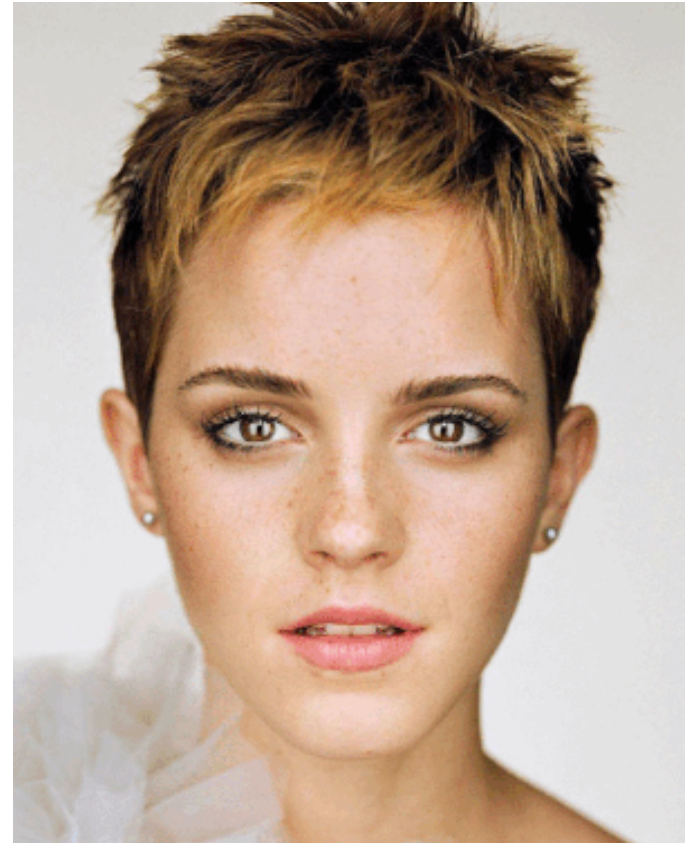


Image Morphing



© Rachel Albert, CS194-26, Fall 2015

CS184: Computer Graphics
Alexei Efros, UC Berkeley, Fall 2016

Image Warping in Biology

D'Arcy Thompson

<http://www-groups.dcs.st-and.ac.uk/~history/Miscellaneous/darcy.html>

http://en.wikipedia.org/wiki/D'Arcy_Thompson

Importance of shape and structure in evolution

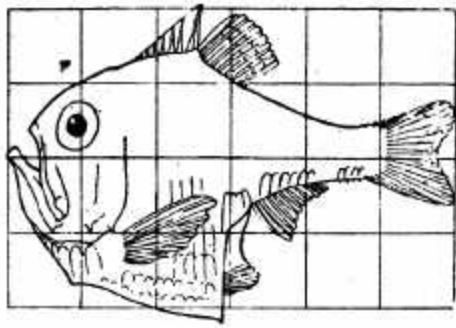
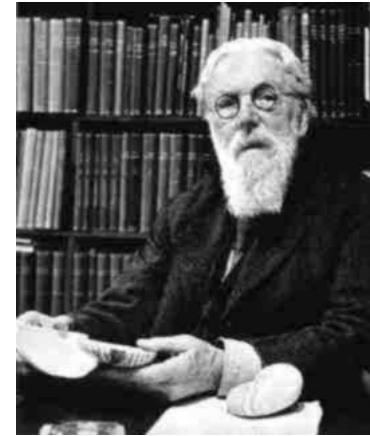
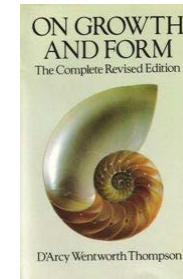


Fig. 517. *Argyropelecus Olfersi*.

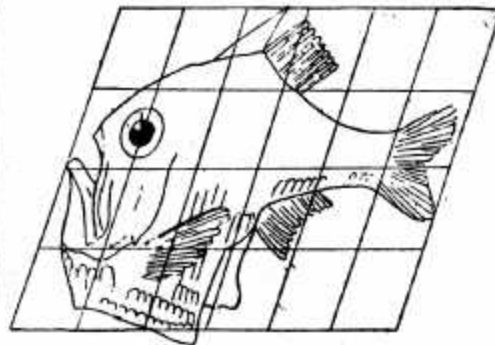
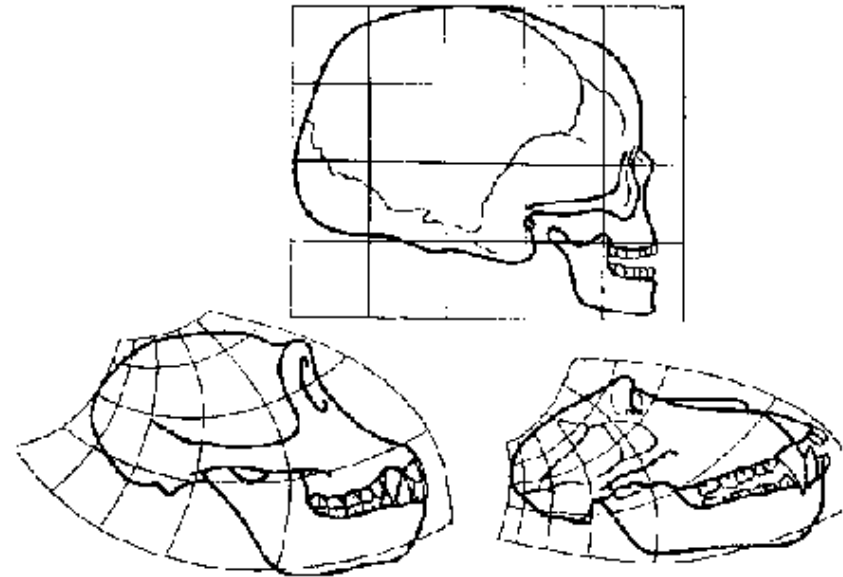


Fig. 518. *Sternoptyx diaphana*.



Skulls of a human, a chimpanzee and a baboon and transformations between them

Morphing = Object Averaging



The aim is to find “an average” between two objects

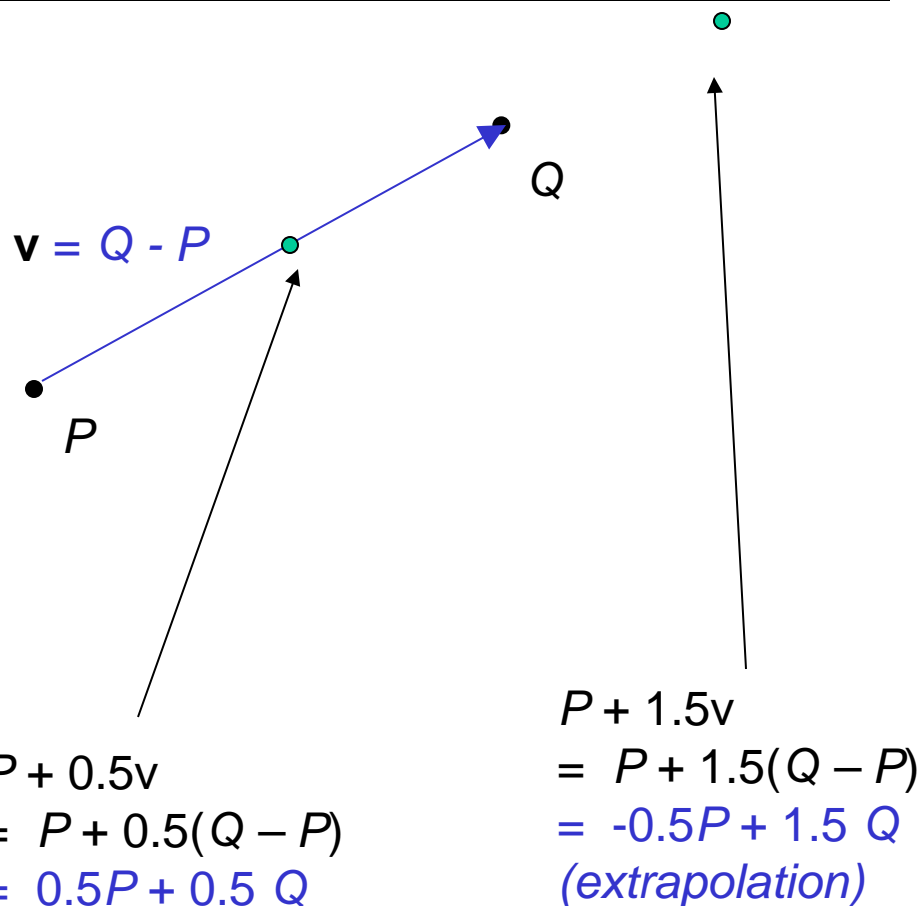
- Not an average of two images of objects...
- ...but an image of the average object!
- How can we make a smooth transition in time?
- Do a “weighted average” over time t !

How do we know what the average object looks like?

- We haven't a clue!
- But we can often fake something reasonable
 - Usually requires user/artist input

Averaging Points

What's the average
of P and Q?



Linear Interpolation
(Affine Combination):
New point $aP + bQ$,
defined only when $a+b = 1$
So $aP+bQ = aP+(1-a)Q$

P and Q can be anything:

- points on a plane (2D) or in space (3D)
- Colors in RGB or HSV (3D)
- Whole images (m-by-n D)... etc.

Idea #1: Cross-Dissolve



Interpolate whole images:

$$\text{Image}_{\text{halfway}} = (1-t) \cdot \text{Image}_1 + t \cdot \text{Image}_2$$

This is called **cross-dissolve** in film industry

But what if the images are not aligned?

