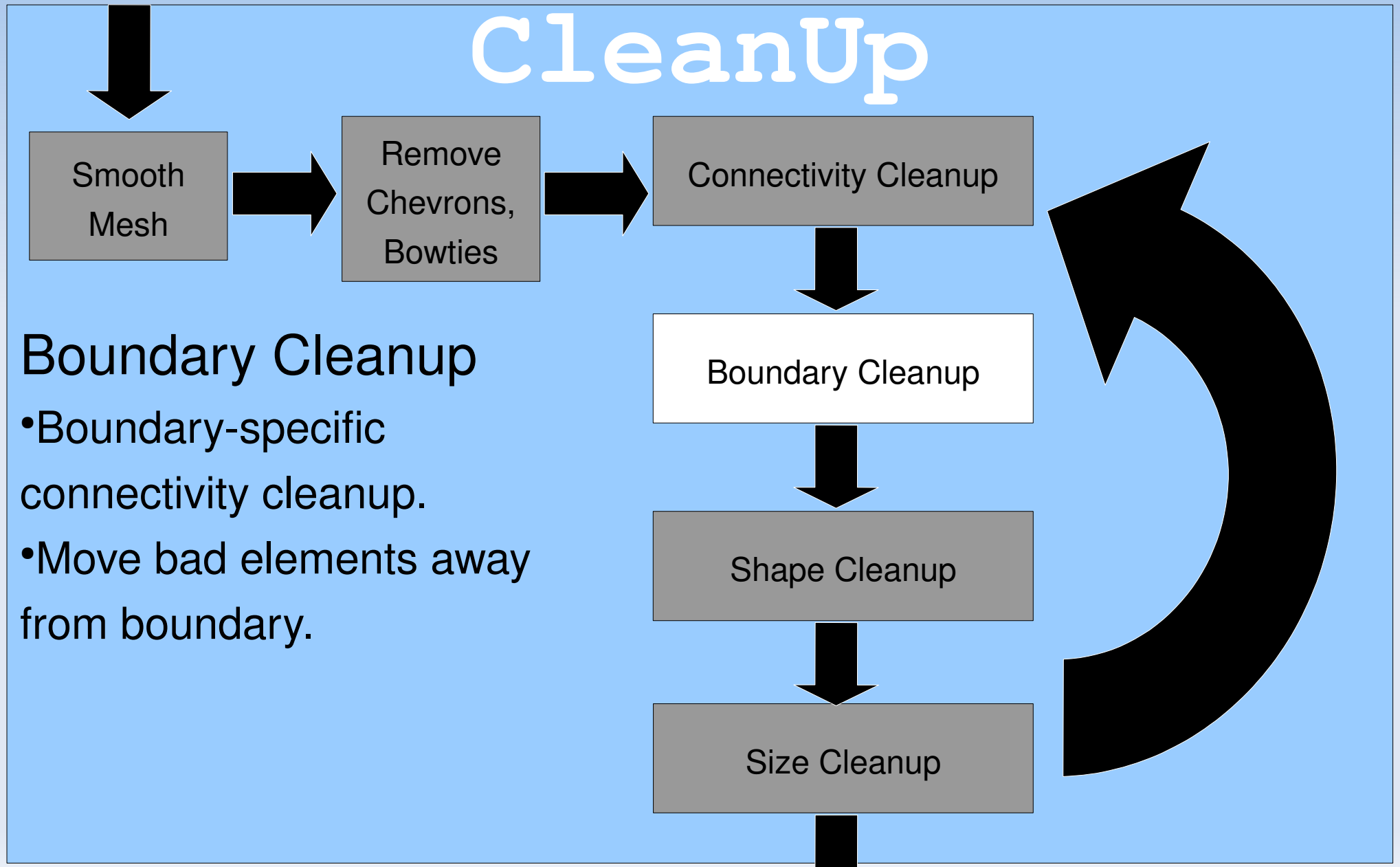
Case 5 - 3030000000



Complete Remeshing



Algorithm Framework



Boundary Cleanup

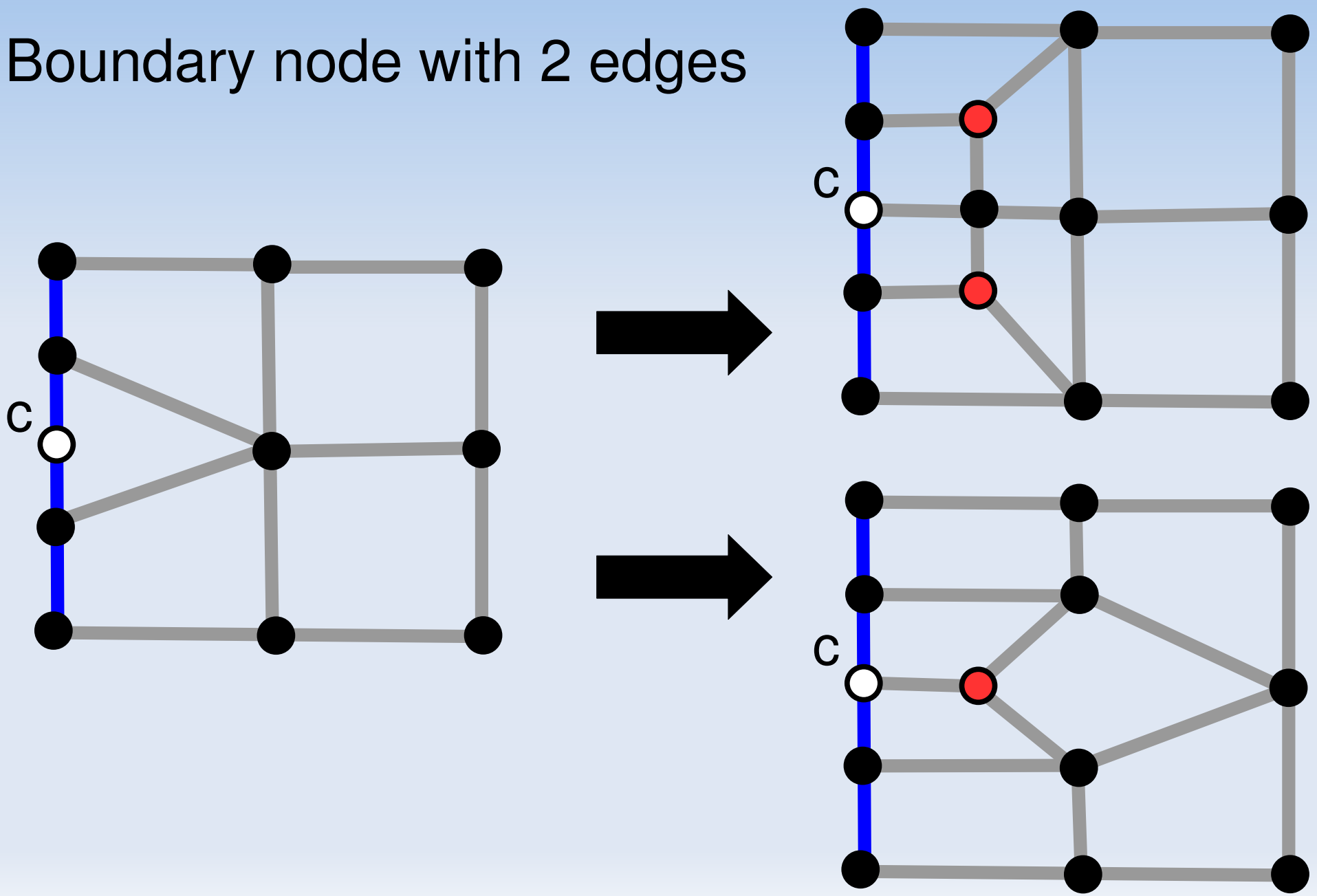
- Boundary-specific connectivity cleanup.
- Move bad elements away from boundary.

