# CS-184: Computer Graphics

## Lecture #9: Scan Conversion

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V2014-S-09-1.0

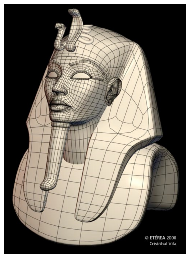
With additional slides based on those of Maneesh Agrawala

## Today

- 2D Scan Conversion
  - Drawing Lines
  - Drawing Curves
  - Filled Polygons
  - Filling Algorithms

# Drawing a Line

- Basically, its easy... but for the details
- Lines are a basic primitive that needs to be done well...

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# Drawing a Line

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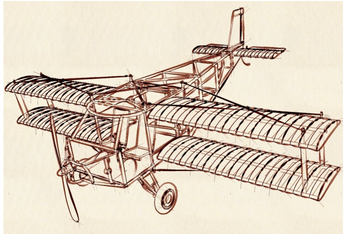

From "A Procedural Approach to Style for NPR Line Drawing from 3D models," by Grabli, Durand, Turquin, Sillion

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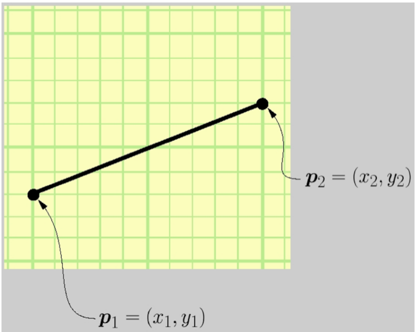
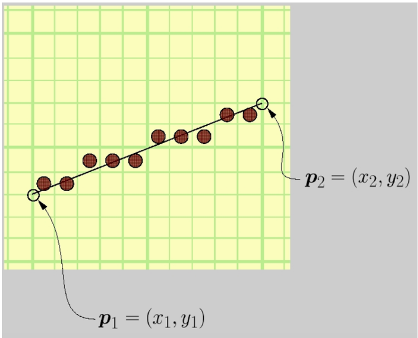
## Drawing a Line



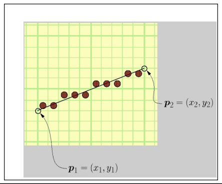

## Drawing a Line



## Drawing a Line

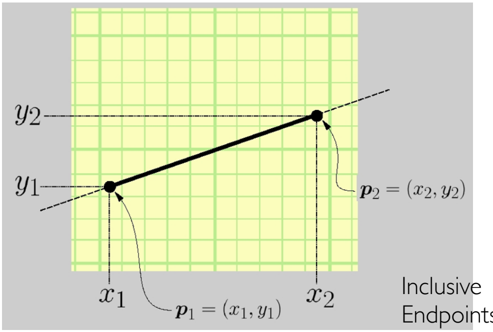
- Some things to consider
  - How thick are lines?
  - How should they join up?
  - Which pixels are the right ones?

For example:






## Drawing a Line



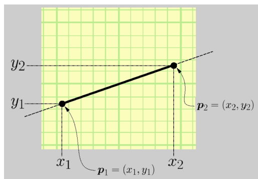

Inclusive Endpoints

## Drawing a Line

$y = m \cdot x + b, x \in [x_1, x_2]$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$b = y1 - m \cdot x_1$

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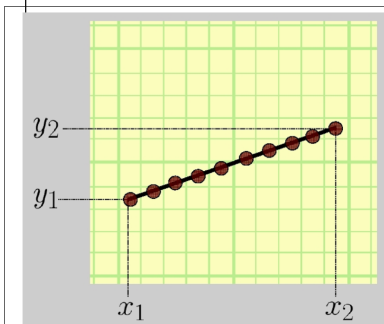
## Drawing a Line




$\Delta x = 1$
$\Delta y = m \cdot \Delta x$

```
x=x1
y=y1
while(x<=x2)
  plot(x,y)
  x++
  y+=Dy
```