Idea #2: Align, then cross-dissolve



Align first, then cross-dissolve

- Alignment using global warp – picture still valid

Parametric (global) warping

Examples of parametric warps:



translation



rotation



aspect



affine

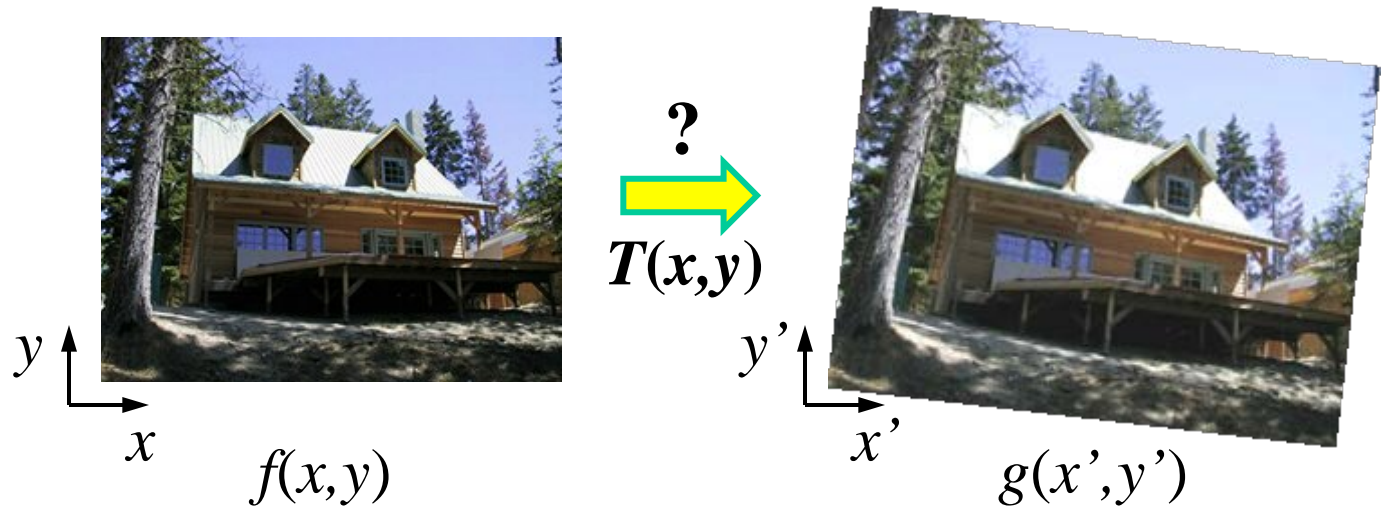


perspective



cylindrical

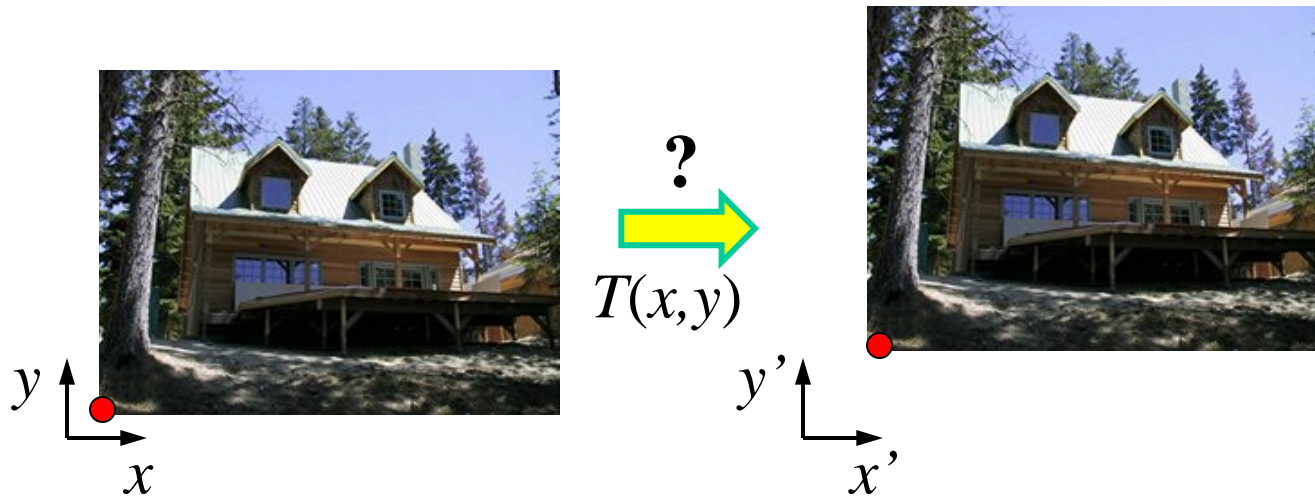
Recovering Transformations



What if we know f and g and want to recover the transform T ?

- willing to let user provide correspondences
 - How many do we need?

Translation: # correspondences?



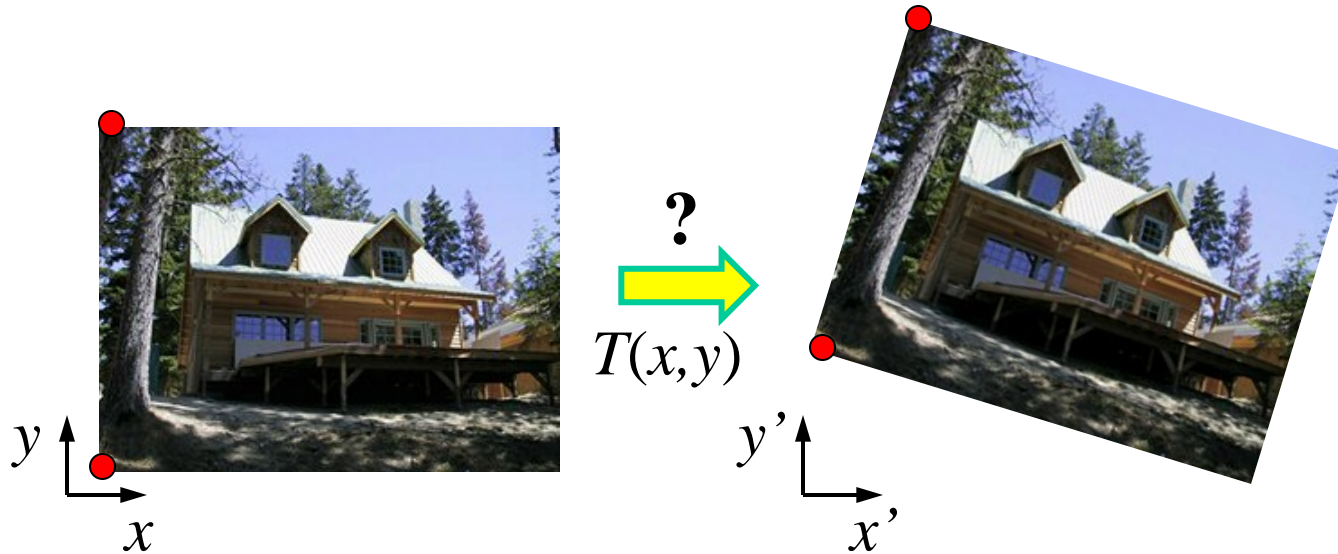
How many correspondences needed for translation?

How many Degrees of Freedom?

What is the transformation matrix?

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & p'_x - p_x \\ 0 & 1 & p'_y - p_y \\ 0 & 0 & 1 \end{bmatrix}$$

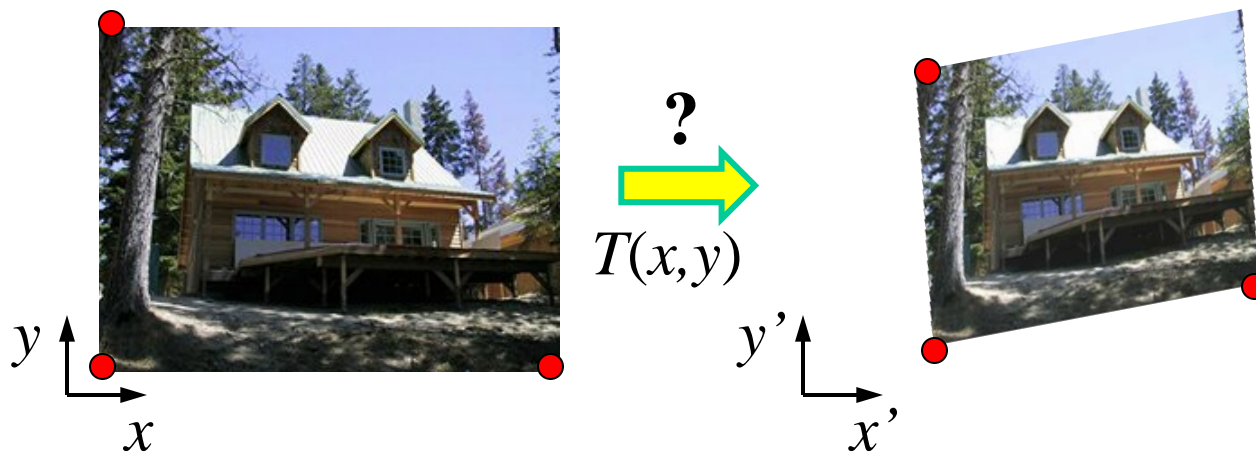
Euclidian: # correspondences?



How many correspondences needed for translation+rotation?

How many DOF?

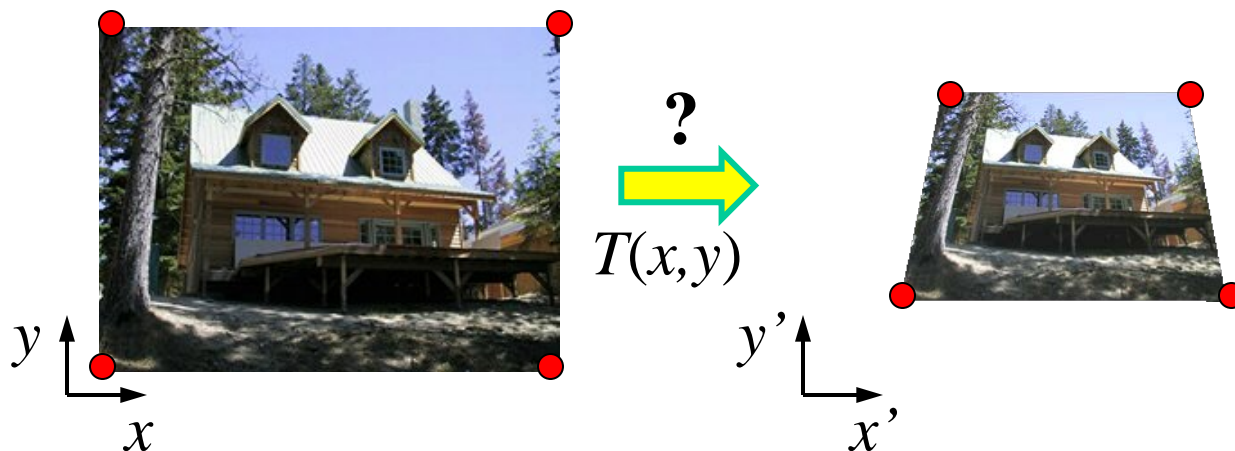
Affine: # correspondences?



