CS-184: Computer Graphics

Lecture #8: Projection

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Today

- Windowing and Viewing Transformations
  - Windows and viewports
  - Orthographic projection
  - Perspective projection

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Screen Space

- Monitor has some number of pixels
  - e.g. 1024 x 768
- Some sub-region used for given program
  - You call it a window
  - Let's call it a viewport instead

[1024, 768]
[1024, 768]
[690, 705]
[60, 350]
[0, 0]
[0, 0]

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Screen Space

- May not really be a "screen"
  - Image file
  - Printer
  - Other
- Little pixel details
- Sometimes odd
  - Upside down
  - Hexagonal

From Shirley textbook.

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Screen Space

- Viewport is somewhere on screen
  - You probably don't care where
  - Window System likely manages this detail
  - Sometimes you care exactly where
- Viewport has a size in pixels
  - Sometimes you care (images, text, *etc.*)
  - Sometimes you don't (using high-level library)

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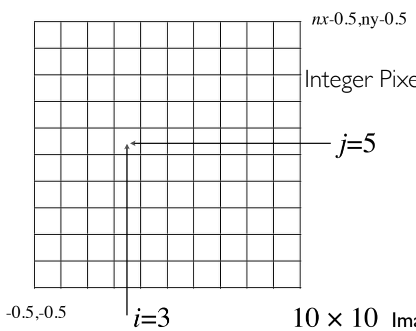
Screen Space



10×10 Image Resolution

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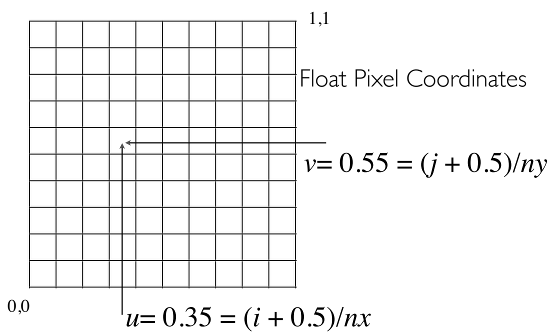
Screen Space



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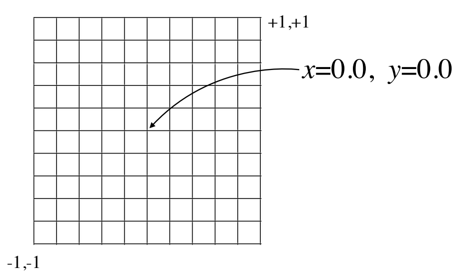
Canonical View Space

- Canonical view region
  - 2D: [-1,-1] to [+1,+1]



From Shirley textbook

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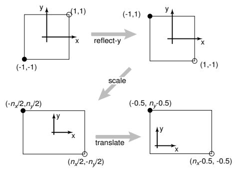
Canonical View Space

- Canonical view region
  - 2D: [-1,-1] to [+1,+1]



From Shirley textbook.
(Image coordinates are up-side-down.)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & \frac{n_x-1}{2} \\ 0 & \frac{-n_y}{2} & \frac{n_y-1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Remove minus for right-side-up

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Canonical View Space

- Canonical view region
  - 2D: [-1,-1] to [+1,+1]
- Define arbitrary ***window*** and define objects
- Transform window to canonical region
- Do other things (we'll see clipping latter)
- Transform canonical to screen space
- Draw it.

From Shirley textbook.

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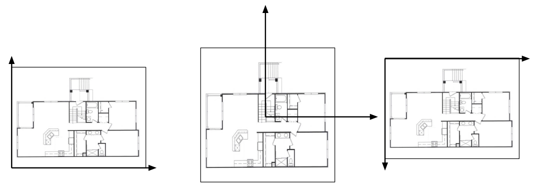
Canonical View Space



World Coordinates (Meters) | Canonical | Screen Space (Pixels)

Note distortion issues...

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Projection

- Process of going from 3D to 2D
- Studies throughout history (**e.g.** painters)
- Different types of projection
  - Linear
    - Orthographic
    - Perspective
  - Nonlinear

Many special cases in books just one of these two...

Orthographic is special case of perspective...