Boundary Cleanup

- Valence patterns are used to detect some of the improvable situations in the mesh.
  - Unlike before, valence patterns may not be oriented arbitrarily.
- Additional goal: Move irregular nodes away from the boundary.

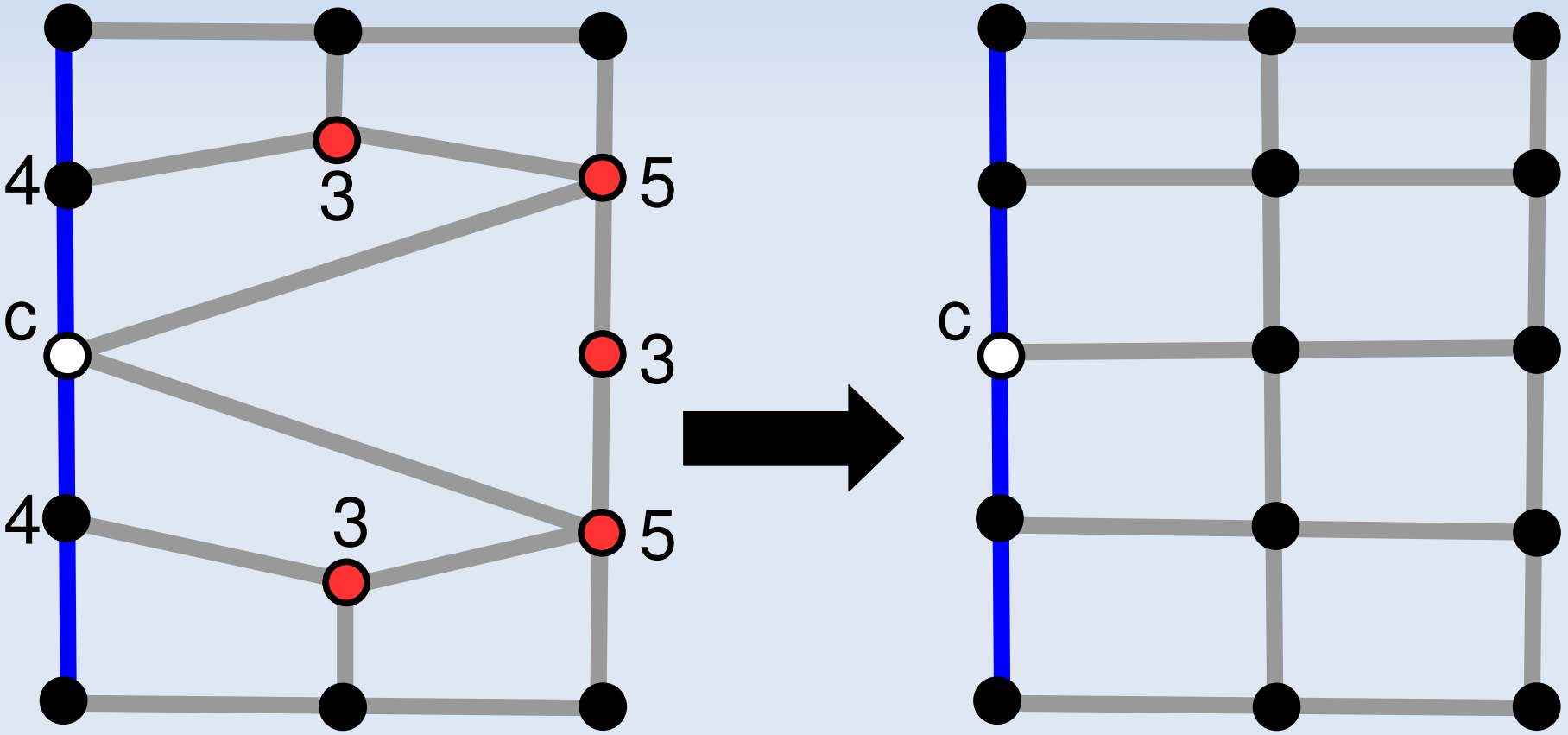
Boundary Cleanup

Boundary node with 2 edges



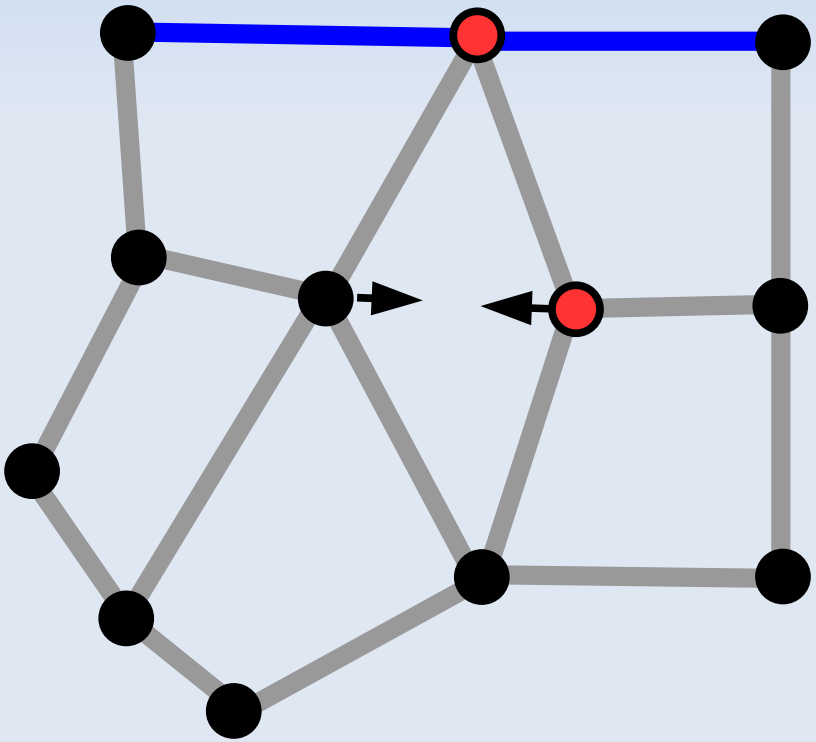
