CS 274
Computational Geometry (Spring 2005)
Homework 5

Homework 5 is due (under my office door or in my CS mailbox) **at 5 pm on Friday, May 13, 2005**.

Where appropriate, you may use algorithms learned in class as subroutines without re-explaining them.

[1] Overlapping intervals (7 points).

(i) Let S be a set of n intervals. Describe a data structure that uses $\mathcal{O}(n)$ storage and returns the *number* of intervals in S that intersect any query interval I in $\mathcal{O}(\log n)$ time. (Note that I didn't say "$\mathcal{O}(k+\log n)$ time.") Hint: my answer modifies one of the data structures taught in class, but I can't think of any way to do the query fast enough with an interval tree.

(ii) Explain how to use a two-dimensional layered range tree to answer the same counting query, also in $\mathcal{O}(\log n)$ time. (Assume you have a correct solution to problem [5] of Homework 4; there's no need to explain that solution again. Observe that a two-dimensional range tree generally requires $\Theta(n \log n)$ space, so this isn't an answer to part (i).)

(iii) Let D be a disk, and let L be a set of l lines in the plane. Describe an algorithm that returns, in $\mathcal{O}(l \log l)$ time, the number of pairs of lines that intersect each other inside D. For simplicity, you may assume that no two lines intersect each other precisely on the boundary of D. Hint: as best I can figure, this can be done with the range tree but not with the answer to part (i).

[2] Voronoi diagrams (4 points). The worst-case complexity of a Voronoi diagram is $\Theta(n^2)$ in three dimensions, and $\Theta(n^{\lceil d/2 \rceil})$ in E^d.

(i) Prove that a planar cross-section of a d-dimensional Voronoi diagram—in other words, the planar subdivision formed by intersecting a 2-flat with a Voronoi diagram—has complexity no greater than $\mathcal{O}(n)$. Hint: What do you know about the properties of Voronoi cells?

(ii) Does a planar cross-section of a d-dimensional Delaunay triangulation also always have a complexity in $\mathcal{O}(n)$? Explain.

[3] Point-rectangle pairs (4 points). Let P be a set of m points in the plane, and let R be a set of n axis-parallel rectangles (possibly intersecting).

Section 10.3 of the Dutch Book includes an algorithm INSERTSEGMENTTREE that inserts an interval into a segment tree (that already contains the endpoints of the interval) in $\mathcal{O}(\log n)$ time. An interval can also be deleted in $\mathcal{O}(\log n)$ time, if a little extra information is stored. Recall that each interval is stored in lists in up to $\mathcal{O}(\log n)$ nodes of the tree. By making these lists doubly-linked, and by maintaining for each interval a list of pointers to the interval's nodes in these doubly-linked lists, the interval can be removed from each list in constant time per list.

Describe an $\mathcal{O}(k + (m + n) \log n)$-time algorithm that reports all pairs $\langle p \in P, r \in R \rangle$ such that $p \in r$, where k is the number of such pairs.

[4] Deterministic BSPs (5 points). Give a deterministic (no randomness) algorithm that always constructs a BSP tree of size $\mathcal{O}(n \log n)$ for a set of n non-crossing line segments in the plane. Hint: Forget about auto-partitions. Use free splits wherever possible and vertical splitting lines otherwise. Expect the analysis to be a bit tricky.

[5] Minkowski sums (5 points). Suppose we want to automatically machine a two-dimensional part by cutting away the region around it with a circular drill bit. Let D be a disk whose center is the origin and whose radius is the radius of the drill bit. When machining a shape S, we may place the center of the drill bit at any point not in the interior of $S \oplus D$.

(i) Let P_1 and P_2 be convex polygons with nonintersecting interiors. Show that $P_1 \oplus D$ and $P_2 \oplus D$ are pseudodisks. (Note: because of the special properties of a circle, your proof should be simpler than the proof given for convex polygons in the book or in class.)

(ii) If P is an n-vertex simple polygon (not necessarily convex), what is the worst-case asymptotic complexity of $P \oplus D$? (The complexity is the total number of vertices, line segments, and circular arcs needed to represent $P \oplus D$.) Briefly justify your answer. (A rigorous proof is not necessary.)