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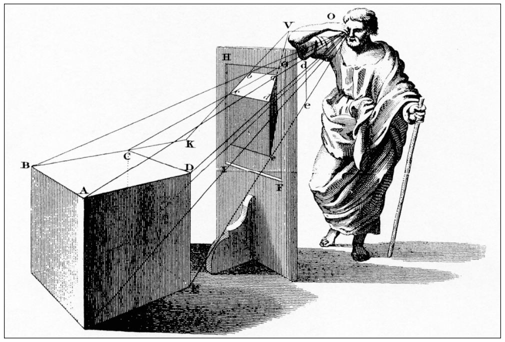
Perspective Projections



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Ray Generation vs. Projection

Viewing in ray tracing

- start with image point
- compute ray that projects to that point
- do this using geometry

Viewing by projection

- start with 3D point
- compute image point that it projects to
- do this using transforms

Inverse processes

- ray gen. computes the preimage of projection

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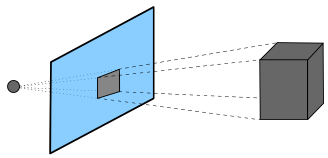
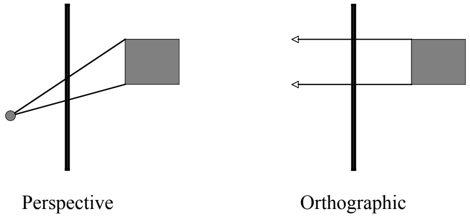
Linear Projection

- Projection onto a planar surface
- Projection directions either
  - Converge to a point
  - Are parallel (converge at infinity)

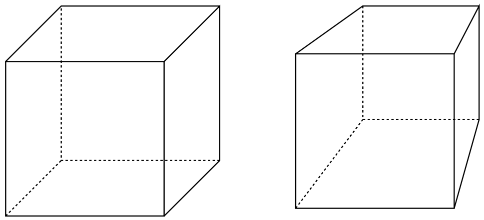


Linear Projection

- A 2D view

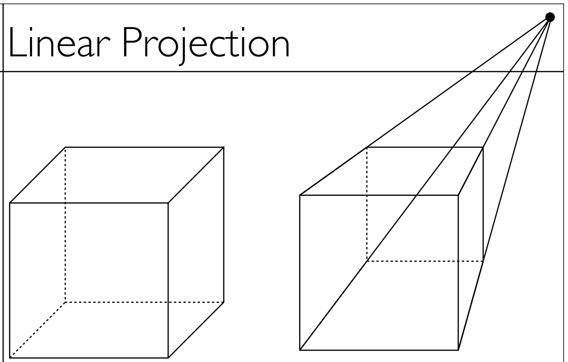


Linear Projection



Orthographic Perspective

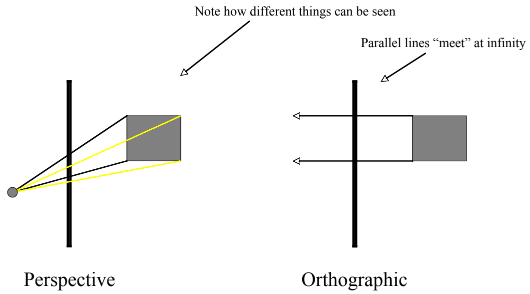
Linear Projection



Orthographic Perspective

Linear Projection

- A 2D view



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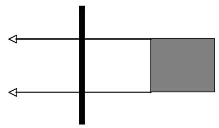
Orthographic Projection

- No foreshortening
- Parallel lines stay parallel
- Poor depth cues



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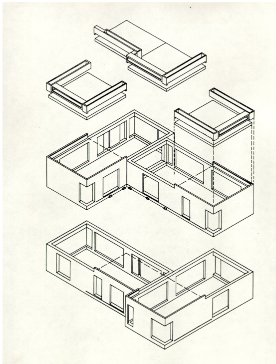
Orthographic Projection



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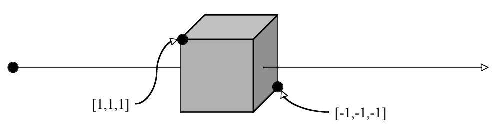
Canonical View Space

- Canonical view region
  - 3D: [-1,-1,-1] to [+1,+1,+1]
- Assume looking down $-Z$ axis
  - Recall that "Z is in your face"



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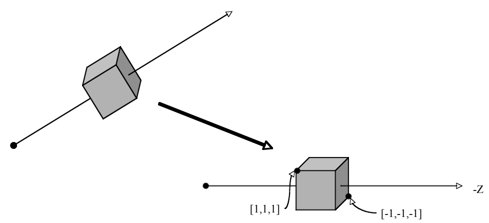
Orthographic Projection

• Convert arbitrary view volume to canonical



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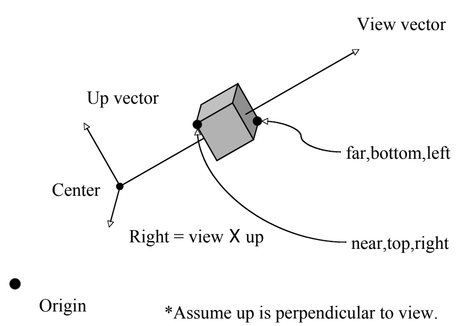
Orthographic Projection