How many correspondences needed for affine?

How many DOF?

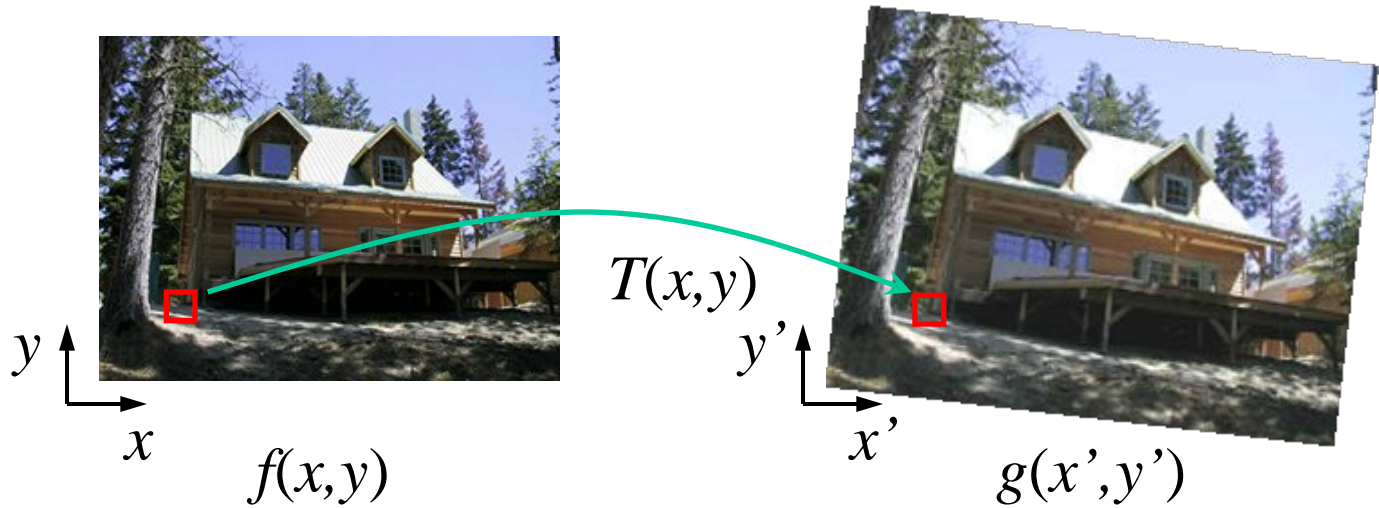
Projective: # correspondences?



How many correspondences needed for projective?

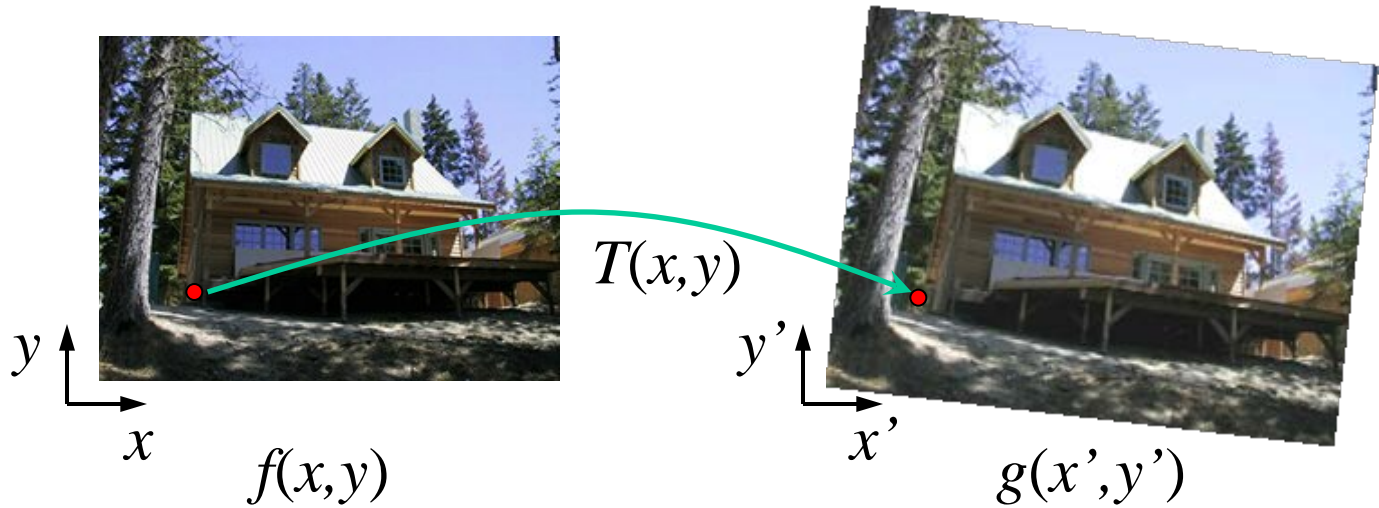
How many DOF?

Image warping



Given a coordinate transform $(x',y') = T(x,y)$ and a source image $f(x,y)$, how do we compute a transformed image $g(x',y') = f(T(x,y))$?

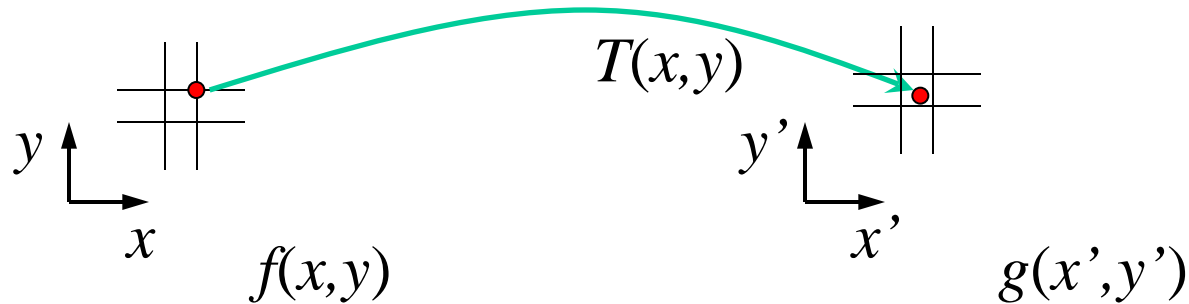
Forward warping



Send each pixel $f(x,y)$ to its corresponding location
 $(x',y') = T(x,y)$ in the second image

Q: what if pixel lands “between” two pixels?

Forward warping



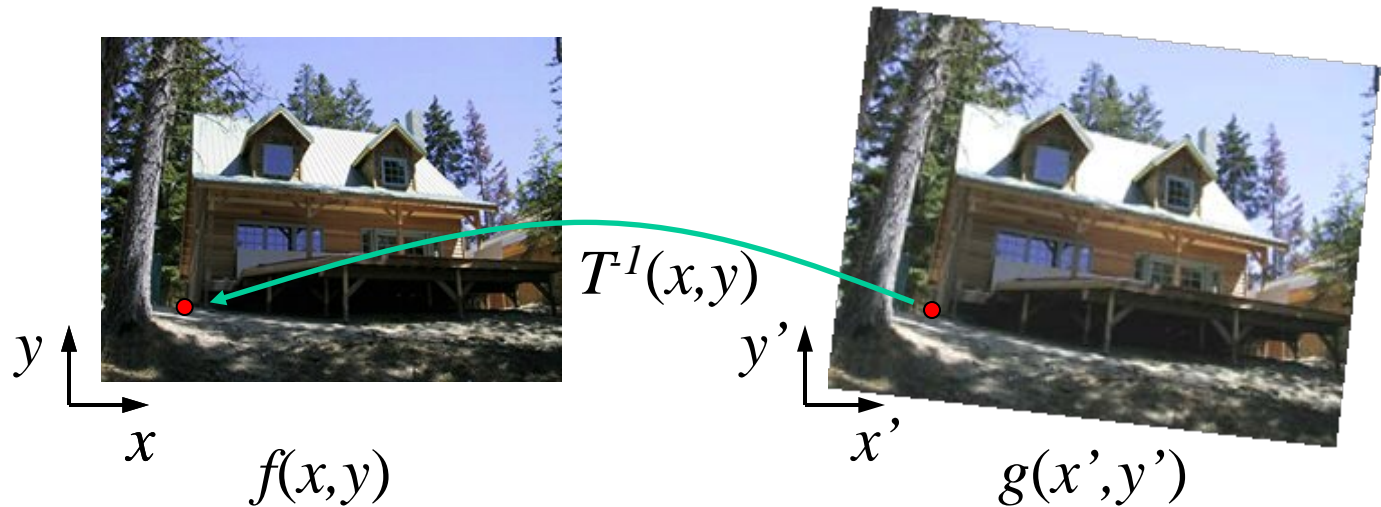
Send each pixel $f(x, y)$ to its corresponding location
 $(x', y') = T(x, y)$ in the second image

Q: what if pixel lands “between” two pixels?

A: distribute color among neighboring pixels (x', y')

- Known as “splatting”
- Check out `griddata` in Matlab

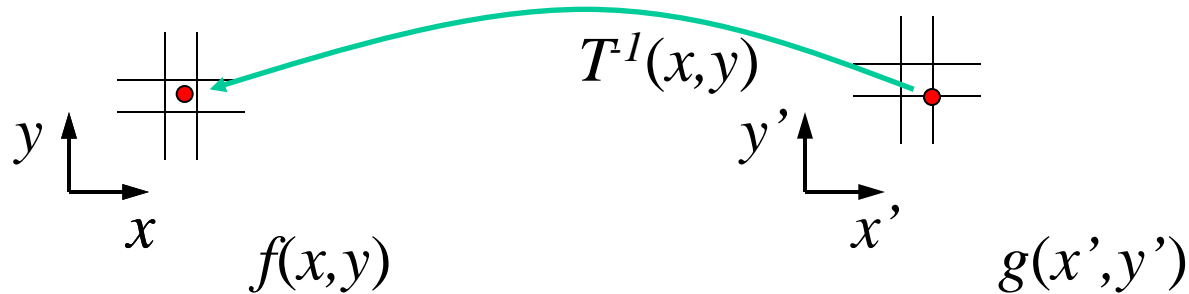
Inverse warping



Get each pixel $g(x', y')$ from its corresponding location
 $(x, y) = T^{-1}(x', y')$ in the first image

Q: what if pixel comes from “between” two pixels?