Boundary Cleanup

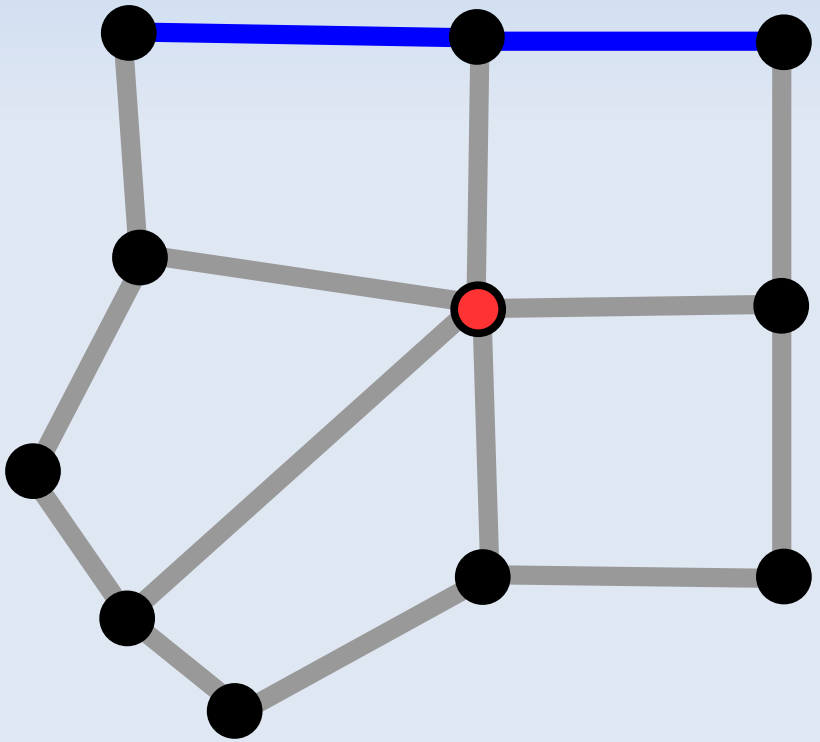
There are more than 12 boundary-specific connectivity cleanup cases. Here's one:



Boundary Cleanup

Boundary “diamonds” mean the boundary node's degree is too large.

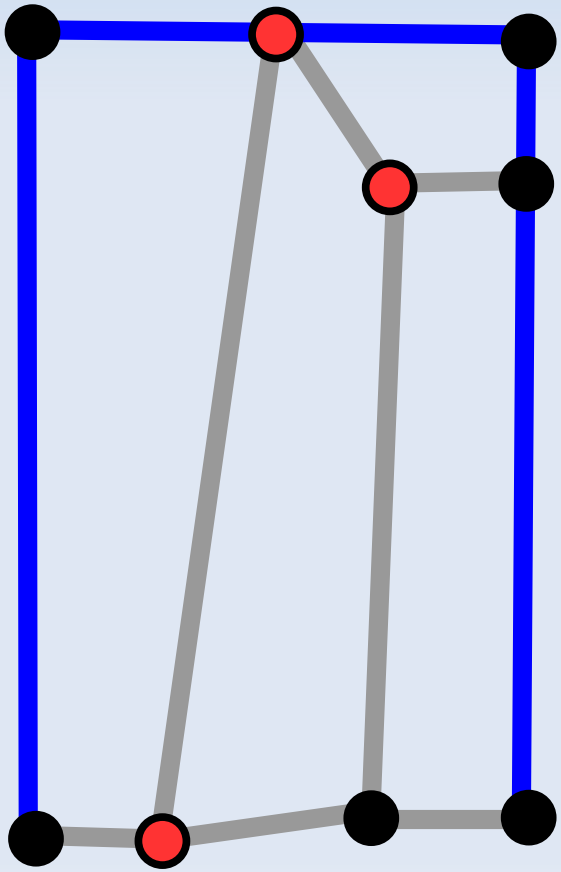


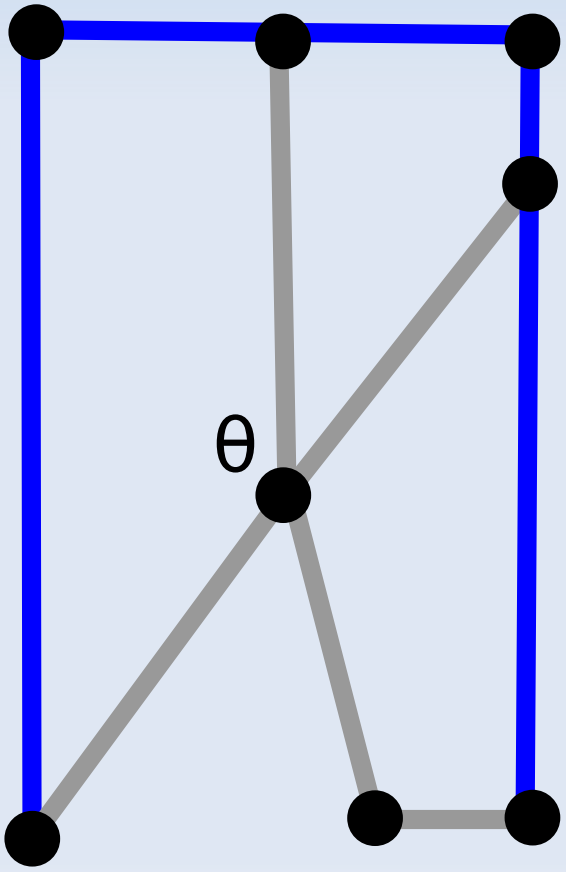


Boundary Cleanup

But some boundary diamonds should not be collapsed.