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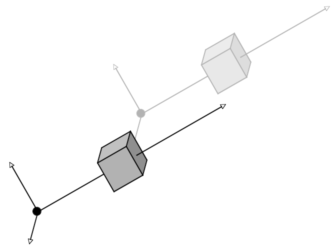
Orthographic Projection

- Step 1: translate center to origin



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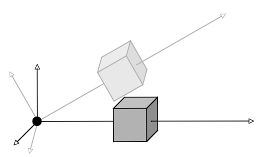
Orthographic Projection

- Step 1: translate center to origin
- Step 2: rotate ***view*** to **-Z** and ***up*** to **+Y**



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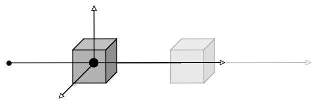
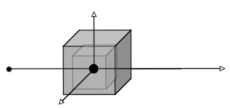
Orthographic Projection

- Step 1: translate center to origin
- Step 2: rotate ***view*** to **-Z** and ***up*** to **+Y**
- Step 3: center view volume



Orthographic Projection

- Step 1: translate center to origin
- Step 2: rotate ***view*** to **-Z** and ***up*** to **+Y**
- Step 3: center view volume
- Step 4: scale to canonical size

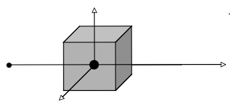


Orthographic Projection

- Step 1: translate center to origin
- Step 2: rotate ***view*** to **-Z** and ***up*** to **+Y**
- Step 3: center view volume
- Step 4: scale to canonical size

$$\mathbf{M} = \underbrace{\mathbf{S} \cdot \mathbf{T}_2}_{\mathbf{M}_o} \cdot \underbrace{\mathbf{R} \cdot \mathbf{T}_1}_{\mathbf{M}_v}$$

$$\mathbf{M} = \mathbf{M}_o \cdot \mathbf{M}_v$$



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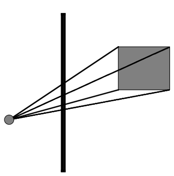
Perspective Projection

- Foreshortening: further objects appear smaller
- Some parallel line stay parallel, most don't
- Lines still look like lines
- **Z ordering preserved (where we care)**



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Perspective Projection

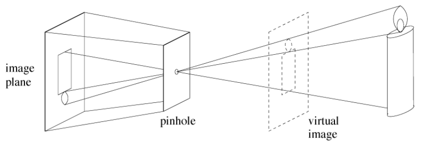

Image from D. Forsyth

Pinhole ***a.k.a*** center of projection

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Perspective Projection

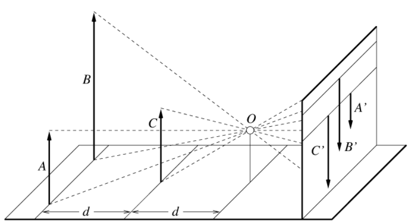

Image from D. Forsyth

Foreshortening: distant objects appear smaller

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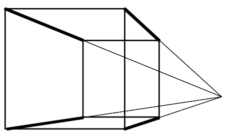
Perspective Projection

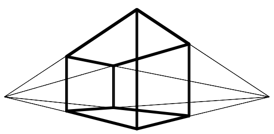
- Vanishing points
  - Depend on the scene
  - Not intrinsic to camera



"One point perspective"

Perspective Projection

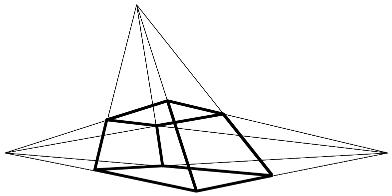
- Vanishing points
  - Depend on the scene
  - Nor intrinsic to camera



"Two point perspective"

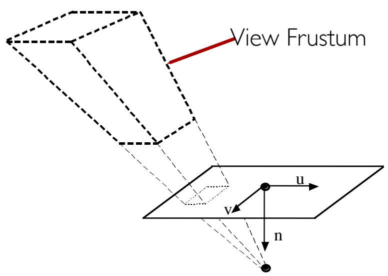
Perspective Projection

- Vanishing points
  - Depend on the scene
  - Not intrinsic to camera

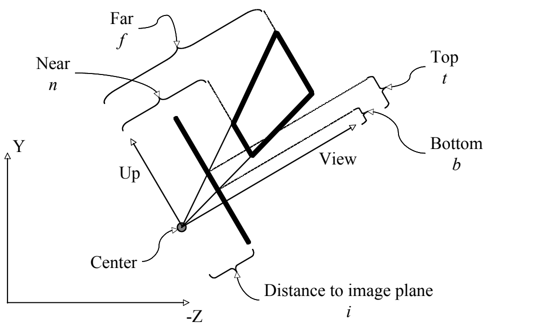


"Three point perspective"

