Problem Set 9 for CS 170

Problem 1 [Interleaving]

Given a string x consisting of 0s and 1s, we write x^k to denote k copies of x concatenated together. We say that a string x' is a *repetition* of x if it is a prefix of x^k for some number k. So $x' = 10110110110$ is a repetition of $x = 101$.

We say that a string s is an *interleaving* of x and y if its symbols can be partitioned into two (not necessarily contiguous) subsequences s' and s'', so that s' is a repetition of x and s'' is a repetition of y. (So each symbol in s must belong to exactly one of s' or s''). For example, if $x = 101$ and $y = 00$, then $s = 100010101$ is an interleaving of x and y, since characters 1,2,5,7,8,9 form 101101—a repetition of x—and the remaining characters form 000—a repetition of y.

Give an efficient algorithm that takes strings s, x and y and decides if s is an interleaving of x and y.

Problem 2 [Sensor Networks]

Consider a network of power-limited sensors that are asleep most of the time, but can be awoken when some central authority decides that something interesting is happening. Suppose that we organize the sensors into a rooted tree, with the central authority at the root. Let us organize the awakening process into rounds. In one round, each sensor can contact and thereby awaken one of its direct subordinates in the tree. The number of rounds that it takes to awaken all of the sensors depends on the sequence in which each sensor awakens its direct subordinates.

Give an efficient algorithm that determines the minimum number of rounds needed to awaken all of the sensors, and outputs a sequence of contacts that achieves this minimum number of rounds.

Problem 3 [Number of Paths]

Suppose that we are given a directed graph $G = (V, E)$ with costs on the edges; the costs may be positive or negative, but every cycle in the graph has strictly positive cost. We are also given two nodes, $v, w \in V$. Give an efficient algorithm that computes the number of shortest $v - w$ paths in G.

Problem 4 [Salad]

A salad is any combination of the following ingredients: (1) tomato; (2) lettuce; (3) spinach; (4) carrot and (5) oil. Each salad must contain: (A) at least 15 grams of protein; (B) at least 2 and at most 6 grams of fat; (C) at least 4 grams of carbohydrates; (D) at most 100 milligrams of sodium. Furthermore, (E) you do not want your salad to be more than 50% greens, by mass. The nutritional contents of these ingredients (per 100 grams) are

ingredient	energy (kcal)	protein (grams)	fat (grams)	carbohydrate (grams)	sodium (milligrams)
tomato	21	0.85	0.33	4.64	9.00
lettuce	16	1.62	0.20	2.37	8.00
spinach	371	12.78	1.58	74.69	7.00
carrot	346	8.39	1.39	80.70	508.20
oil	884	0.00	100.00	0.00	0.00

Find a linear programming applet on the web, and use it to make the salad with the fewest calories under the nutritional constraints. Describe your linear programming formulation, the optimal solution (the quantity of each ingredient and the value). Cite the web resources that you used.