CS-184: Computer Graphics

Lecture #14: Subdivision

Prof. James O'Brien
University of California, Berkeley

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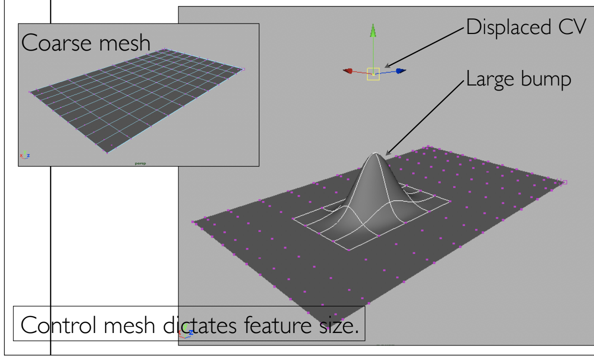
Subdivision

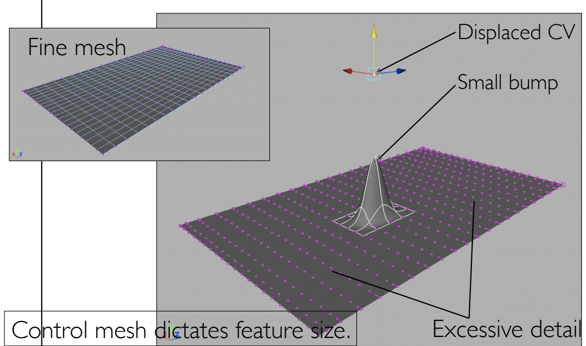
- Start with:
  - Given control points for a curve or surface, find new control points for a sub-section of curve/surface
- Key extension to basic idea:
  - Generalize to non-regular surfaces

Consider NURBS Surface



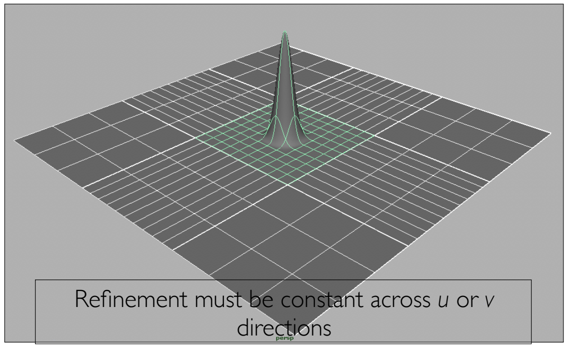
Control mesh dictates feature size.

Consider NURBS Surface



Control mesh dictates feature size.

Tensor Product Surface Refinement



Refinement must be constant across *u* or *v* directions

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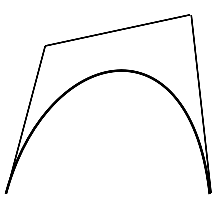
Bézier Subdivision



$u \in [0..1]$

$$\mathbf{x}(u) = \sum_i b_i(u)\,\mathbf{p}_i$$

$$\mathbf{x}(u) = [1, u, u^2, u^3]\,\beta_Z\,\mathbf{P}$$

Vector of control points

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

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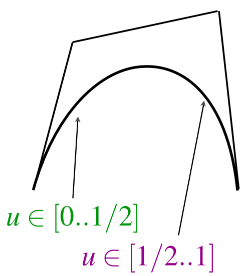
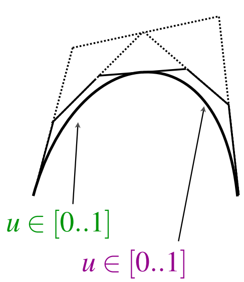
Bézier Subdivision



$$\mathbf{x}(u) = [1, u, u^2, u^3]\, \beta_Z \mathbf{P}$$

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

Bézier Subdivision

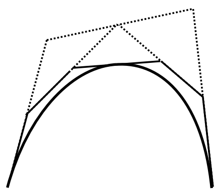


$$\mathbf{x}(u) = [1, u, u^2, u^3]\, \beta_Z \mathbf{P}$$

Can't change these...

