Problem Set 2 for CS 170

Problem 1 [Adjacency Matrices]

Let A denote the *adjacency matrix* of a directed graph $G = (V, E)$ (a binary matrix such that $a_{ij} = 1$ iff $(i, j) \in E$).

a Prove that the (i, j)th entry of the matrix A^k is the count of the number of paths from node i to node j that have exactly k edges.

b Under the assumption that G is acyclic, give an efficient algorithm for obtaining the *ancestor matrix* (a binary matrix B such that $b_{ij} = 1$ iff i is an ancestor of j) from the adjacency matrix A.

Problem 2 [Cliques]

A *clique* is a set of vertices in an undirected graph $G = (V, E)$ such that every pair of vertices in the set is connected by an edge. Given a clique $C \subseteq V$, show that all the vertices in C lie in a single root-to-leaf path in a depth-first search tree. Must the clique vertices be adjacent to one another in the path?

Problem 3 [Bipartite Graph Test]

An undirected graph $G = (V, E)$ is *bipartite* if and only if its vertices can be partitioned into sets V_1 and V_2 such that every edge has one vertex in V_1 and the other in V_2. Give a linear time algorithm based on depth-first search for determining if a given graph is bipartite.

Problem 4 [Semiconnectedness]

A directed graph $G = (V, E)$ is said to be *semiconnected* if for any pair (u, v) of vertices there is a path either from u to v, or from v to u (or both).

a Give an efficient algorithm to determine whether a given graph is semiconnected.

b Show that G is semiconnected if and only if the DAG of its strongly connected components have a unique topologically sorted order.