Stat 153 Homework Assignment 1 (due Wednesday, Sept. 12)

1. Let us define a *drunkard's walk* along the x-axis as follows. The drunkard starts at the origin at time zero; thus, $x_0 = 0$. At each time step t the drunk tosses a fair coin, stepping forward one unit if the result of the coin toss is heads and stepping backward one unit if the result is tails.

   Let w_t denote the result of the coin toss at time t. We assume that w_t is independent of w_s for $t \neq s$.

   (a) Write a mathematical formula for x_t.

   (b) Calculate the mean of x_t and the variance of x_t.

   (c) Is the drunkard's walk a stationary process? Why or why not?

2. Let $\{x_t\}$ be the moving average process given by:

$$x_t = w_t + \theta w_{t-2},$$

   where $\{w_t\}$ is WN(0,1). Find the autocovariance and autocorrelation functions for this process when $\theta = 0.8$.

3. Let $\{w_t\}$ be WN$(0, \sigma^2)$, and let a, b, and c be constants. Which, if any, of the following processes are stationary? For each stationary process specify the mean and autocovariance function.

   (a) $x_t = a + bw_t + cw_{t-2}$

   (b) $x_t = a + bw_0$

   (c) $x_t = w_0 \cos(ct)$

   (d) $x_t = w_t w_{t-1}$

4. Let w_1 and w_2 be independent random variables, each with mean 0 and variance σ^2 and let c be a constant. Calculate the mean and autocovariance function of the following process:

$$x_t = w_1 \cos(ct) + w_2 \sin(ct).$$

   Is this process stationary?

5. Do problem 1.10 in Shumway and Stoffer.

6. Do problem 1.6(a) in Shumway and Stoffer.