$$\beta_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

Bézier Subdivision



$$\mathbf{x}(u) = [1, u, u^2, u^3]\,\beta_Z \mathbf{P} \quad u \in [0..\tfrac{1}{2}]$$

$$\mathbf{x}(u) = [1, \frac{u}{2}, \frac{u^2}{4}, \frac{u^3}{8}]\beta_Z \mathbf{P} \quad u \in [0..1]$$

$$\mathbf{x}(u) = [1, u, u^2, u^3]\mathbf{S}_1 \beta_Z \mathbf{P}$$

$$\mathbf{S}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1/8 \end{bmatrix}$$

$$\mathbf{x}(u) = [1, u, u^2, u^3]\beta_Z \beta_Z^{-1} \mathbf{S}_1 \beta_Z \mathbf{P}$$

$$\mathbf{x}(u) = [1, u, u^2, u^3]\beta_Z \mathbf{H}_{Z1} \mathbf{P}$$

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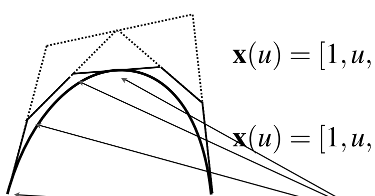
Bézier Subdivision



$$\mathbf{x}(u) = [1, u, u^2, u^3]\beta_Z \mathbf{H}_{Z1} \mathbf{P}$$

$$\mathbf{x}(u) = [1, u, u^2, u^3]\beta_Z \mathbf{P}_1$$

$$\mathbf{P}_1 = \mathbf{H}_{Z1} \mathbf{P}$$

$$\mathbf{H}_{Z1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{bmatrix}$$

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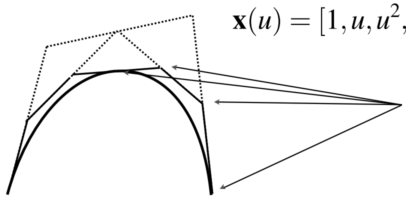
Bézier Subdivision

$$\mathbf{x}(u) = [1, u, u^2, u^3]\beta_Z \mathbf{P}_2$$



$$\mathbf{P}_2 = \mathbf{H}_{Z2}\mathbf{P}$$

$$\mathbf{H}_{Z2} = \begin{bmatrix} \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{S}_2 = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{8} \\ 0 & 0 & \frac{1}{4} & \frac{3}{8} \\ 0 & 0 & 0 & \frac{1}{8} \end{bmatrix}$$

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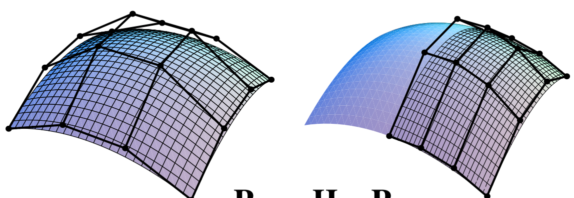
Bézier Subdivision



$$\mathbf{P}_{2\cdot} = \mathbf{H}_{Z2}\,\mathbf{P}$$

$$\mathbf{x}(u,v) = [1, u, u^2, u^3]\beta_Z \mathbf{P} \beta_Z^{\mathsf{T}} [1, v, v^2, v^3]^{\mathsf{T}}$$

4 x 4 matrix of control points

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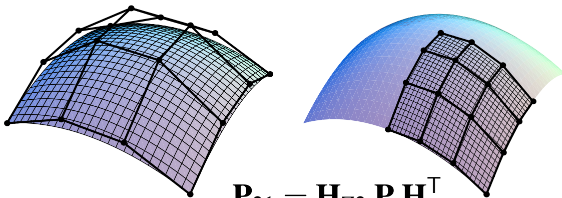
Bézier Subdivision

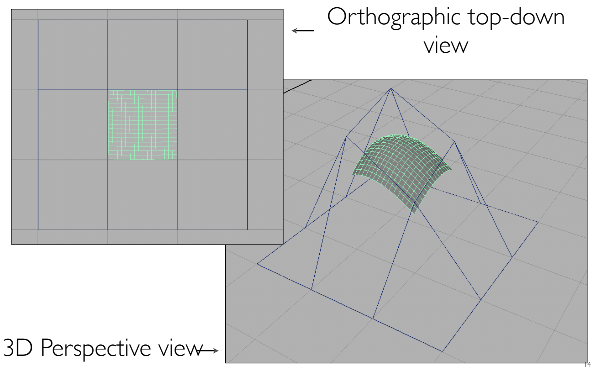


$$\mathbf{P}_{21} = \mathbf{H}_{Z2}\, \mathbf{P}\, \mathbf{H}_{Z1}^{\top}$$

$$\mathbf{x}(u,v) = [1, u, u^2, u^3]\beta_Z \mathbf{P} \beta_Z^{\top} [1, v, v^2, v^3]^{\top}$$