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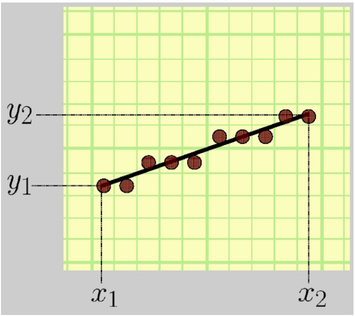
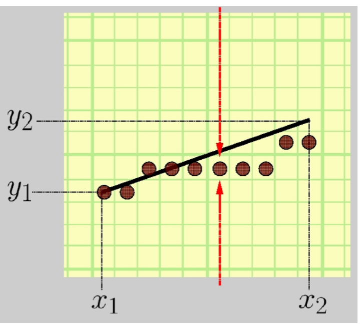
# Drawing a Line




$\Delta x = 1$
$\Delta y = m \cdot \Delta x$

After rounding

# Drawing a Line

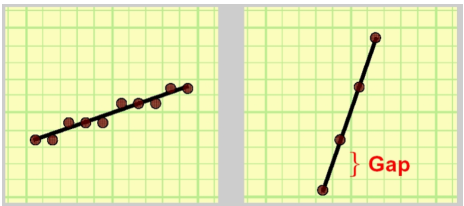



$\Delta x = 1$
$\Delta y = m \cdot \Delta x$
$y {+}{=} \Delta y$

Accumulation of roundoff errors

How slow is float-to-int conversion?

## Drawing a Line




$|m| \leq 1$ $|m| > 1$

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## Drawing a Line

```
void drawLine-Error1(int x1,x2, int y1,y2)

  float m = float(y2-y1)/(x2-x1)
  int x = x1
  float y = y1

  while (x <= x2)

    setPixel(x,round(y),PIXEL_ON)

    x += 1
    y += m
```

Not exact math

Accumulates errors

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# Drawing a Line

```
void drawLine-Error2(int x1,x2, int y1,y2)

  float m = float(y2-y1)/(x2-x1)
  int x = x1
  int y = y1
  float e = 0.0

  while (x <= x2)

    setPixel(x,y,PIXEL_ON)

    x += 1
    e += m
    if (e >= 0.5)
      y+=1
      e-=1.0
```

No more rounding

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# Drawing a Line

```
void drawLine-Error3(int x1,x2, int y1,y2)

  int x = x1
  int y = y1
  float e = -0.5

  while (x <= x2)

    setPixel(x,y,PIXEL_ON)

    x += 1
    e += float(y2-y1)/(x2-x1)
    if (e >= 0.0)
      y+=1
      e-=1.0
```

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## Drawing a Line

```
void drawLine-Error4(int x1,x2, int y1,y2)

  int x = x1
  int y = y1
  float e = -0.5*(x2-x1)          // was -0.5

  while (x <= x2)

    setPixel(x,y,PIXEL_ON)

    x += 1
    e += y2-y1                    // was /(x2-x1)
    if (e >= 0.0)                 // no change
      y+=1
      e-=(x2-x1)                  // was 1.0
```

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## Drawing a Line

```
void drawLine-Error5(int x1,x2, int y1,y2)

  int x = x1
  int y = y1
  int e = -(x2-x1)                // removed *0.5

  while (x <= x2)

    setPixel(x,y,PIXEL_ON)

    x += 1
    e += 2*(y2-y1)                // added 2*
    if (e >= 0.0)                 // no change
      y+=1
      e-=2*(x2-x1)                // added 2*
```

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## Drawing a Line

```
void drawLine-Bresenham(int x1,x2, int y1,y2)

  int x = x1
  int y = y1
  int e = -(x2-x1)

  while (x <= x2)

    setPixel(x,y,PIXEL_ON)

    x += 1
    e += 2*(y2-y1)
    if (e >= 0.0)
      y+=1
      e-=2*(x2-x1)
```

Faster
Not wrong

$0 \leq m \leq 1$
$x_1 \leq x_2$

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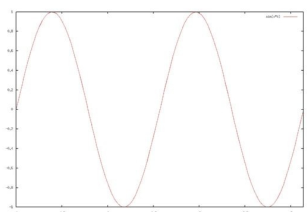
## Drawing Curves



$y = f(x)$

Only one value of $y$ for each value of $x$...