Perspective Projection



Perspective Projection



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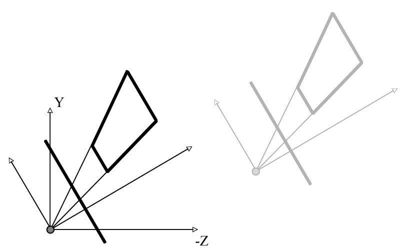
Perspective Projection

- Step 1: Translate ***center*** to origin



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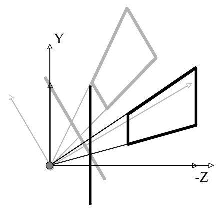
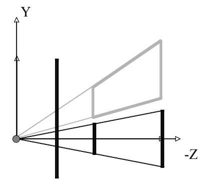
Perspective Projection

- Step 1: Translate ***center*** to origin
- Step 2: Rotate ***view*** to **-Z**, ***up*** to **+Y**



Perspective Projection

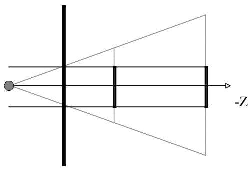
- Step 1: Translate ***center*** to origin
- Step 2: Rotate ***view*** to **-Z**, ***up*** to **+Y**
- Step 3: Shear center-line to **-Z** axis



Perspective Projection

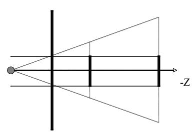
- Step 1: Translate ***center*** to origin
- Step 2: Rotate ***view*** to **-Z**, ***up*** to **+Y**
- Step 3: Shear center-line to **-Z** axis
- Step 4: Perspective

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{i+f}{i} & f \\ 0 & 0 & \frac{-1}{i} & 0 \end{bmatrix}$$



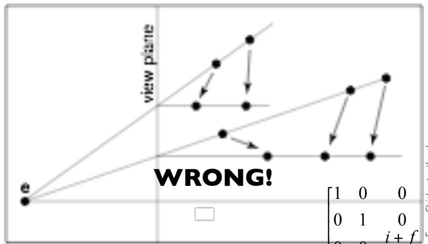
Perspective Projection

- Step 4: Perspective
  - Points at z=-i stay at z=-i
  - Points at z=-f stay at z=-f
  - Points at z=0 goto z=±∞
  - Points at z=-∞ goto z=-(i+f)
  - **x and y values divided by** -z/i
  - Straight lines stay straight
  - Depth ordering preserved in [-i,-f]
  - Movement along lines distorted



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{i+f}{i} & f \\ 0 & 0 & \frac{-1}{i} & 0 \end{bmatrix}$$

Perspective Projection



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{i+f}{i} & f \\ 0 & 0 & \frac{-1}{i} & 0 \end{bmatrix}$$

From Shirley textbook.

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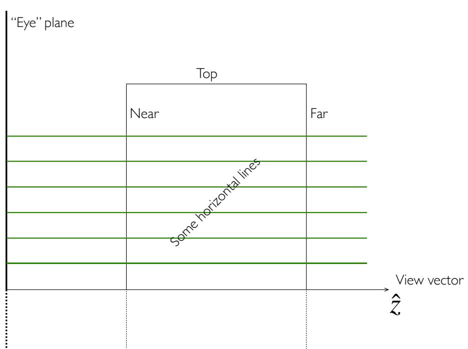
Perspective Projection



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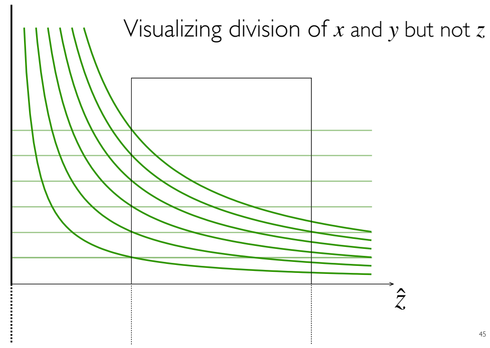
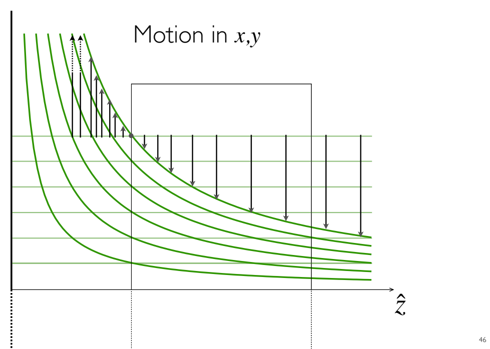
Perspective Projection

