CS170 | Problem Set 13 | Out: Apr 21, 2001
Due: Apr 27, 2001

1. Prove that the intersection point of a line $\overleftrightarrow{p_1p_2}$ and a line $\overleftrightarrow{p_3p_4}$ is given by

$$p = p_1 + \alpha(p_2 - p_1),$$

where

$$\alpha = \frac{\begin{vmatrix} x_3 - x_1 & y_3 - y_1 \\ x_4 - x_1 & y_4 - y_1 \end{vmatrix}}{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_4 - x_3 & y_4 - y_3 \end{vmatrix}}.$$

Hint: The intersection point must be expressible as both $p_1+\alpha(p_2-p_1)$ and $p_3+\beta(p_4-p_3)$. Write the resulting system of linear equations in matrix form (with unknowns α and β), then solve for α using Cramer's Rule (which you should remember from linear algebra).

2. Let p_1, p_2, and p_3 be three points in the plane.

   (a) If the line $\overleftrightarrow{p_1p_2}$ is not vertical, it can be expressed using the classic line equation $y = mx + b$. What are the values of m and b? (Write your answer in terms of x_1, y_1, x_2, and y_2.)

   (b) Prove that the signed area of the triangle $\triangle p_1p_2p_3$ is

$$\frac{\begin{vmatrix} x_2 - x_1 & x_3 - x_1 \\ y_2 - y_1 & y_3 - y_1 \end{vmatrix}}{2}.$$

   Note that this is half the signed area of the parallelepiped formed by the vectors $p_2 - p_1$ and $p_3 - p_1$.
   Hint: Assume without loss of generality that $x_1 \leq x_3 \leq x_2$ (we can achieve this by rotating the labels, without changing the signed area). Compute by integration the area between the triangle's "upper" contour and its "lower" contour, where the "upper" contour consists of the edges $\overline{p_1p_3}$ and $\overline{p_3p_2}$, and the "lower" contour consists of the edge $\overline{p_1p_2}$ (so that the integral will turn out positive if p_3 is above $\overline{p_1p_2}$, and negative if p_3 is below). You can express the contours as functions of x using your answer to part a.

3. CLR Exercise 35.1-4.

4. CLR Exercise 35.1-6.

5. CLR Exercise 35.2-1.