Inverse warping



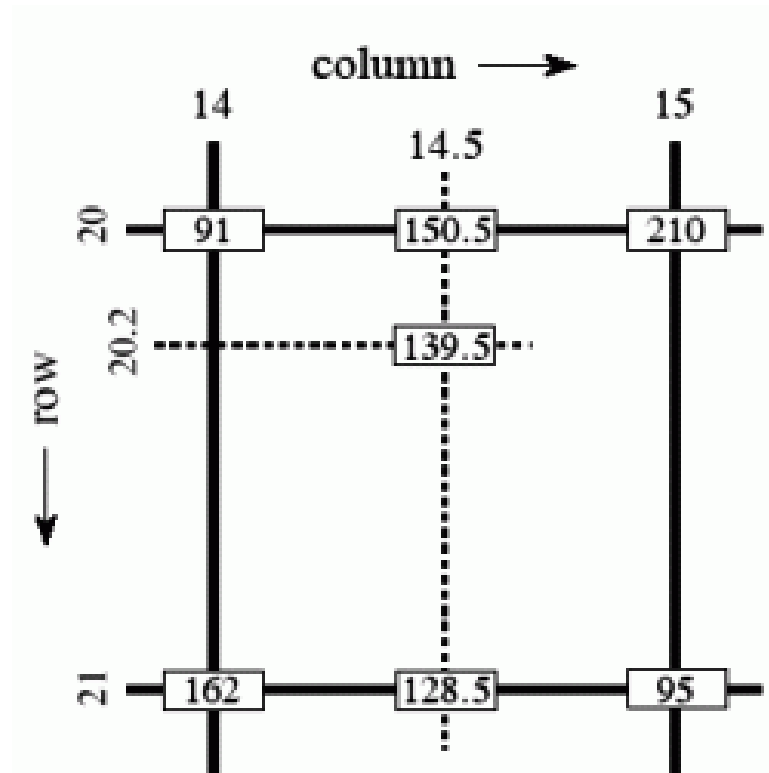
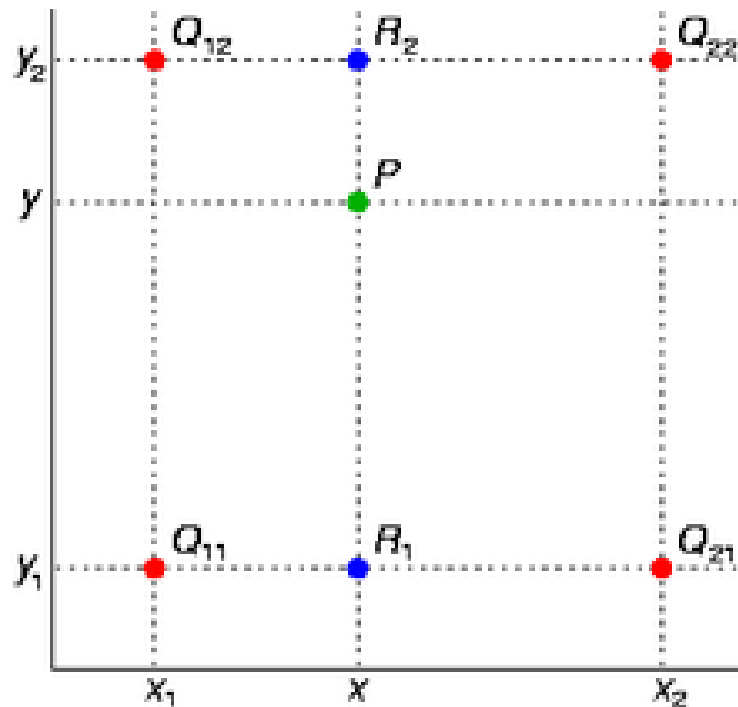
Get each pixel $g(x', y')$ from its corresponding location $(x, y) = T^{-1}(x', y')$ in the first image

Q: what if pixel comes from “between” two pixels?

A: *Interpolate* color value from neighbors

- nearest neighbor, bilinear, Gaussian, bicubic
- Check out `interp2` in Matlab

Bilinear Interpolation



Forward vs. inverse warping

Q: which is better?

A: usually inverse—eliminates holes

- however, it requires an invertible warp function—not always possible...

Global warp not always enough!



What to do?

- Cross-dissolve doesn't work
- Global alignment doesn't work
 - Cannot be done with a global transformation (e.g. affine)
- Any ideas?

Feature matching!

- Nose to nose, tail to tail, etc.
- This is a local (non-parametric) warp

Local (non-parametric) Image Warping

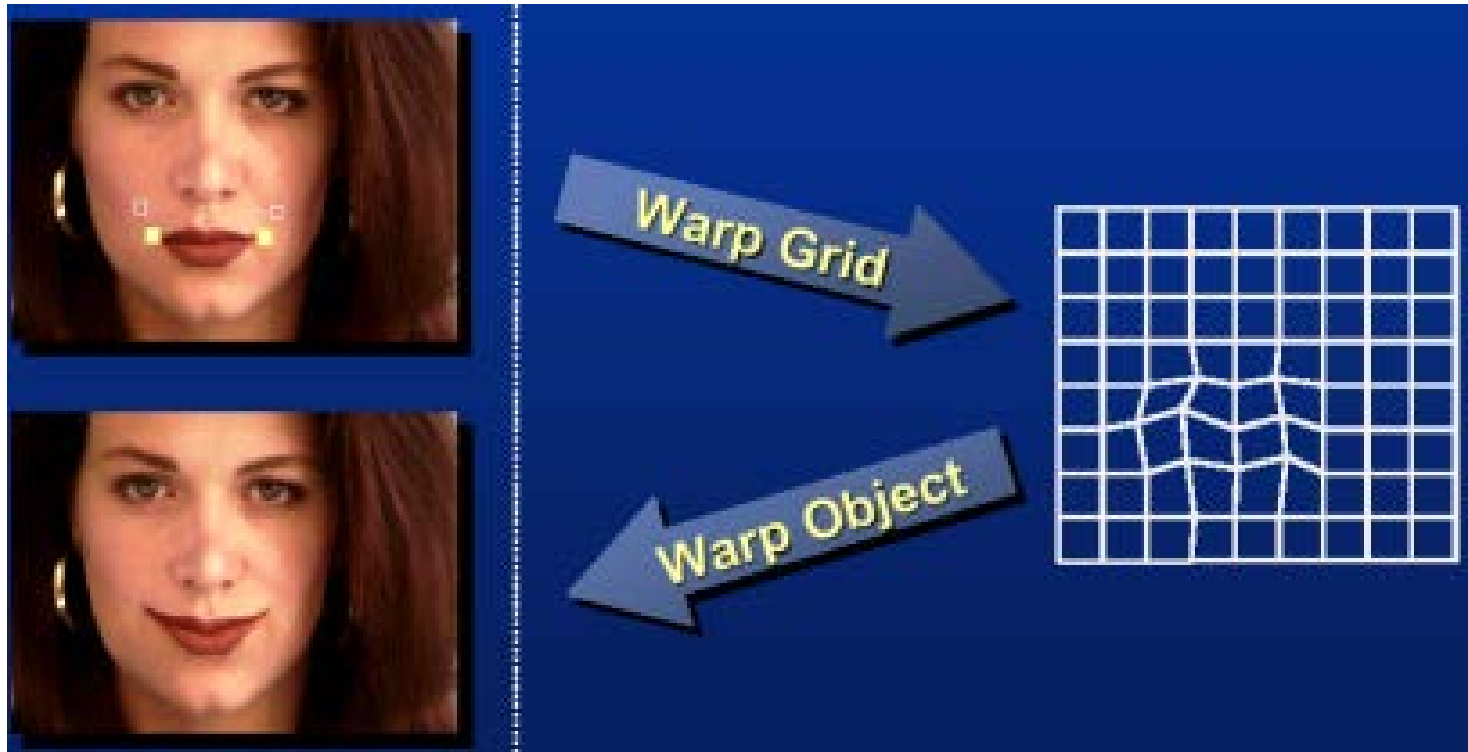


Need to specify a more detailed warp function

- Global warps were functions of a few (2,4,8) parameters
- Non-parametric warps $u(x,y)$ and $v(x,y)$ can be defined independently for every single location x,y !
- Once we know vector field u,v we can easily warp each pixel (use backward warping with interpolation)