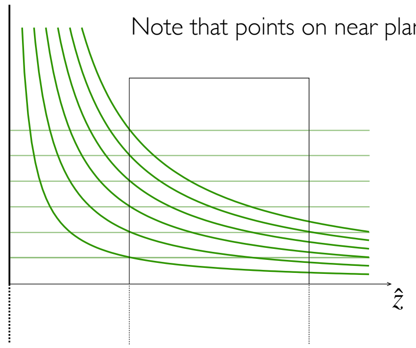


Perspective Projection



Perspective Projection

Note that points on near plane fixed



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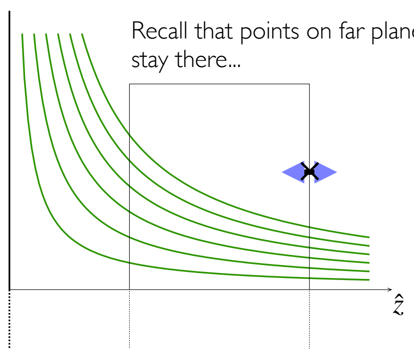
Perspective Projection

Recall that points on far plane will stay there...



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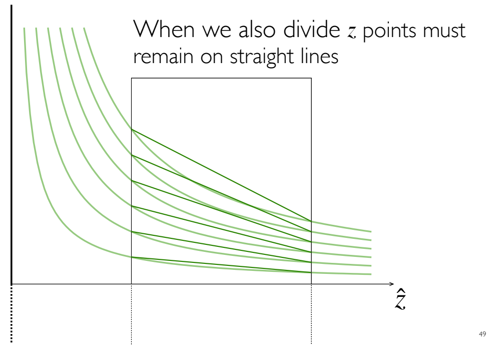
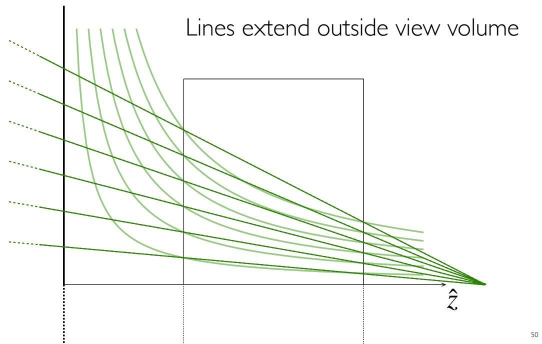
Perspective Projection