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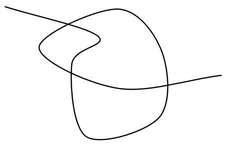
## Drawing Curves

- Parametric curves
  - Both *x* and *y* are a function of some third parameter

$$x = f(u)$$
$$y = f(u)$$



$$\mathbf{x} = \mathbf{f}(u)$$

$$u \in [u_0 \ldots u_1]$$

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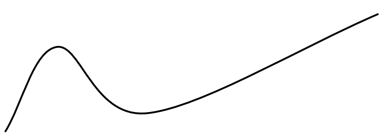
## Drawing Curves



$$\mathbf{x} = \mathbf{f}(u) \qquad u \in [u_0 \ldots u_1]$$

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# Drawing Curves

- Draw curves by drawing line segments
  - Must take care in computing end points for lines
  - How long should each line segment be?

$$\mathbf{x} = \mathbf{f}(u) \qquad u \in [u_0 \ldots u_1]$$

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# Drawing Curves

- Draw curves by drawing line segments
  - Must take care in computing end points for lines
  - How long should each line segment be?
  - Variable spaced points

$$\mathbf{x} = \mathbf{f}(u) \qquad u \in [u_0 \ldots u_1]$$

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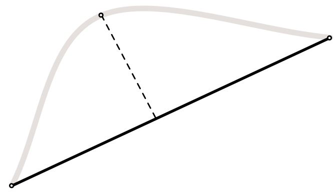
## Drawing Curves

- Midpoint-test subdivision



$$|\mathbf{f}(u_{mid}) - \mathbf{l}(0.5)|$$

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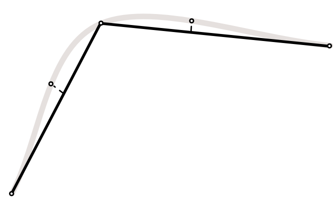
## Drawing Curves

- Midpoint-test subdivision



$$|\mathbf{f}(u_{mid}) - \mathbf{l}(0.5)|$$

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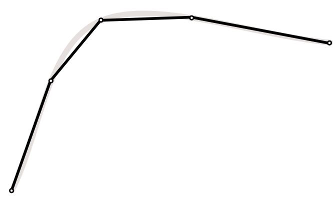
# Drawing Curves

- Midpoint-test subdivision



$$|\mathbf{f}(u_{mid}) - \mathbf{l}(0.5)|$$

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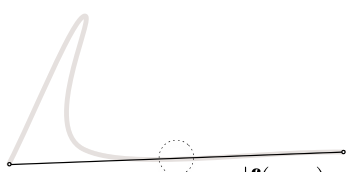
# Drawing Curves

- Midpoint-test subdivision
  - Not perfect
  - We need more information for a guarantee...



$$|\mathbf{f}(u_{mid}) - \mathbf{l}(0.5)|$$

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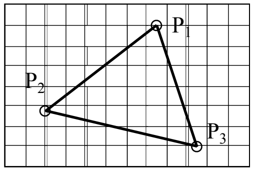
# Filling Triangles

- Render an image of a geometric primitive by setting pixel colors

```
void SetPixel(int x, int y, Color rgba)
```

- Example: Filling the inside of a triangle

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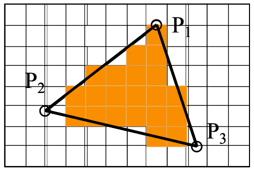
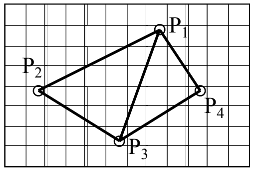


# Triangle Scan Conversion

- Properties of a good algorithm
  - Symmetric
  - Straight edges
  - Antialiased edges
  - No cracks between adjacent primitives
  - MUST BE FAST!