The angle θ could become arbitrarily close to 180.





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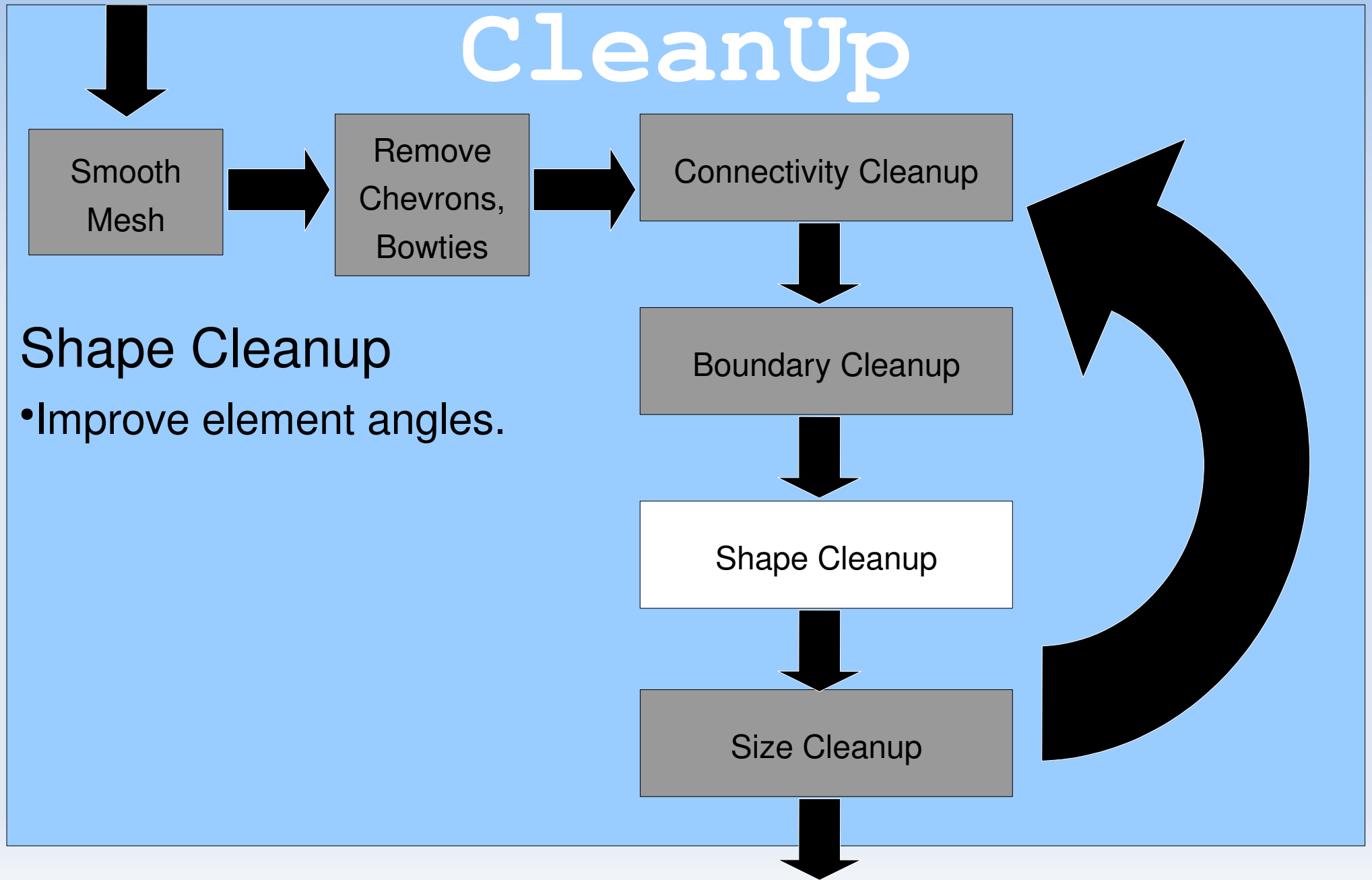


Shape Cleanup

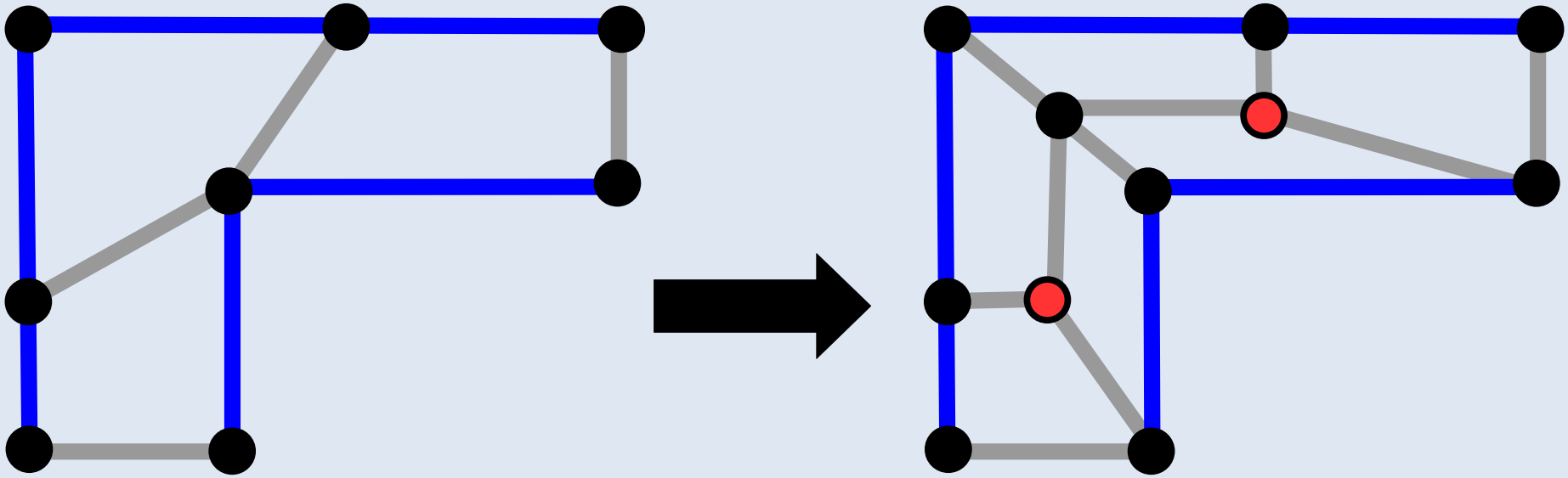
- Improve element angles.