When we also divide z points must remain on straight lines

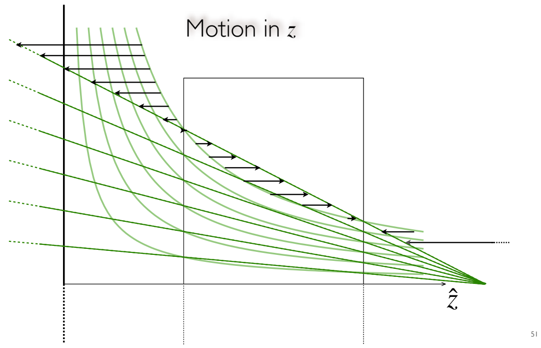


Perspective Projection

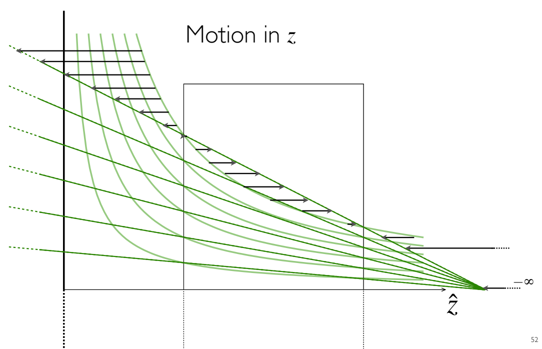
Lines extend outside view volume



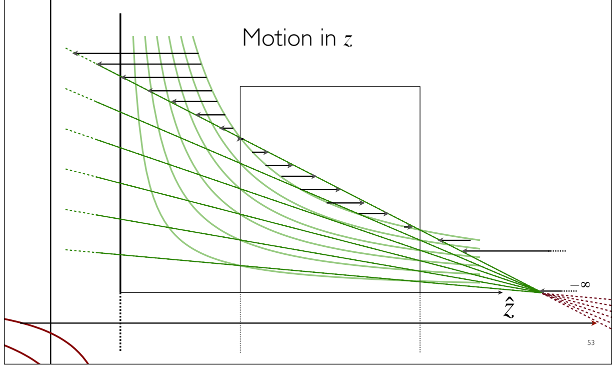
Perspective Projection



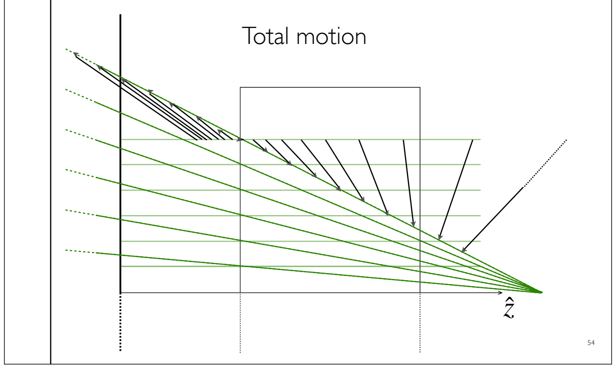
Perspective Projection



Perspective Projection

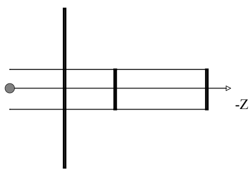


Perspective Projection



Perspective Projection

- Step 1: Translate ***center*** to orange
- Step 2: Rotate ***view*** to **-Z**, ***up*** to **+Y**
- Step 3: Shear center-line to **-Z** axis
- Step 4: Perspective
- Step 5: center view volume
- Step 6: scale to canonical size



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Perspective Projection

- Step 1: Translate ***center*** to orange
- Step 2: Rotate ***view*** to **-Z**, ***up*** to **+Y**

$\} \mathbf{M}_v$

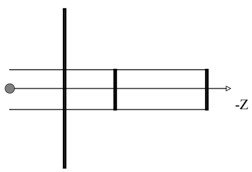
- Step 3: Shear center-line to **-Z** axis
- Step 4: Perspective

$\} \mathbf{M}_p$

- Step 5: center view volume
- Step 6: scale to canonical size

$\} \mathbf{M}_o$

$$\mathbf{M} = \mathbf{M}_o \cdot \mathbf{M}_p \cdot \mathbf{M}_v$$



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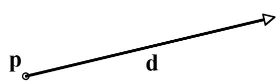
Perspective Projection

- There are other ways to set up the projection matrix
  - View plane at $z=0$ zero
  - Looking down another axis
  - *etc...*
- Functionally equivalent

Vanishing Points

- Consider a ray:

$$\mathbf{r}(t) = \mathbf{p} + t\,\mathbf{d}$$



Vanishing Points

- Ignore **Z** part of matrix
- **X and Y will give location in image plane**
- Assume image plane at $z=-i$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ & \text{whatever} & & \\ 0 & 0 & -1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} I_x \\ I_y \\ I_w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Vanishing Points

$$\begin{bmatrix} I_x \\ I_y \\ I_w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z \end{bmatrix}$$

$$\begin{bmatrix} I_x / I_w \\ I_y / I_w \end{bmatrix} = \begin{bmatrix} -x/z \\ -y/z \end{bmatrix}$$

Vanishing Points

- Assume

$$d_z = -1$$

$$\begin{bmatrix} I_x / I_w \\ I_y / I_w \end{bmatrix} = \begin{bmatrix} -x/z \\ -y/z \end{bmatrix} = \begin{bmatrix} \dfrac{p_x + td_x}{-p_z + t} \\ \dfrac{p_y + td_y}{-p_z + t} \end{bmatrix}$$

$$\lim_{t \to \pm\infty} = \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

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Vanishing Points

$$\lim_{t \to \pm\infty} = \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

- All lines in direction $\mathbf{d}$ converge to same point in the image plane -- the vanishing point
- Every point in plane is a v.p. for some set of lines
- Lines parallel to image plane ($d_z = 0$) vanish at infinity

What's a horizon?