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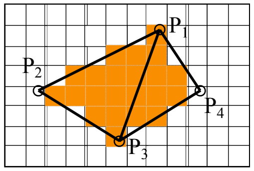
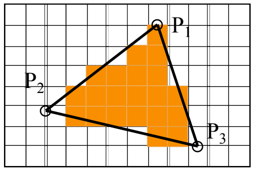


## Simple Algorithm

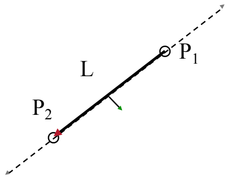
- Color all pixels inside triangle

```
void ScanTriangle(Triangle T, Color rgba){
  for each pixel P at (x,y){
      if (Inside(T, P))
          SetPixel(x, y, rgba);
  }
}
```



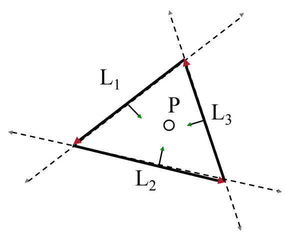

## Line Defines Two Halfspaces

- Implicit equation for a line
  - On line: $ax + by + c = 0$
  - On right: $ax + by + c < 0$
  - On left: $ax + by + c > 0$



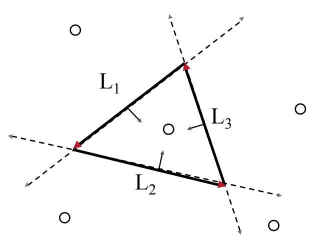
## Inside Triangle Test

- Point is inside triangle if it is in positive halfspace of all three boundary lines
  - Triangle vertices are ordered counter-clockwise
  - Point must be on the left side of every boundary line




## Inside Triangle Test

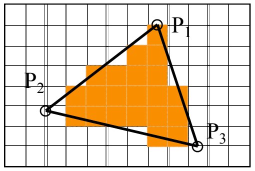
```
Boolean Inside(Triangle T, Point P)
{
  for each boundary line L of T {
     Scalar d = L.a*P.x + L.b*P.y + L.c;
     if (d < 0.0) return FALSE;
  }
  return TRUE;
}
```



## Simple Algorithm

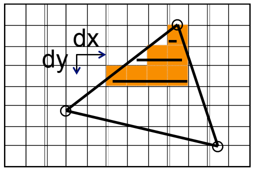
- What is bad about this algorithm?

```
void ScanTriangle(Triangle T, Color rgba){
  for each pixel P at (x,y){
     if (Inside(T, P))
        SetPixel(x, y, rgba);
  }
}
```




## Triangle Sweep-Line Algorithm

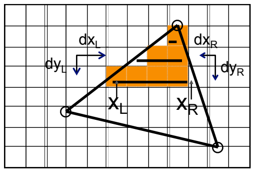
- Take advantage of spatial coherence
  - Compute which pixels are inside using horizontal spans
  - Process horizontal spans in scan-line order
- Take advantage of edge linearity
  - Use edge slopes to update coordinates incrementally



# Triangle Sweep-Line Algorithm

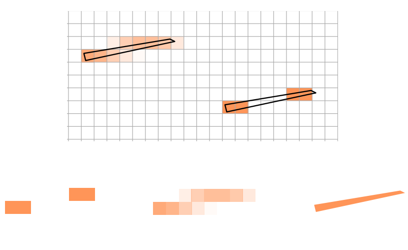
```
void ScanTriangle(Triangle T, Color rgba){
    for each edge pair {
        initialize xL, xR;
        compute dxL/dyL and dxR/dyR;
        for each scanline at y
            for (int x = ceil(xL); x <= xR; x++)
                SetPixel(x, y, rgba);
        xL += dxL/dyL;
        xR += dxR/dyR;
    }
}
```

Bresenham's algorithm works the same way, but uses only integer operations!