Shape Cleanup

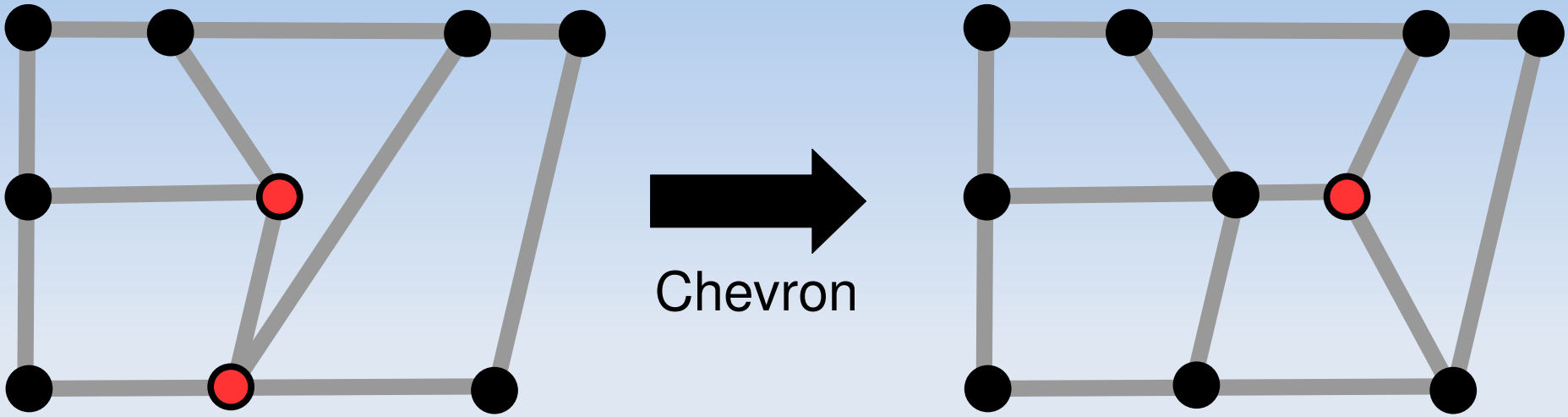
- Shape cleanup targets angles > 160°.
- Remove chevrons (arrowheads) and bowties.
- Important: smooth before and after each shape cleanup operation

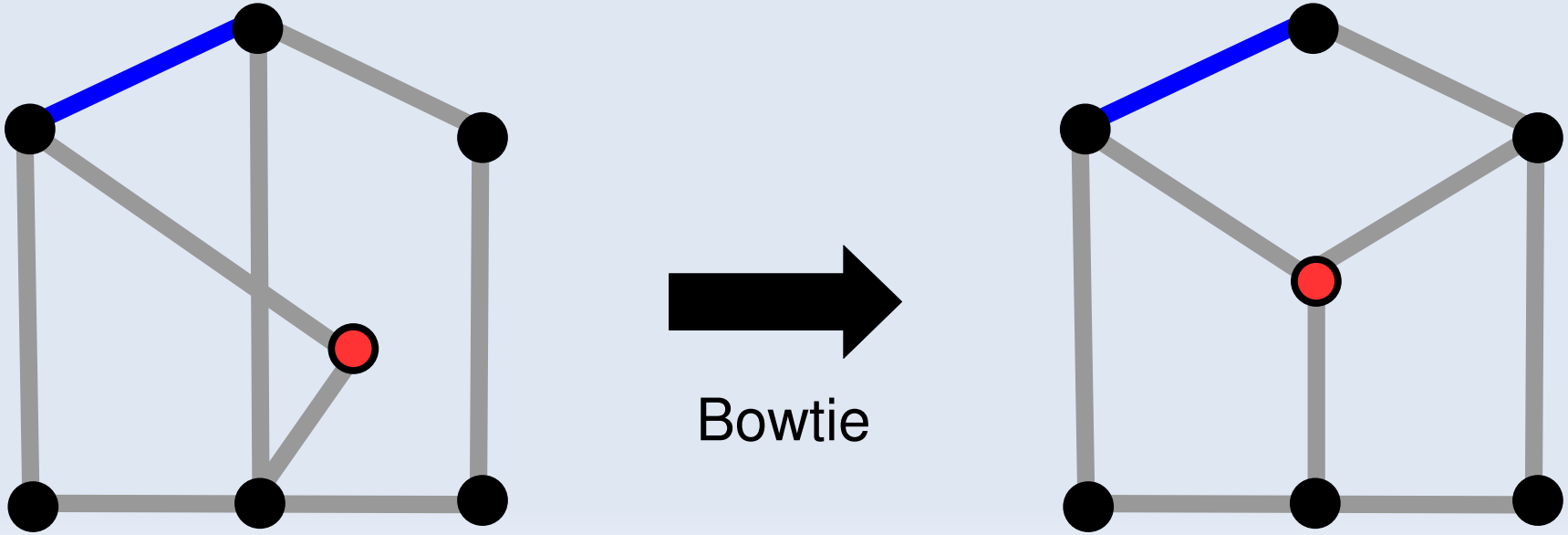
Eliminating Large Angles

Large angles are removed by combining neighbors and remeshing.