4 x 4 matrix of control points

Regular B-Spline Subdivision

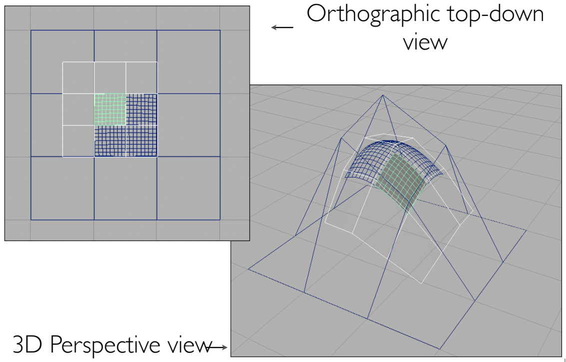


Orthographic top-down view

3D Perspective view

Regular B-Spline Subdivision

← Orthographic top-down view



3D Perspective view→

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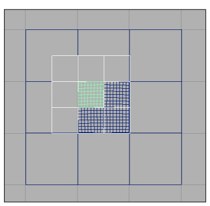
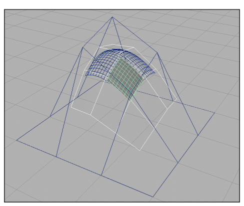
Regular B-Spline Subdivision





$$\mathbf{x}(u,v) = [1,u,u^2,u^3]\beta_B\,\mathbf{P}\,\beta_B^{\top}[1,v,v^2,v^3]^{\top}$$

$$\mathbf{P}_{11} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B1}^{\top}$$

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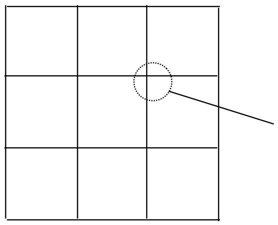
Regular B-Spline Subdivision



$$\mathbf{P}_{11} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B1}^{\mathsf{T}}$$

In this parametric view these knot points are collocated.

The 3D control points are not.

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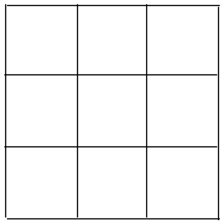
Regular B-Spline Subdivision



$$\mathbf{P}_{11} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B1}^{\mathsf{T}}$$
$$\mathbf{P}_{12} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B2}^{\mathsf{T}}$$

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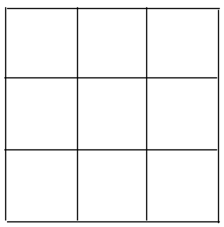
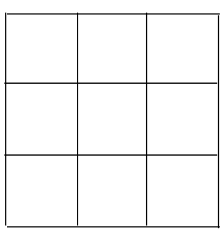
Regular B-Spline Subdivision



$$\mathbf{P}_{11} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B1}^{\mathsf{T}}$$
$$\mathbf{P}_{12} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B2}^{\mathsf{T}}$$
$$\mathbf{P}_{22} = \mathbf{H}_{B2}\,\mathbf{P}\,\mathbf{H}_{B2}^{\mathsf{T}}$$

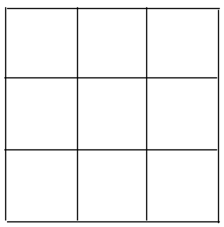
Regular B-Spline Subdivision



$$\mathbf{P}_{11} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B1}^{\mathsf{T}}$$
$$\mathbf{P}_{12} = \mathbf{H}_{B1}\,\mathbf{P}\,\mathbf{H}_{B2}^{\mathsf{T}}$$
$$\mathbf{P}_{22} = \mathbf{H}_{B2}\,\mathbf{P}\,\mathbf{H}_{B2}^{\mathsf{T}}$$
$$\mathbf{P}_{21} = \mathbf{H}_{B2}\,\mathbf{P}\,\mathbf{H}_{B1}^{\mathsf{T}}$$

$$\mathbf{H}_{B1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{8} & \frac{3}{4} & \frac{1}{8} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{8} & \frac{3}{4} & \frac{1}{8} \end{bmatrix} \quad \mathbf{H}_{B2} = \begin{bmatrix} \frac{1}{8} & \frac{3}{4} & \frac{1}{8} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{8} & \frac{3}{4} & \frac{1}{8} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Regular B-Spline Subdivision