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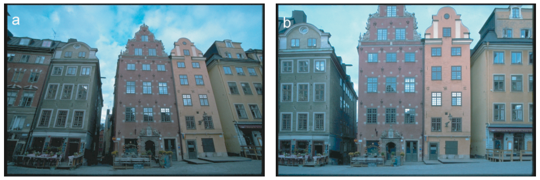
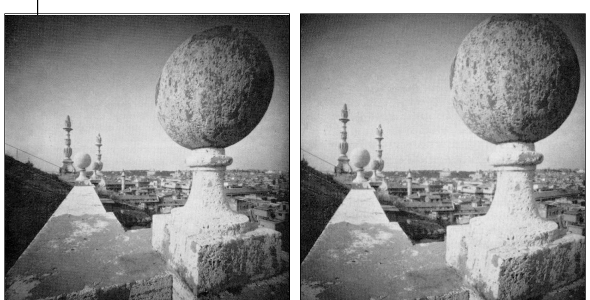
Perspective Tricks

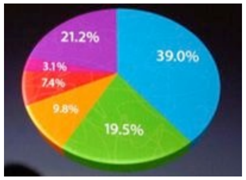


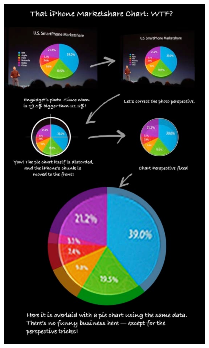
Right Looks Wrong (Sometimes)



From *Correction of Geometric Perceptual Distortions in Pictures*, Zorin and Barr SIGGRAPH 1995

Right Looks Wrong (Sometimes)



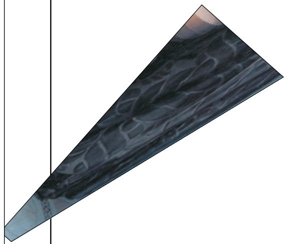


From WIRED Magazine

Strangeness

The Ambassadors
by Hans Holbein the Younger

Strangeness



The Ambassadors
by Hans Holbein the Younger

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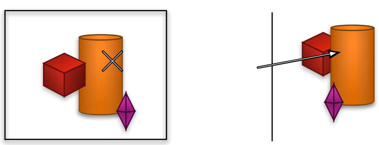
Ray Picking

- Pick object by picking point on screen



- Compute ray from pixel coordinates.

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Ray Picking

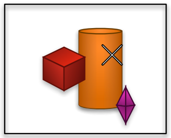
- Transform from World to Screen is:

$$\begin{bmatrix} I_x \\ I_y \\ I_z \\ I_w \end{bmatrix} = \mathbf{M} \begin{bmatrix} W_x \\ W_y \\ W_z \\ W_w \end{bmatrix}$$

- Inverse:

$$\begin{bmatrix} W_x \\ W_y \\ W_z \\ W_w \end{bmatrix} = \mathbf{M}^{-1} \begin{bmatrix} I_x \\ I_y \\ I_z \\ I_w \end{bmatrix}$$

- What $\mathbf{Z}$ value?



Ray Picking

Depends on screen details, YMMV
General idea should translate...

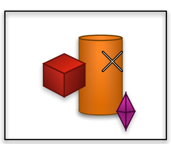
- Recall that:
  - Points at z=-i stay at z=-i
  - Points at z=-f stay at z=-f

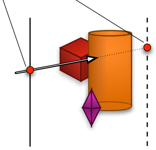
$$\mathbf{r}(t) = \mathbf{p} + t\,\mathbf{d}$$

$$\mathbf{r}(t) = \mathbf{a}_w + t(\mathbf{b}_w - \mathbf{a}_w)$$

$$\mathbf{a}_s = [s_x, s_y, -i]$$

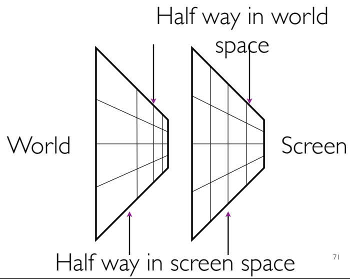
$$\mathbf{b}_s = [s_x, s_y, -f]$$





Depth Distortion

- Recall depth distortion from perspective
  - Interpolating in screen space different than in world
  - Ok, for shading (mostly)
  - Bad for texture



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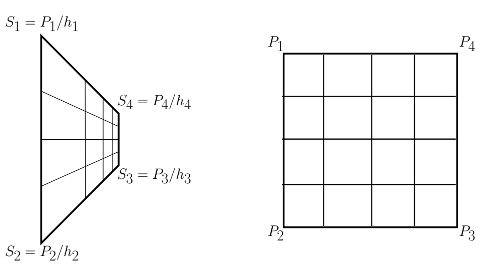
Depth Distortion



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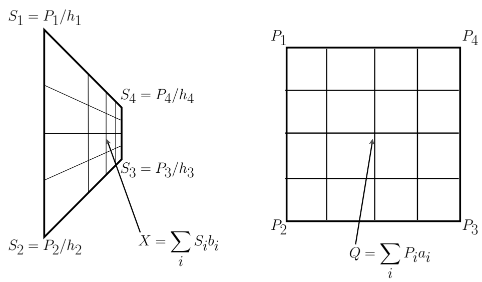
Depth Distortion



We know the S_i , P_i , and b_i , but not the a_i .