Chevrons and Bowties





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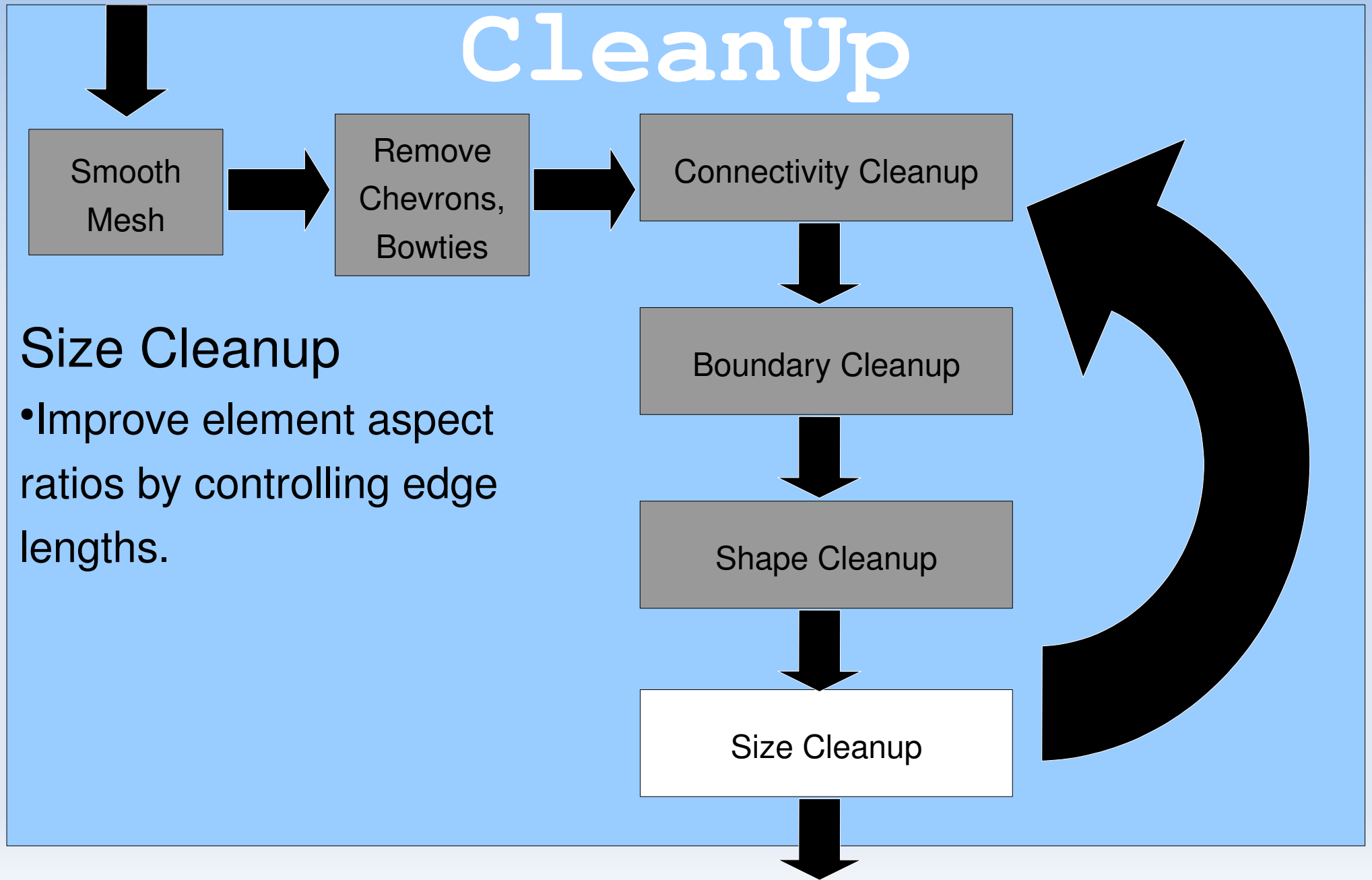


Size Cleanup

- Improve element aspect ratios by controlling edge lengths.

