1. Show that \(|z|^2 = z\bar{z}^2\). Draw a sketch that explains this result.

2. Let \(A(\nu) = (1 - .25e^{-2\pi i \nu} - .125e^{-4\pi i \nu})\).
   (a) Compute the complex numbers \(A(1/6)\) and \(A(1/12)\). Express your result in polar coordinates.
   (b) Compute \(A^2(1/6)\) and \(A^2(1/12)\), again expressing your result in polar coordinates.
   (c) Compute \(|A(1/6)|\) and \(|A(1/12)|\).
   (d) Compute \(|A(1/6)|^2\) and \(|A(1/12)|^2\).

3. Graph and compare the amplitudes of the frequency response function for the two linear filters with coefficients equal to \((1/3, 1/3, 1/3)\) and \((1/4, 1/2, 1/4)\) for \(r = -1, 0, 1\) and zero for \(|r| > 1\). What are their relative advantages and disadvantages as low pass filters? Suppose that a cosine function with a period of 12 time units is passed through the first filter. What is its amplitude, its period, and its phase shift?

4. Consider the following linear filter:

   \[
   a_r = \begin{cases} 
   1 & r = 0 \\
   -1 & r = 12 \\
   0 & \text{otherwise}
   \end{cases}
   \]

   Such a filter is often used to remove the yearly effect in a time series that is measured monthly.
   (a) Compute the amplitude of the frequency response function for \(\nu \in [0, 1/2]\). Plot your result.
   (b) What frequencies are passed by this filter? What frequencies are not passed? Can you explain this result intuitively?

5. Let \(X_t\) and \(Y_t\) be independent stationary time series and let \(Z_t = X_t + Y_t\). Show that the power spectral densities add; i.e.: \(f_Z(\nu) = f_X(\nu) + f_Y(\nu)\).

6. Problem 3.5 in Shumway and Stoffer.