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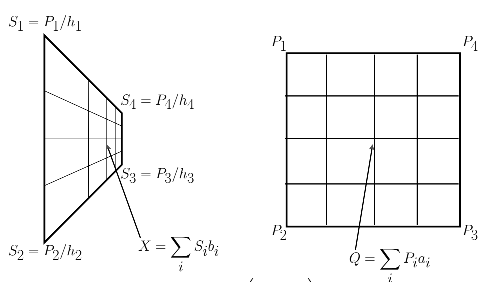
Depth Distortion



$$X = Q/h = \left(\sum_i P_i a_i\right) / \left(\sum_j h_j a_j\right)$$

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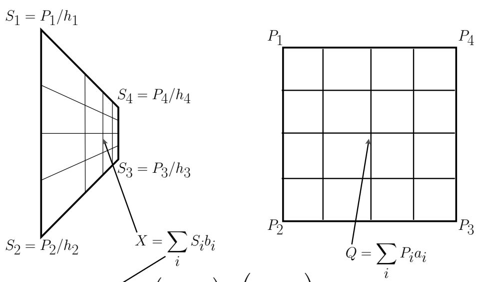
Depth Distortion



$$\sum_i S_i b_i = \left(\sum_i P_i a_i\right) / \left(\sum_j h_j a_j\right)$$

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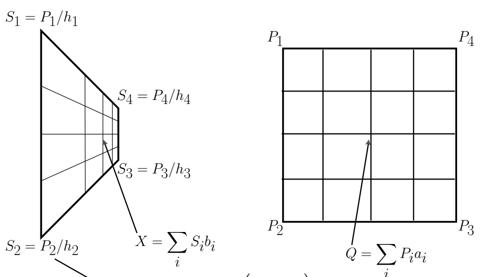
Depth Distortion



$$\sum_i P_i b_i / h_i = \left(\sum_i P_i a_i\right) / \left(\sum_j h_j a_j\right)$$

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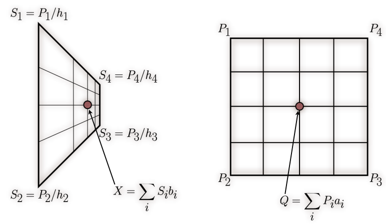
Depth Distortion

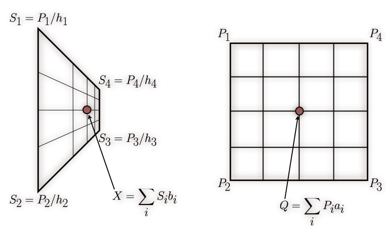


$$\sum_i P_i b_i/h_i = \left(\sum_i P_i a_i\right) / \left(\sum_j h_j a_j\right)$$

Independent of given vertex locations.

$$b_i/h_i = a_i/\left(\sum_j h_j a_j\right) \quad \forall i$$

Depth Distortion

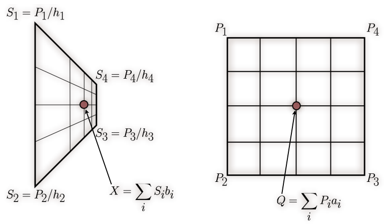


$$b_i/h_i = a_i/\left(\sum_j h_j a_j\right) \quad \forall i$$

Linear equations in the a_i.

$$\left(\sum_j h_j a_j\right) b_i/h_i - a_i = 0 \quad \forall i$$

Depth Distortion



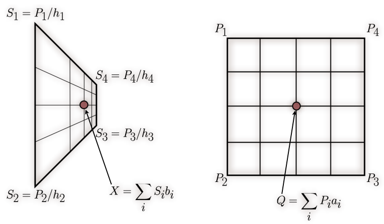
Linear equations in the a_i .

$$\left(\sum_j h_j a_j\right) b_i/h_i - a_i = 0 \quad \forall i$$

Not invertible so add some extra constraints.

$$\sum_i a_i = \sum_i b_i = 1$$

Depth Distortion



For a line: $a_1 = h_2 b_i/(b_1 h_2 + h_1 b_2)$

For a triangle: $a_1 = h_2 h_3 b_1/(h_2 h_3 b_1 + h_1 h_3 b_2 + h_1 h_2 b_3)$

Obvious Permutations for other coefficients.