Stat 153 Homework Assignment 4 (due Wednesday, November 7)

1. Show that $|z|^2 = z\bar{z}$. Draw a sketch that explains this result.

2. Let $A(\nu) = (1 - .25e^{-2\pi i\nu} - .125e^{-4\pi i\nu})$.

   (a) Compute the complex numbers $A(1/6)$ and $A(1/12)$. Express your result in polar coordinates.

   (b) Compute $A^2(1/6)$ and $A^2(1/12)$, again expressing your result in polar coordinates.

   (c) Compute $|A(1/6)|$ and $|A(1/12)|$.

   (d) Compute $|A(1/6)|^2$ and $|A(1/12)|^2$.

3. Graph and compare the amplitudes of the frequency response function for the two linear filters with coefficients equal to $(1/3, 1/3, 1/3)$ and $(1/4, 1/2, 1/4)$ for $r = -1, 0, 1$ and zero for $|r| > 1$. What are their relative advantages and disadvantages as low pass filters? Suppose that a cosine function with a period of 12 time units is passed through the first filter. What is its amplitude, its period, and its phase shift?

4. Consider the following linear filter:

$$a_r = \begin{cases} 1 & r = 0 \\ -1 & r = 12 \\ 0 & \text{otherwise} \end{cases}$$

   Such a filter is often used to remove the yearly effect in a time series that is measured monthly.

   (a) Compute the amplitude of the frequency response function for $\nu \in [0, 1/2]$. Plot your result.

   (b) What frequencies are passed by this filter? What frequencies are not passed? Can you explain this result intuitively?

5. Let X_t and Y_t be independent stationary time series and let $Z_t = X_t + Y_t$. Show that the power spectral densities add; i.e.: $f_Z(\nu) = f_X(\nu) + f_Y(\nu)$.

6. Problem 3.5 in Shumway and Stoffer.