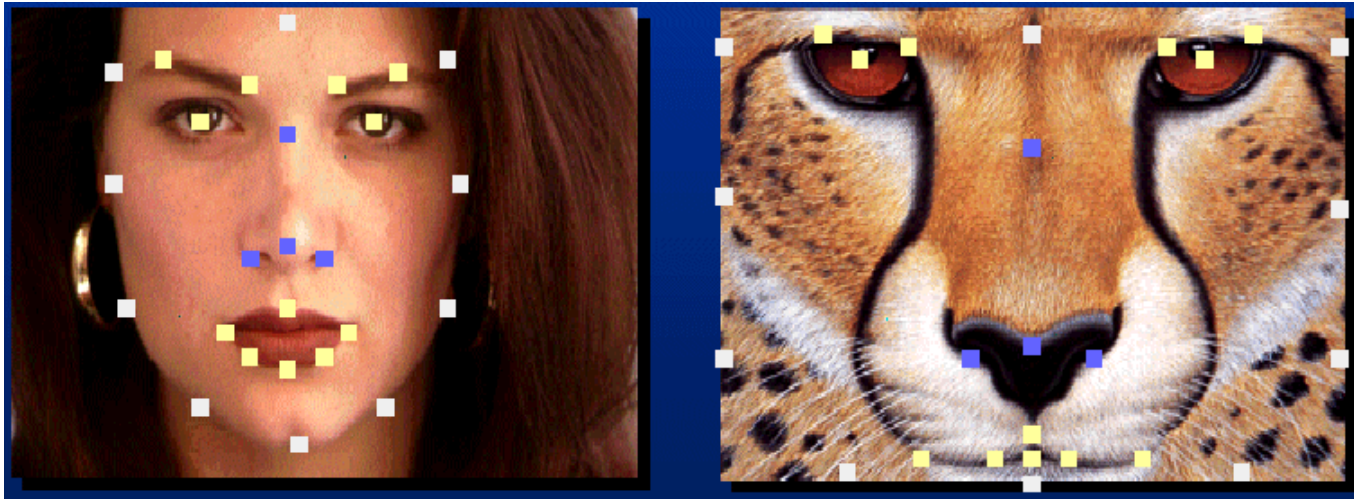
Warp specification -- dense

Define vector field to specify a dense warp



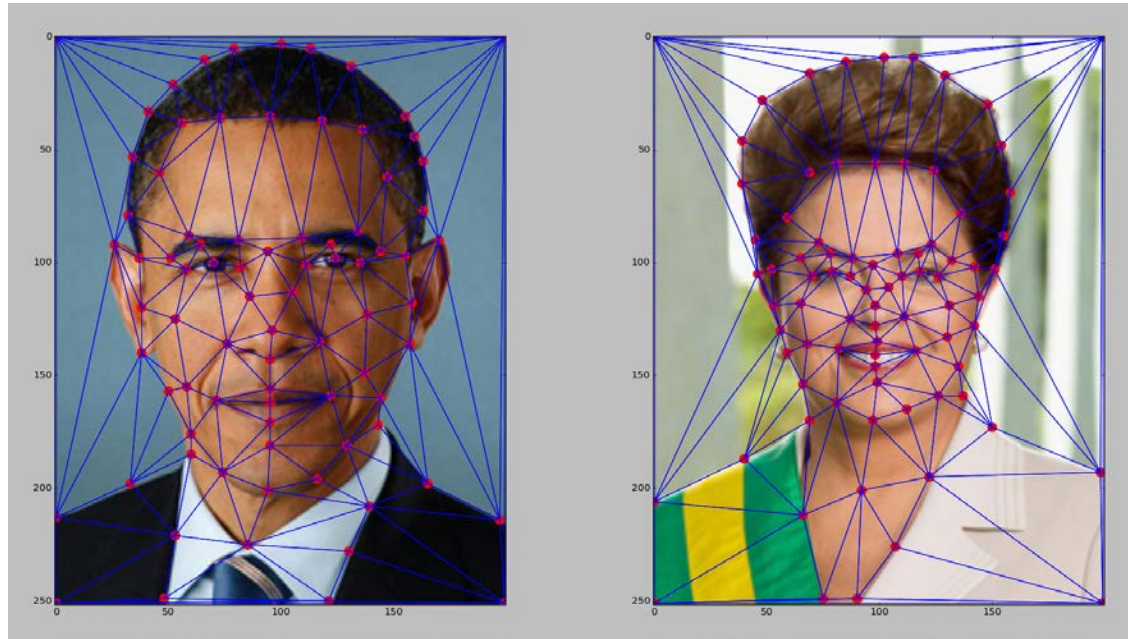
Warp specification - sparse

How can we specify a sparse warp?



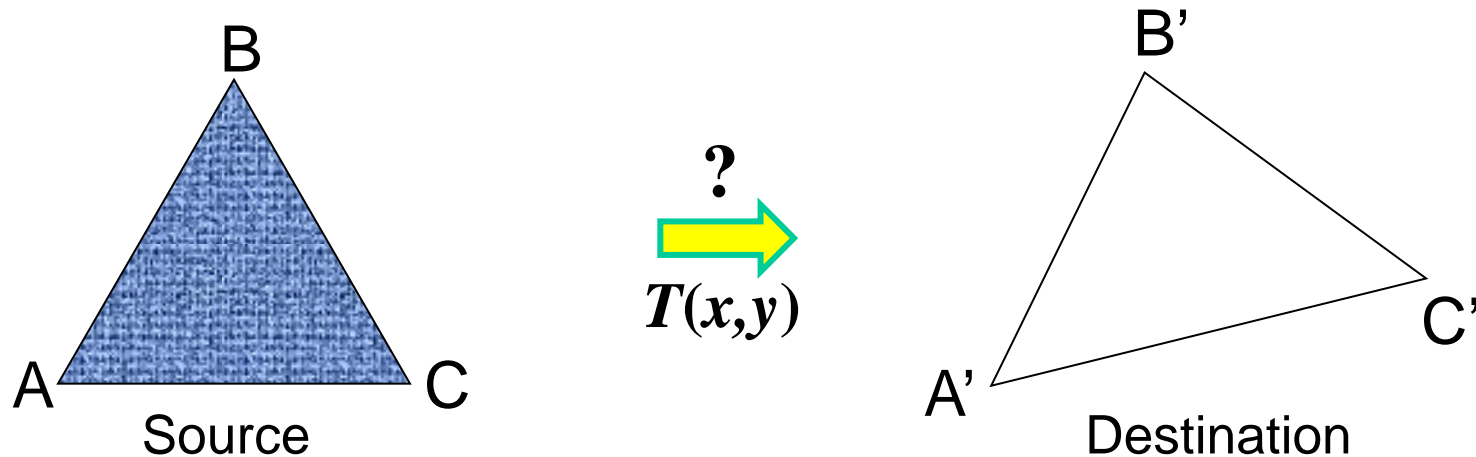
How do we go from feature points to pixels?

Triangular Mesh



1. Input correspondences at key feature points
2. Define a triangular mesh over the points
 - Same mesh in both images!
 - Now we have triangle-to-triangle correspondences
3. Warp each triangle separately from source to destination
 - How do we warp a triangle?

Warping triangles



Given two triangles: ABC and A'B'C' in 2D (12 numbers)
Need to find transform T to transfer all pixels from one to the other.

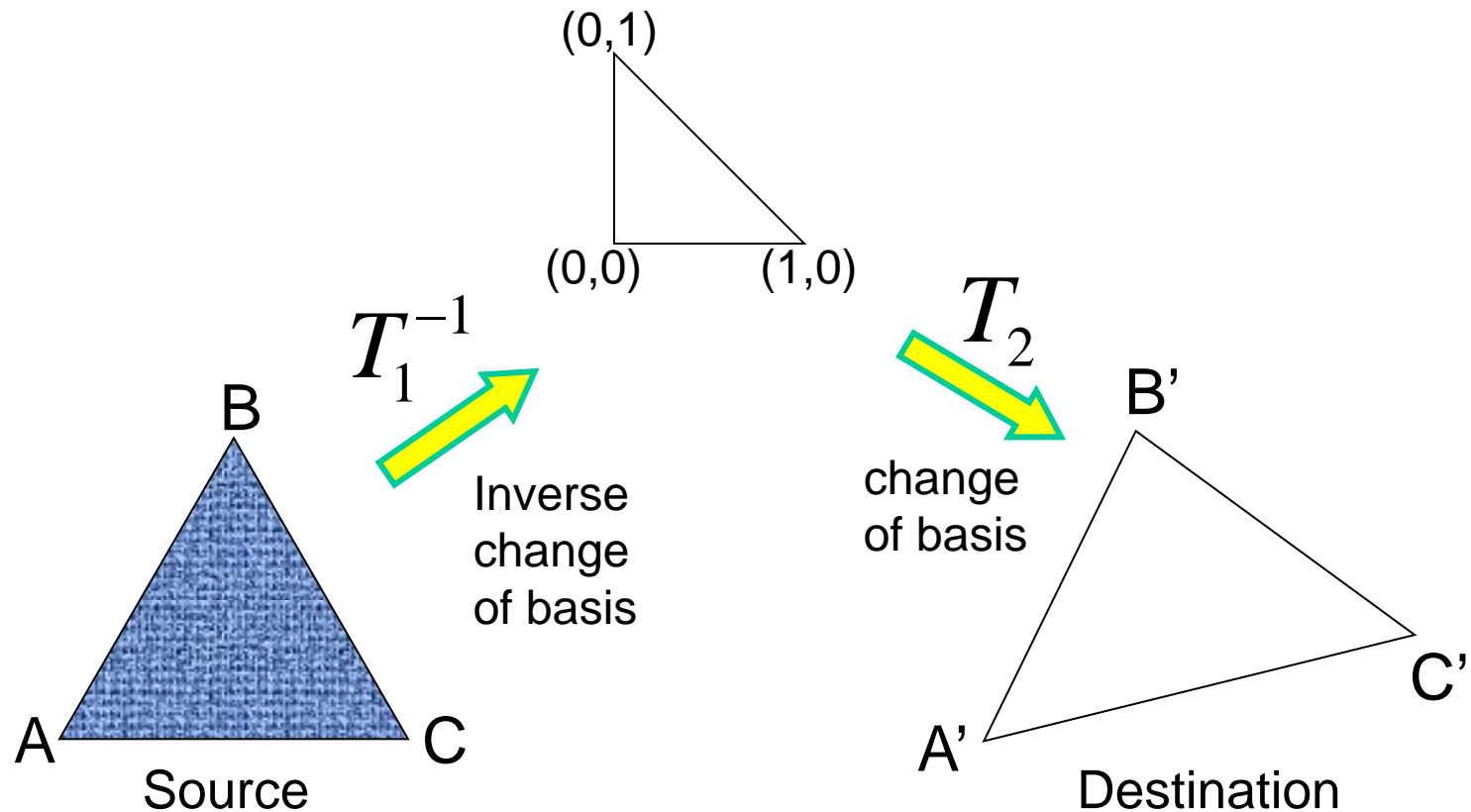
What kind of transformation is T?

How can we compute the transformation matrix:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Two ways:
Algebraic and
geometric

warping triangles (Barycentric Coordinates)

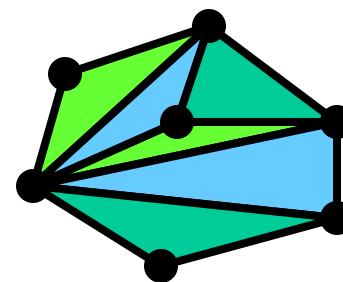
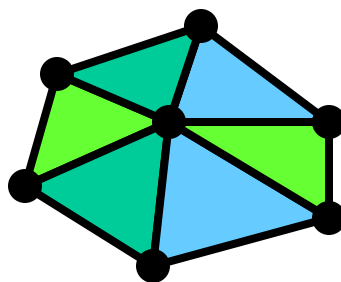
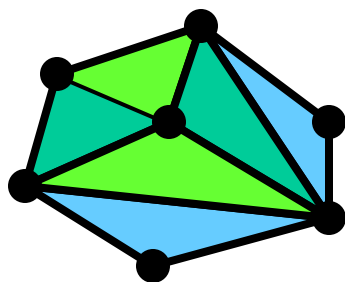


Don't forget to move the origin too!