# Antialiasing

Desired solution of an integral over pixel



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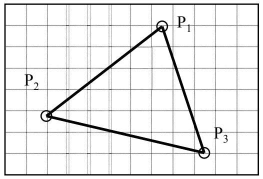
# Hardware Antialiasing

Supersample pixels

- Multiple samples per pixel
- Average subpixel intensities (box filter)
- Trades intensity resolution for spatial resolution

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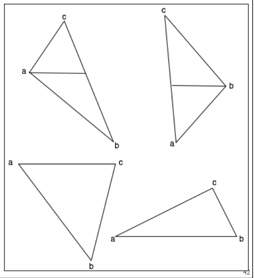
# Optimize for Triangles

- Spilt triangle into two parts
  - Two edges per part
  - Y-span is monotonic
- For each row
  - Interpolate span
- Interpolate barycentric coordinates

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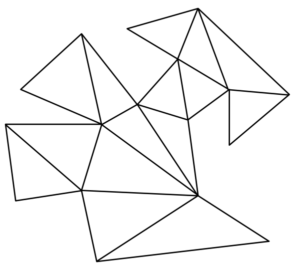
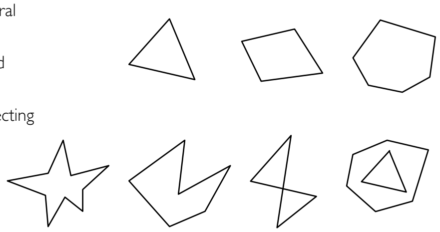
# Hardware Scan Conversion

- Convert everything into triangles
  - Scan convert the triangles



# Polygon Scan Conversion

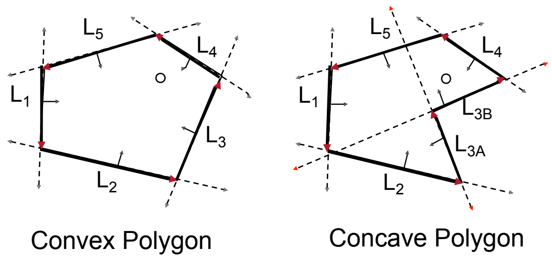
- Fill pixels inside a polygon
  - Triangle
  - Quadrilateral
  - Convex
  - Star-shaped
  - Concave
  - Self-intersecting
  - Holes



What problems do we encounter with arbitrary polygons?

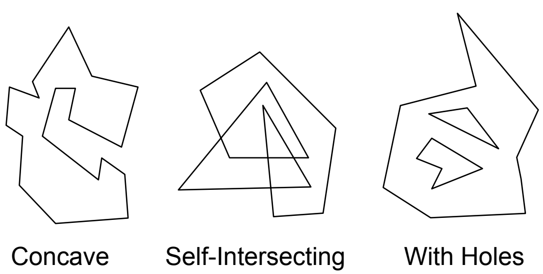
## Polygon Scan Conversion

- Need better test for points inside polygon
  - Triangle method works only for convex polygons



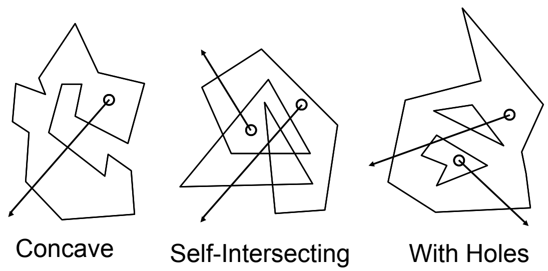

## Inside Polygon Rule

- What is a good rule for which pixels are inside?