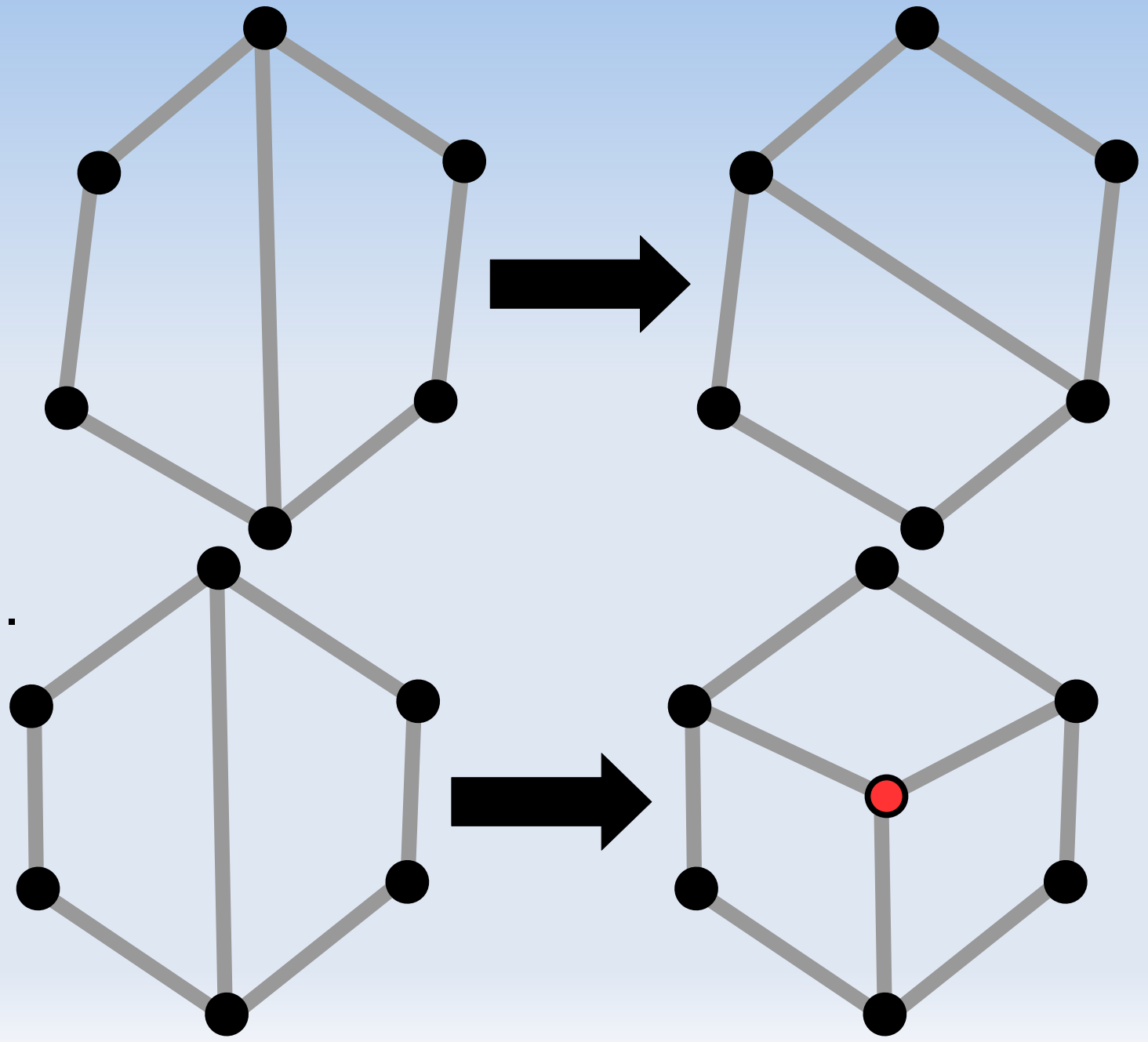
Size Cleanup

- Bring edge lengths closer to some desired size.
- Edge lengths provided by background function.

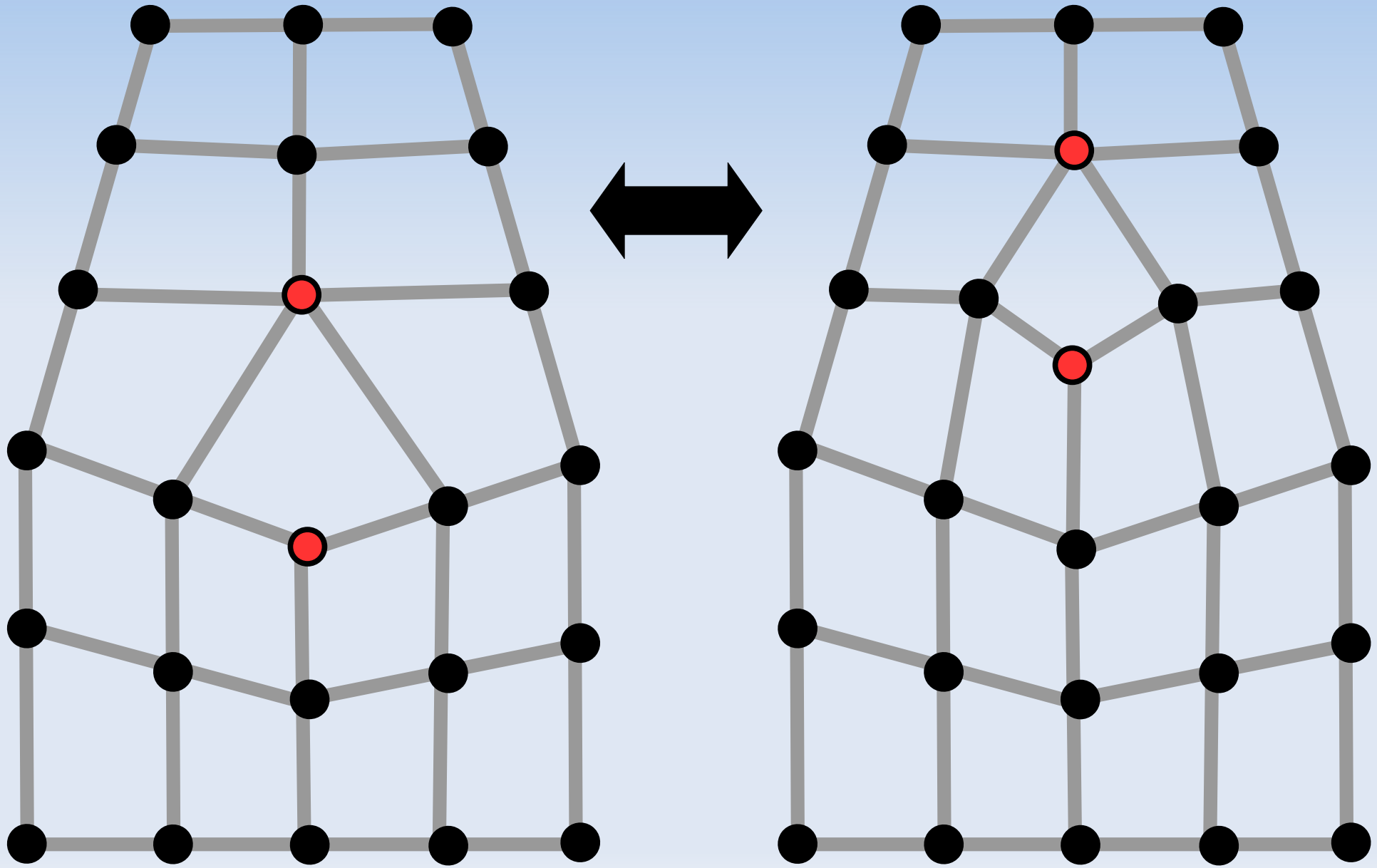
Edge Length Operations

Where edge is >2.5x goal size, combine with neighbor.

Either:
Edge rotation (top).
Three smaller quads (bottom).