Triangulations

A *triangulation* of set of points in the plane is a *partition* of the convex hull to triangles whose vertices are the points, and do not contain other points.

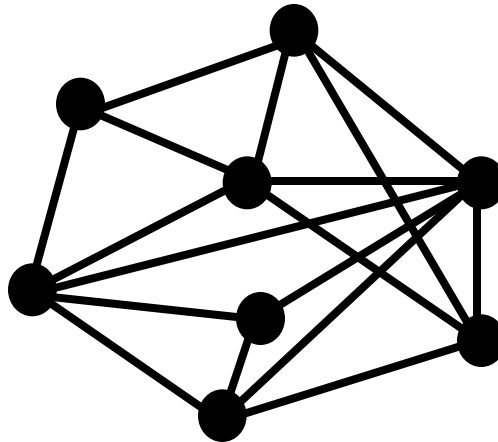
There are an exponential number of triangulations of a point set.



An $O(n^3)$ Triangulation Algorithm

Repeat until impossible:

- Select two sites.
- If the edge connecting them does not intersect previous edges, keep it.



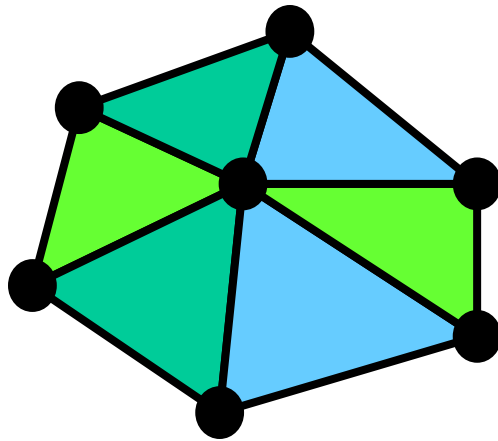
“Quality” Triangulations

Let $\alpha(T) = (\alpha_1, \alpha_2, \dots, \alpha_{3t})$ be the vector of angles in the triangulation T in increasing order.

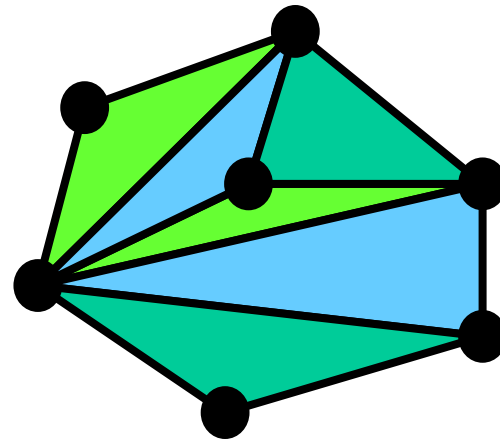
A triangulation T_1 will be “better” than T_2 if $\alpha(T_1) > \alpha(T_2)$ lexicographically.

The Delaunay triangulation is the “best”

- Maximizes smallest angles



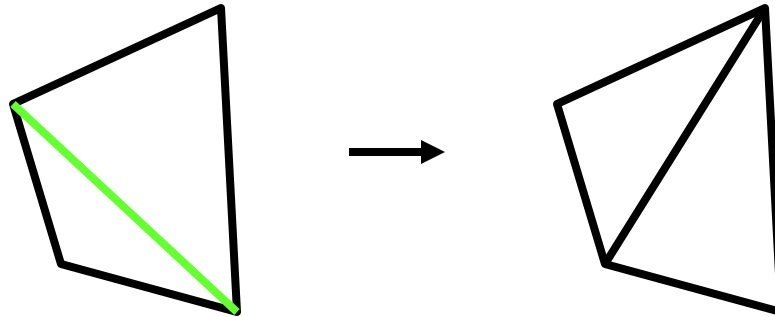
good



bad

Improving a Triangulation

In any convex quadrangle, an *edge flip* is possible. If this flip *improves* the triangulation locally, it also improves the global triangulation.

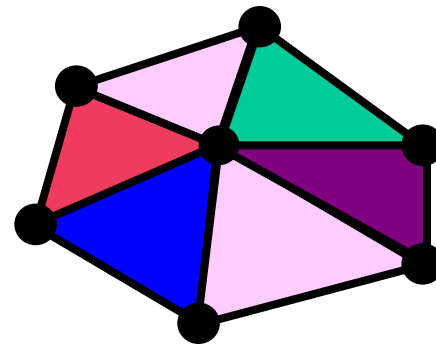
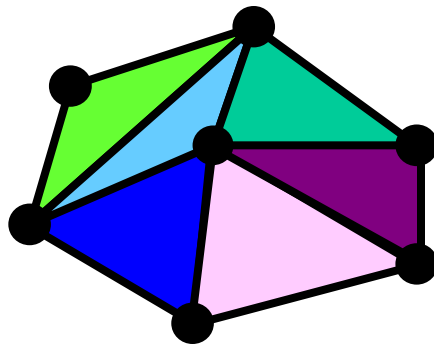
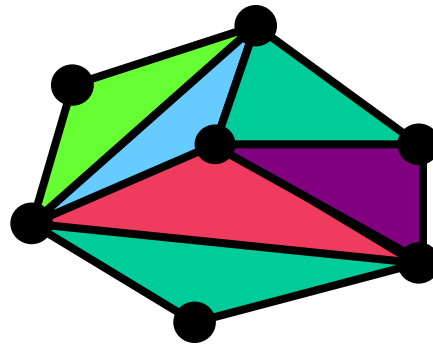
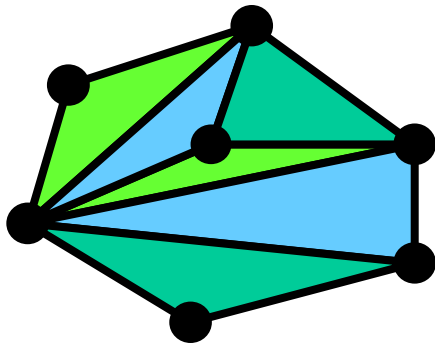


If an edge flip improves the triangulation, the first edge is called *illegal*.

Naïve Delaunay Algorithm

Start with an arbitrary triangulation. Flip any illegal edge until no more exist.

Could take a long time to terminate.



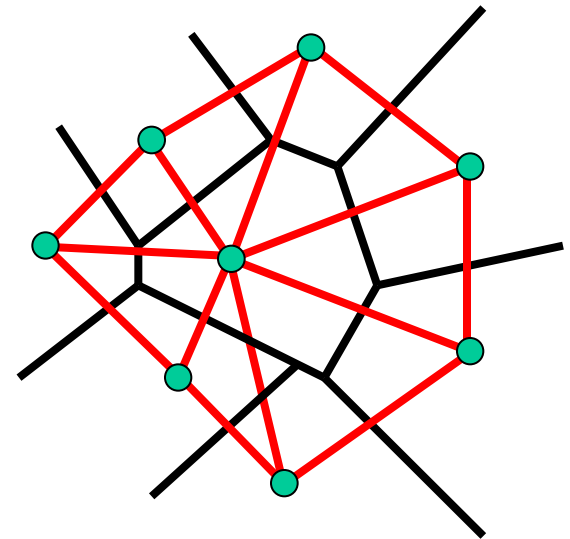
Delaunay Triangulation by Duality

General position assumption: There are no four co-circular points.

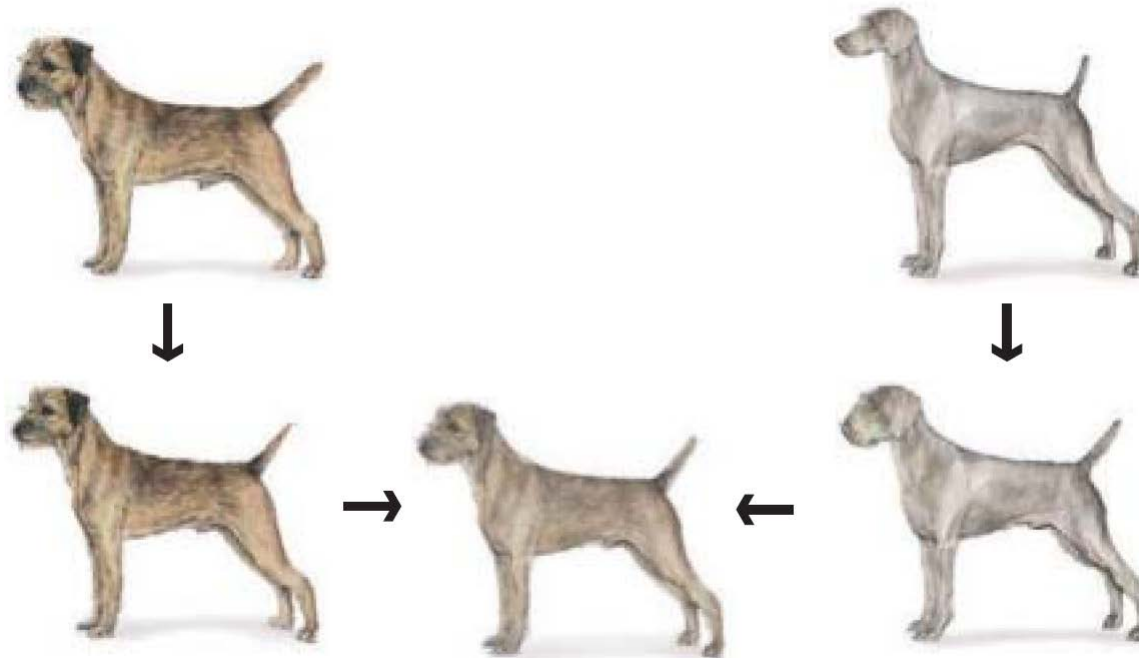
Draw the dual to the Voronoi diagram by connecting each two neighboring sites in the Voronoi diagram.