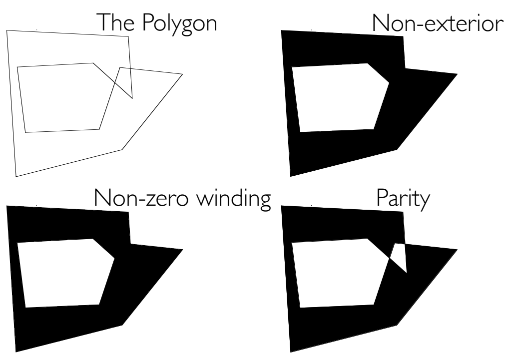


# Inside Polygon Rule

- Odd-parity rule
  - Any ray from P to infinity crosses odd number of edges




# Inside/Outside Testing

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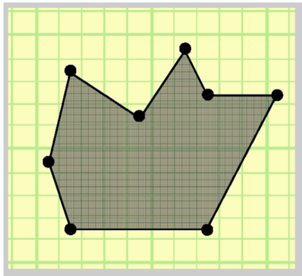
## Filled Polygons



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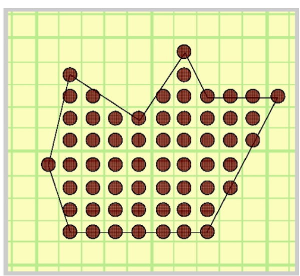
## Filled Polygons



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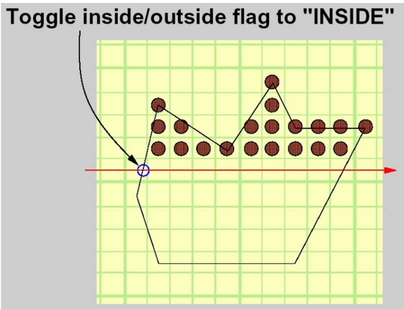
## Filled Polygons

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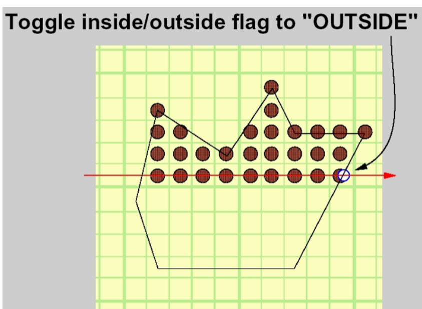
## Filled Polygons

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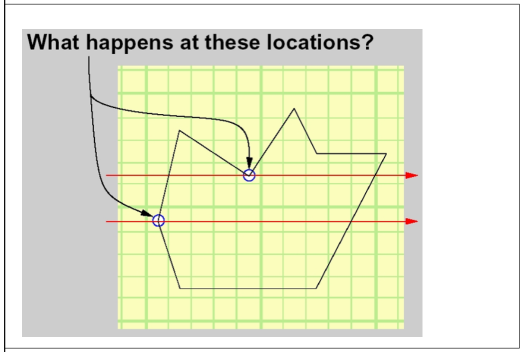
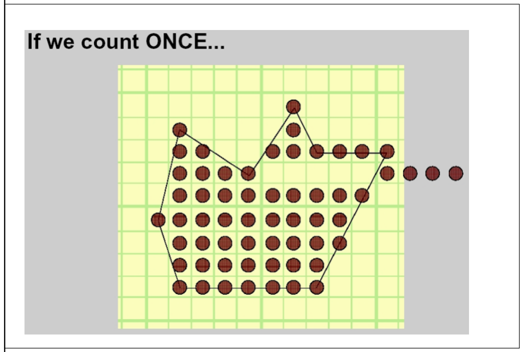
## Filled Polygons

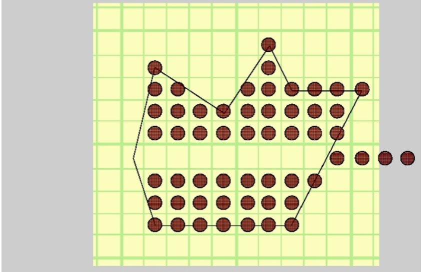



## Filled Polygons



## Filled Polygons

If we count TWICE...