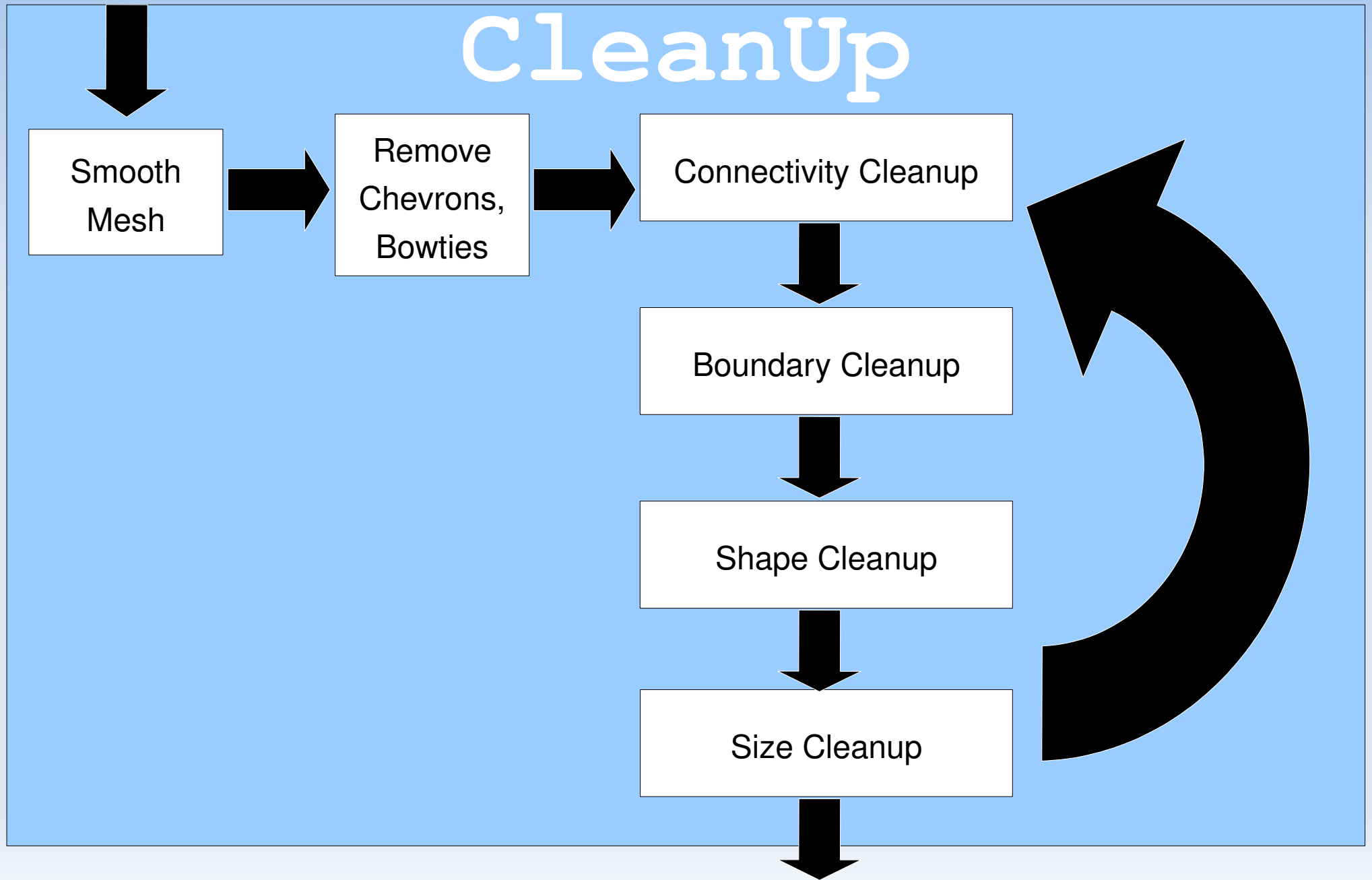
Edge Length Operations



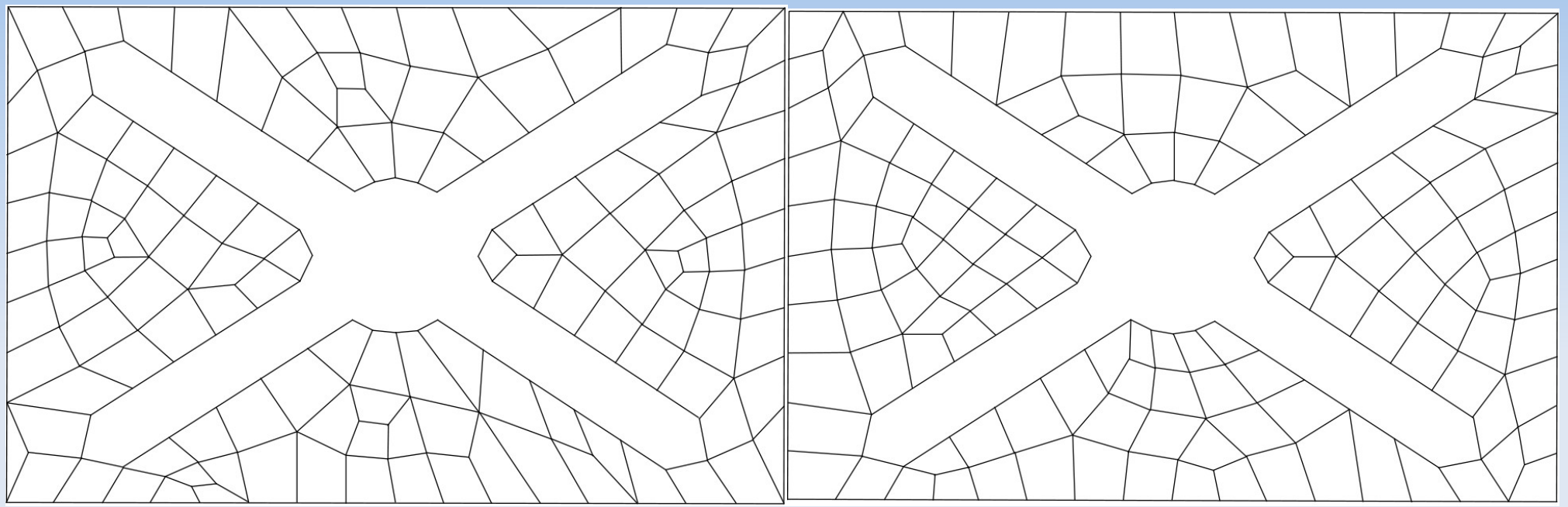
Termination

- Because cleanup stages have different goals, the mesh may oscillate and the program will continue finding work to do.
- The author suggests 3 iterations of the cleanup operations before termination.

Algorithm Framework



Results



Avg. Skew: 27°

Irregular Nodes: 42

4 quads w/ 3 collinear nodes

1 6-valent node

Avg. Skew: 23°

Irregular Nodes: 28

0 quads w/ 3 collinear nodes

0 6-valent nodes

Questions?

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