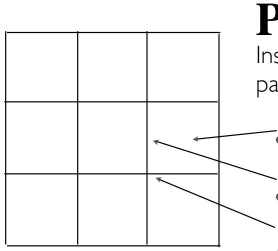
$$\mathbf{P}^{i+1} = \mathbf{H}\,\mathbf{P}^{i}$$

Length 25 vector of fine CPs

25 x 16 subdivision matrix

Length 16 vector of coarse CP

Regular B-Spline Subdivision



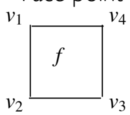
$$\mathbf{P}^{i+1} = \mathbf{H}\,\mathbf{P}^{i}$$

Inspection would reveal a pattern

- Face points
- Edge points
- Vertex points

Regular B-Spline Subdivision

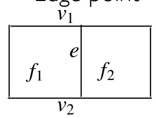
Face point

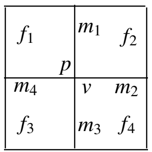


$$f = \frac{v_1 + v_2 + v_3 + v_4}{4}$$

$$e = \frac{v_1 + v_2 + f_1 + f_2}{4}$$

Edge point





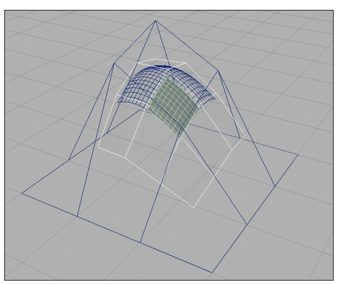
Vertex point

$$v = \frac{f_1 + f_2 + f_3 + f_4 + 2(m_1 + m_2 + m_3 + m_4) + 4p}{16}$$

m midpoint of edge, not "edge point"
p old "vertex point"

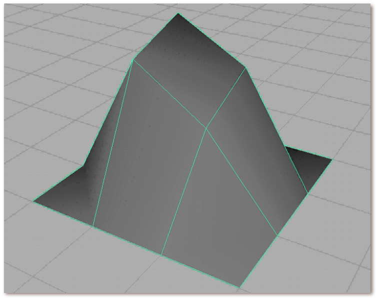
Regular B-Spline Subdivision

- Recall that control mesh approaches surface



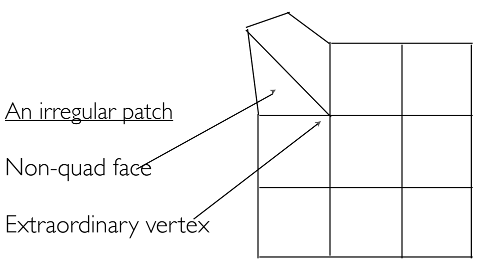
Regular B-Spline Subdivision

- Limit of subdivision is the surface



Irregular B-Spline Subdivision

- Catmull-Clark Subdivision
  - Generalizes regular B-Spine subdivision



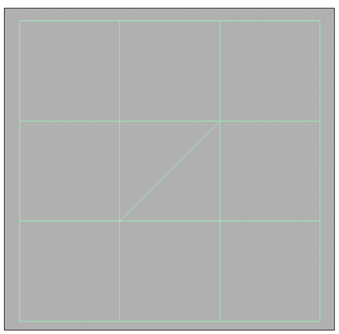
Irregular B-Spline Subdivision

- Catmull-Clark Subdivision
  - Generalizes regular B-Spine subdivision
  - Rules reduce to regular for ordinary vertices/faces

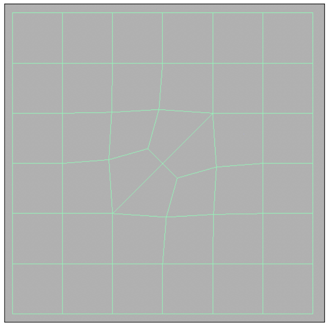
$$f = \text{average of surrounding vertices}$$
$$e = \frac{f_1 + f_2 + v_1 + v_2}{4}$$
$$v = \frac{\bar{f}}{n} + \frac{2\bar{m}}{n} + \frac{p(n-3)}{n}$$