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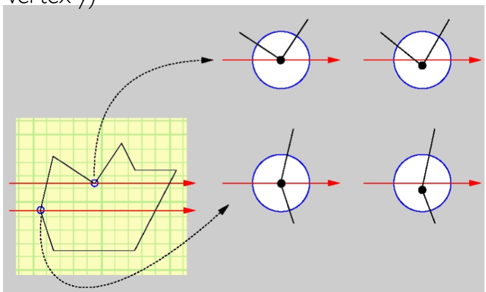
## Filled Polygons

Treat (scan y = vertex y) as (scan y > vertex y)



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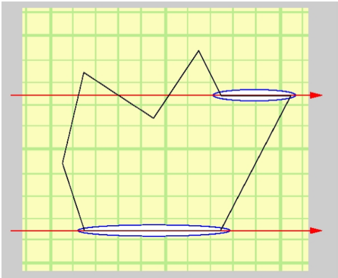
# Filled Polygons

Horizontal edges



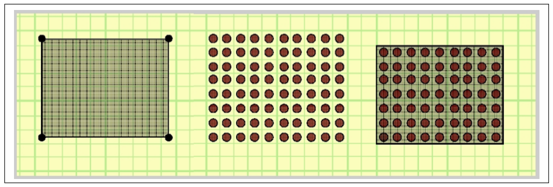


# Filled Polygons

Horizontal edges

# Filled Polygons

- "Equality Removal" applies to all vertices
- Both $x$ and $y$ coordinates



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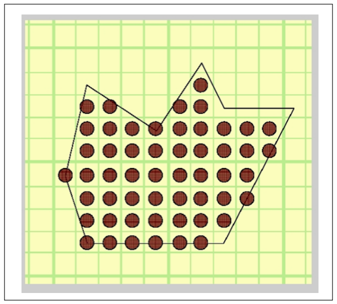
# Filled Polygons

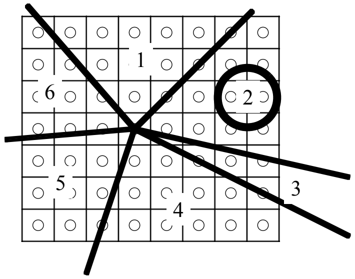
- Final result:



60

# Filled Polygons

- Who does this pixel belong to?

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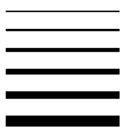
# Drawing a Line

- How thick?



- Ends?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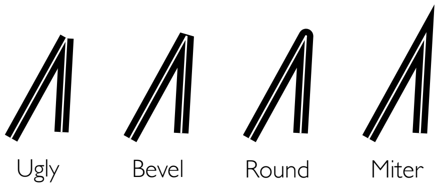
# Drawing a Line

- Joining?

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# Flood Fill

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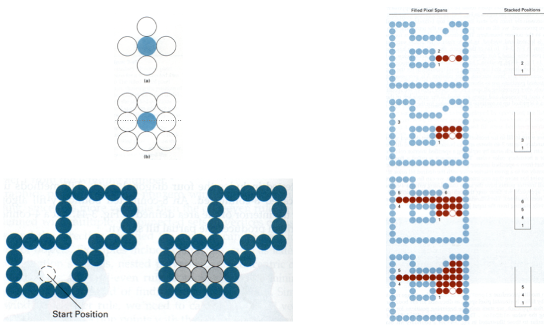
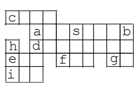
# Flood Fill




# Span-Based Algorithm

Definition: a ***run*** is a horizontal span of identically colored pixels




1. Start at pixel "s", the seed.
2. Find the run containing "s" ("b" to "a").
3. Fill that run with the new color.
4. Search every pixel above run, looking for pixels of interior color
5. For each one found,
6. Find left side of that run ("c"), and push that on a stack.
7. Repeat lines 4-7 for the pixels below ("d").
8. Pop stack and repeat procedure with the new seed

**The algorithm finds runs ending at "e", "f", "g", "h", and "i"**