Corollary: The DT may be constructed in $O(n \log n)$ time.

This is what Matlab's `delaunay` function uses.



Full Morphing Procedure



Morphing procedure:

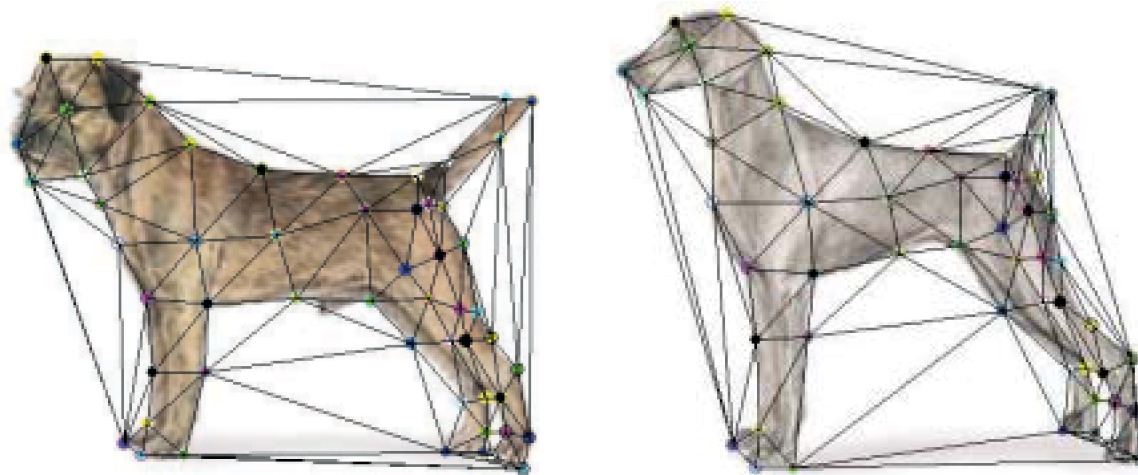
for every t ,

1. Find the average shape (the “mean dog” 😊)
 - local warping
2. Find the average color
 - Cross-dissolve the warped images

1. Create Average Shape

How do we create an intermediate warp at time t ?

- Assume $t = [0,1]$
- Simple linear interpolation of each feature pair
 - $p=(x,y) \rightarrow p'(x,y)$
- $(1-t)*p+t*p'$ for corresponding features p and p'



2. Create Average Color



Interpolate whole images:

$$\text{Image}_{\text{halfway}} = (1-t) * \text{Image} + t * \text{image}'$$

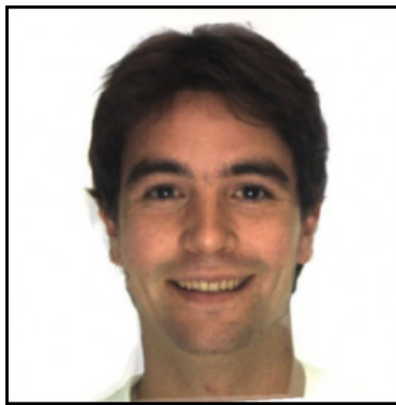
cross-dissolve!

Morphing & matting

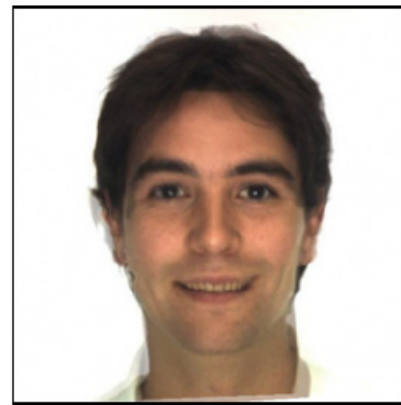
Extract foreground first to avoid artifacts in the background



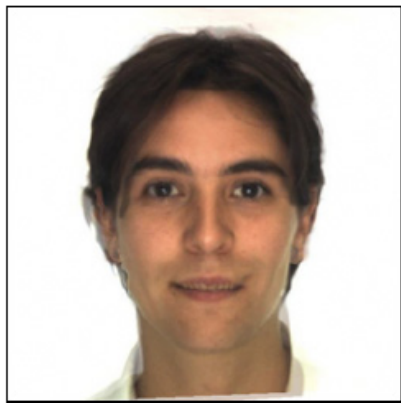
(c) $\alpha = 0.0$



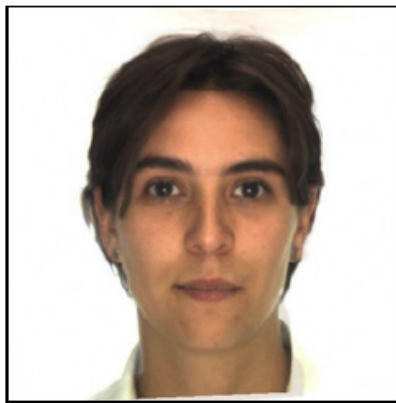
(d) $\alpha = 0.2$



(e) $\alpha = 0.4$



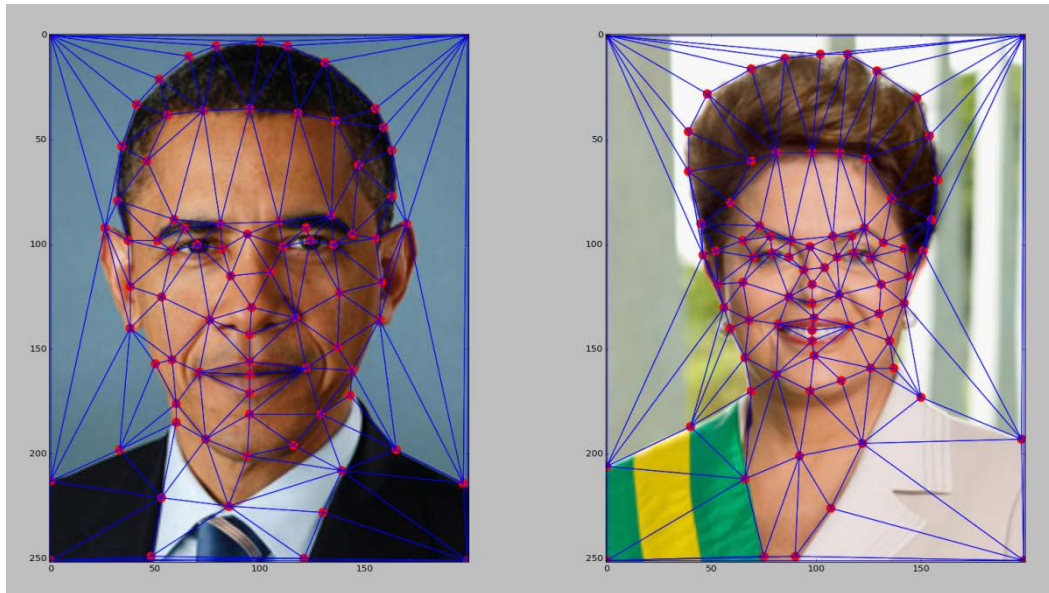
(f) $\alpha = 0.6$



(g) $\alpha = 0.8$



(h) $\alpha = 1.0$

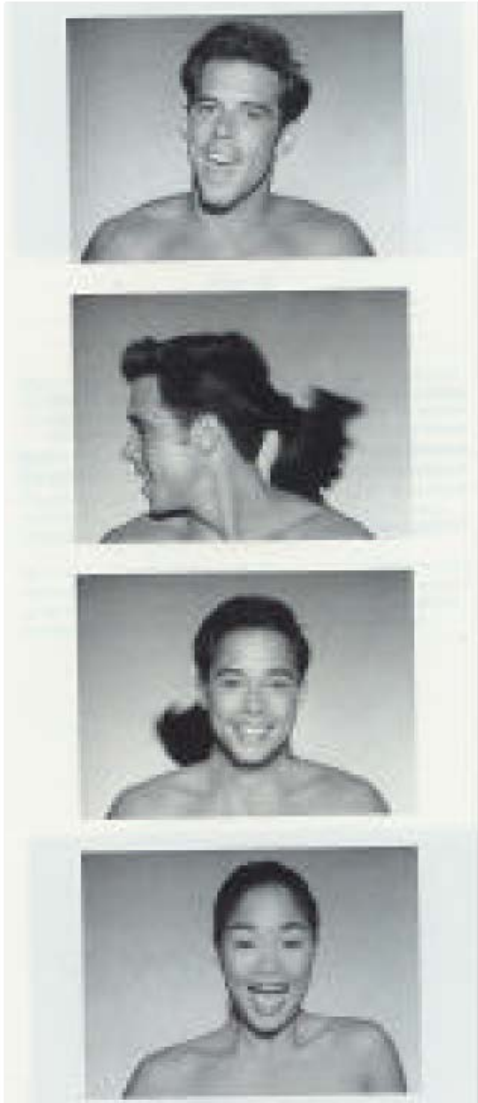


(c) Ian Albuquerque Raymundo da Silva

Summary of morphing

1. Define corresponding points
2. Define triangulation on points
 - Use same triangulation for both images
3. For each $t = 0:\text{step}:1$
 - a. Compute the average shape at t (weighted average of points)
 - b. For each triangle in the average shape
 - Get the affine projection to the corresponding triangles in each image
 - For each pixel in the triangle, find the corresponding points in each image and set value to weighted average (cross-dissolve each triangle)
 - c. Save the image as the next frame of the sequence

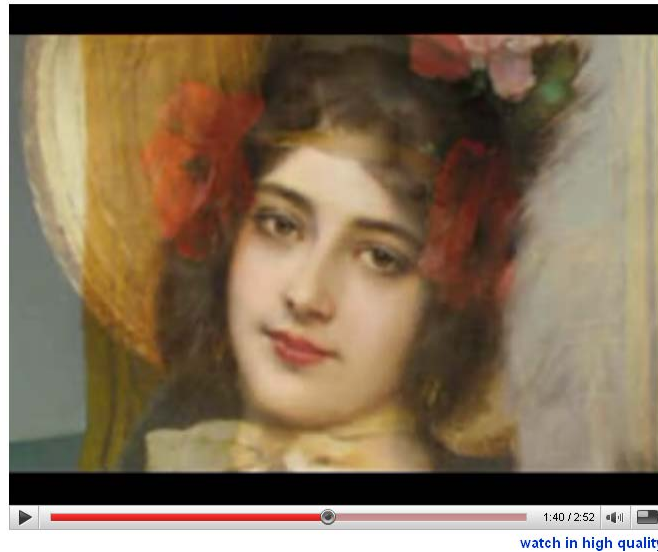
Dynamic Scene (“Black or White”, MJ)



