Problem Set 5 for CS 170

Problem 0 [Feedback]

Name one thing that you would you like to see improved in the lectures or discussion sections. (E.g., more explanation of proofs, more examples, more pseudocode, less explanation of proofs, fewer examples, less pseudocode, go faster, go slower, etc). Overall, is the pace of the class too fast, too slow, or just about right?

Problem 1 [Carving out an MST]

(a) Consider a weighted undirected graph G, in which all of the edge costs are distinct. Let C be any cycle in G, and let $e = (v, w)$ be the most expensive edge belonging to C. Show that e does not belong to any minimum spanning tree of G.

(b) Consider the following algorithm for finding a minimum spanning tree in G. Sort the edges E. Start with the full graph G and remove edges, considering them in order of *decreasing* cost, as long as they do not disconnect the current graph. Use the result in part (a) to prove that this algorithm yields a minimum spanning tree.

Problem 2 [MSTs for Dynamical Graphs]

Consider a weighted undirected graph $G = (V, E)$. Assume you are given a minimum spanning tree T in G. Now assume that a new edge is added to G, connecting nodes $v, w \in V$ with cost `weight`(v, w).

(a) Give an efficient algorithm to test if T remains the minimum-cost spanning tree with the new edge added to G (but not to the tree T). Make your algorithm run in time $O(|E|)$. Please note any assumptions you make about what data structure is used to represent the tree T and the graph G.

(b) Suppose T is no longer the minimum-cost spanning tree. Give a linear-time algorithm to update the tree T to the new minimum-cost spanning tree.