Stat 153 Homework Assignment 2 (due Friday, Sept. 28)

1. Find the autocovariance and autocorrelation functions of the moving average process:

$$x_t = w_t + .6w_{t-1} - .2w_{t-2}$$

where w_t is WN(0,σ^2).

2. Write down the $\phi(B)$ and the $\theta(B)$ polynomials for each of the following models. Determine which models are causal and which are invertible.

   (a) $x_t + 0.2x_{t-1} - 0.48x_{t-2} = w_t$

   (b) $x_t + 1.9x_{t-1} + 0.88x_{t-2} = w_t + 0.2w_{t-1} + 0.7w_{t-2}$

   (c) $x_t + 0.6x_{t-1} = w_t + 1.2w_{t-1}$

   (d) $x_t + 1.8x_{t-1} + 0.81x_{t-2} = w_t$

   (e) $x_t + 1.6x_{t-1} = w_t - 0.4w_{t-1} + 0.04w_{t-2}$

3. Find the poles and zeros associated with the following ARMA model, plotting them in the complex plane:

$$x_t = 2.333x_{t-1} - 0.5x_{t-2} - 0.667x_{t-3} + 0.667x_{t-4} + w_t - 0.1w_{t-1} - 0.2w_{t-2}$$

   Is this a causal model? Is it invertible?

4. For the causal processes in Problem 2, calculate the first six coefficients, ψ_i, in the linear process representation of the model.

5. Calculate the first five autocovariances for an AR(2) process with:

   (a) $\phi_1 = 0.6$ and $\phi_2 = -0.2$

   (b) $\phi_1 = -0.6$ and $\phi_2 = 0.2$

6. Calculate the first five autocovariances for the ARMA model:

$$x_t + 0.2x_{t-1} = w_t - 0.8w_{t-1}$$