Stat 153 Homework Assignment 3 (due Wednesday, Oct. 24)

1. Consider a zero-mean stationary time series $\{x_t\}$ that is additionally *Gaussian*. In particular, each pair of variables, (x_t, x_{t+h}), are assumed to be bivariate Gaussian with covariance $\gamma(h)$ and variances $\gamma(0)$. Compute the conditional mean of x_{t+h} given x_t. (This exercise should help to convince you that linear forecasting is not only reasonable, but also optimal, in some cases).

2. Suppose that you have observed a time series with the three data points $(1.5, -0.8, 0.9)$.

   (a) Calculate the log likelihood of this sequence under an AR(1) model with $\phi = 0.5$.

   (b) Calculate the log likelihood of this sequence under an MA(1) model with $\theta = 0.3$.

3. Suppose that in a sample of size 100 from an AR(1) process with mean μ, $\phi = .6$ and $\sigma_w^2 = 2$, we obtain $\bar{x} = .271$. Construct an approximate 95% confidence interval for μ. Do the data suggest that $\mu = 0$?

4. Two hundred observations of a time series, $x_1, \ldots, x_200$ gave the following sample statistics:

sample mean:	$\bar{x} = 3.82$
sample variance:	$\hat{\gamma}(0) = 1.15$
sample ACF:	$\hat{\rho}(1) = .427$
	$\hat{\rho}(2) = .475$
	$\hat{\rho}(3) = .169$

   (a) Based on these sample statistics, is it reasonable to suppose that $\{x_t - \mu\}$ is white noise?

   (b) Assuming that $\{x_t - \mu\}$ can be modeled as the AR(2) process:

$$(x_t - \mu) = \phi_1(x_{t-1} - \mu) + \phi_2(x_{t-2} - \mu) + w_t,$$

   where w_t is white noise, find estimates of μ, ϕ_1, ϕ_2, and σ_w^2.

   (c) Would you conclude that $\mu = 0$?

   (d) Construct 95% confidence intervals for ϕ_1 and ϕ_2.

5. Use your implementation of the Durbin-Levinson algorithm to find the Yule-Walker estimates for the parameters of AR(1), AR(2), and AR(3) models for the following data sequence:

$$-0.2,\ 3.1,\ -3.9,\ 4.5,\ -2.9,\ 2.7,\ -1.8,\ -0.7,\ 0.2,\ -1.0$$

   What are the PACF's for lags 1, 2 and 3? Based on these PACF's, which model would you (very tentatively) select to represent these data?