$\bar{m}$ = average of adjacent midpoints
$\bar{f}$ = average of adjacent face points
n = valence of vertex

Catmull-Clark Subdivision



Catmull-Clark Subdivision



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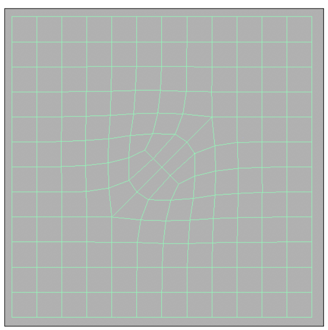
Catmull-Clark Subdivision



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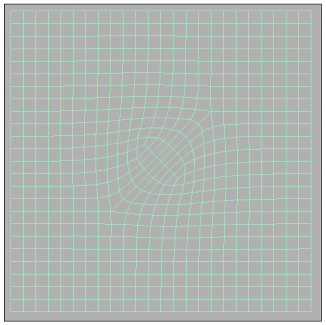
Catmull-Clark Subdivision



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Continuity of Catmull-Clark

- In "ordinary" regions
  - Surface is fully C^2 everywhere except extraordinary points
  - Fast evaluation by matrix exponentiation
    - See "*Exact Evaluation Of Catmull-Clark Subdivision Surfaces At Arbitrary Parameter Values*" by Jos Stam, SIGGRAPH 1998.
- At extraordinary points
  - Surface is at least C^1
  - Curvature is Lipschitz continuous at extraordinary points

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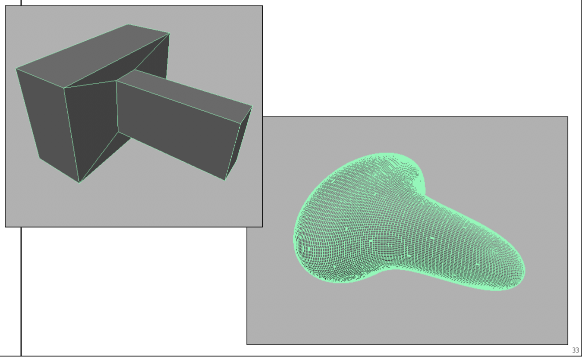
Catmull-Clark Subdivision



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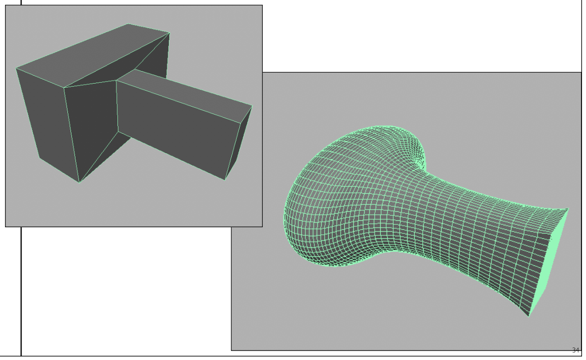
Catmull-Clark Subdivision



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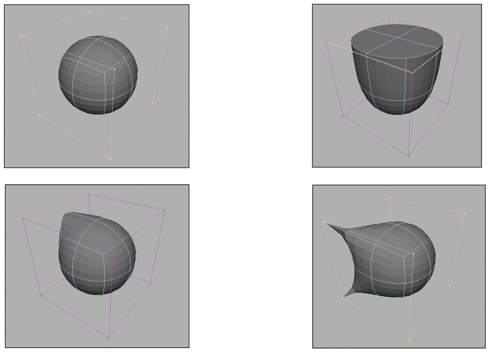
Catmull-Clark Subdivision



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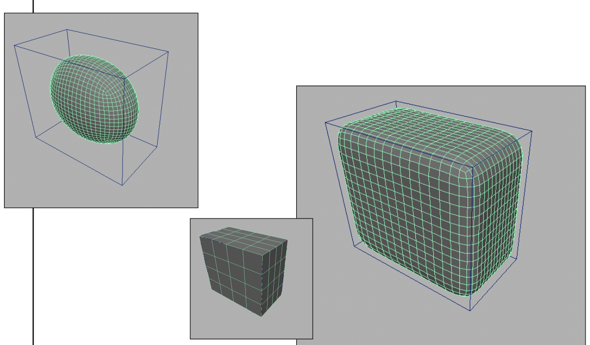
Catmull-Clark Subdivision



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