<http://www.youtube.com/watch?v=R4kLKv5gtxc>

Women in Art video

Women In Art

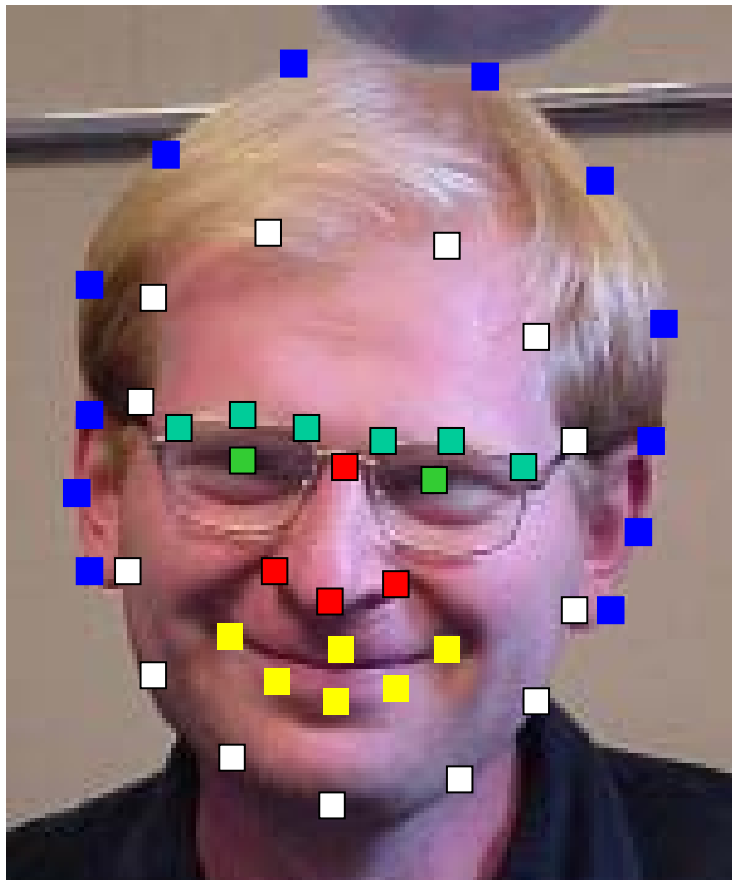


http://youtube.com/watch?v=nUDIoN-_Hxs

Multi-image alignment / averaging

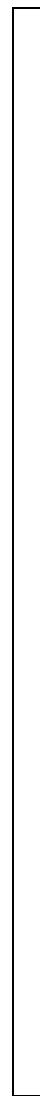


Shape Vector



Provides alignment!

=



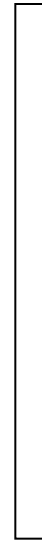
43

Appearance Vectors vs. Shape Vectors

Appearance
Vector



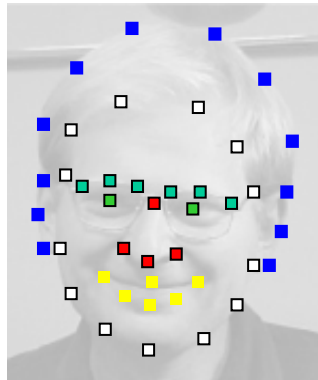
200*150 pixels (RGB)



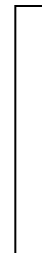
Vector of
 $200*150*3$
Dimensions

- Requires Annotation
- Provides alignment!

Shape
Vector



43 coordinates (x,y)

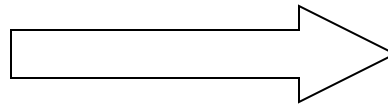


Vector of
 $43*2$
Dimensions

Average Face



1. Warp to mean shape
2. Average pixels



Subpopulation means

Examples:

- Male vs. female
- Happy vs. sad
- Average Kids
- Happy Males
- Etc.
- <http://www.faceresearch.org>



Average female



Average kid



Average happy male



Average male

Average Women of the world



Central African

Burmese

Cambodian

English

Ethiopian

Filipino



Greek

Indian

Iranian

Irish

Israeli

Italian



Peruvian

Polish

Romanian

Russian

Samoan

South African

Average Men of the world



AUSTRIA



AFGHANISTAN



ARGENTINA



BURMA (MYANMAR)



GERMANY



GREECE



CAMBODIA



ENGLAND



ETHIOPIA



FRANCE



IRAQ



IRELAND



MONGOLIA



PERU



POLAND



PUERTO RICO



UZBEKISTAN



AFRICAN AMERICAN

Deviations from the mean



Image X



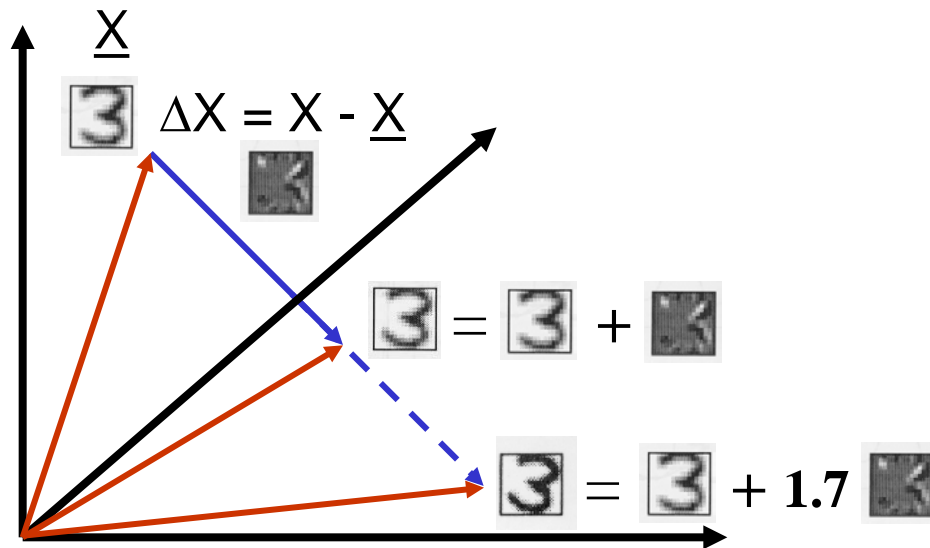
Mean \underline{X}

=



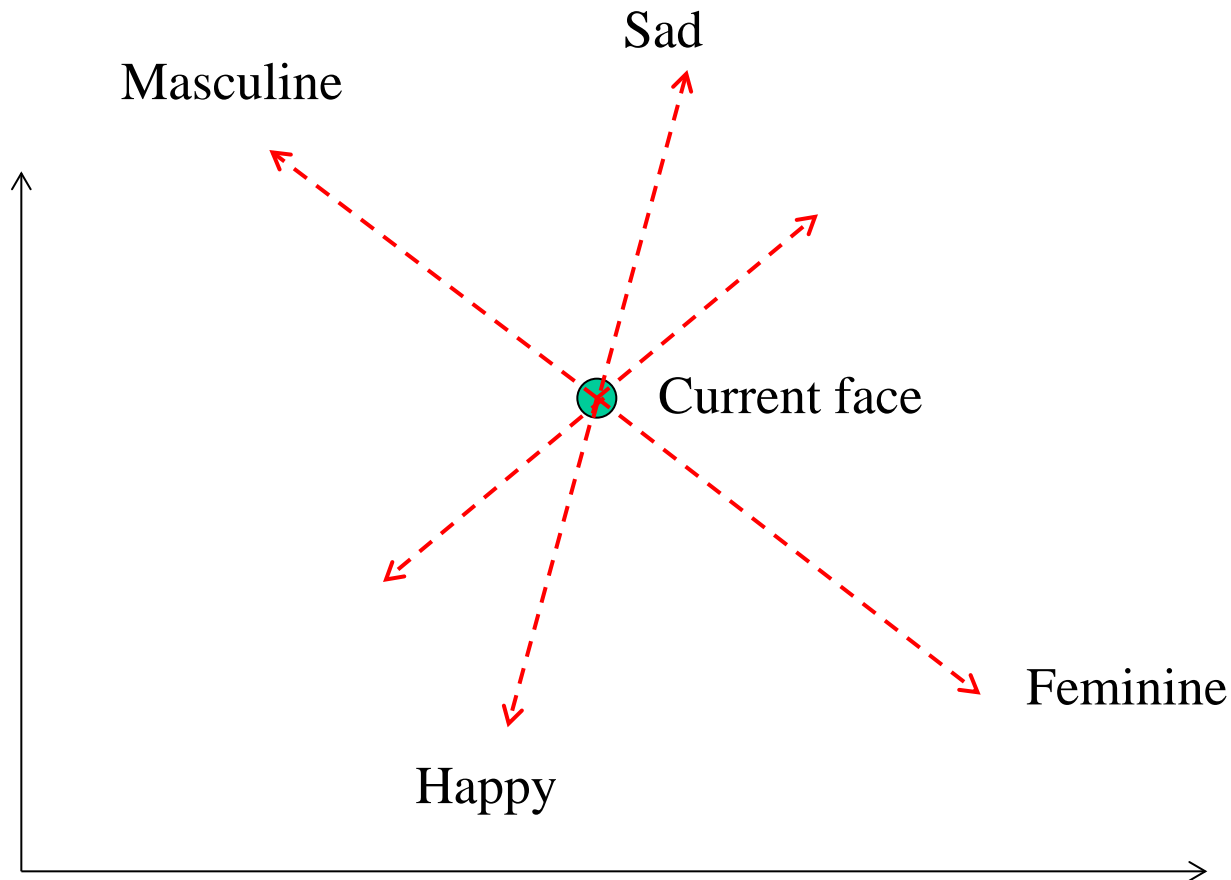
$$\Delta X = X - \underline{X}$$

Deviations from the mean



Extrapolating faces

- We can imagine various meaningful directions.



Manipulating faces

- How can we make a face look more female/male, young/old, happy/sad, etc.?
- <http://www.faceresearch.org/demos/transform>

