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CS-184: Computer Graphics

Lecture #18: Simulation Basics

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V2016-F-18-1.0

(With some slides from Prof. Ren Ng who is really an awesome guy.)

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Today

- Introduction to Simulation
  - Basic particle systems
  - Time integration (simple version)

Physically Based Animation

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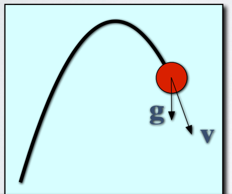
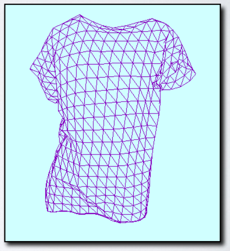
- Generate motion of objects using numerical simulation methods



$$\mathbf{x}^{t+\Delta t} = \mathbf{x}^t + \Delta t\, \mathbf{v}^t + \frac{1}{2}\Delta t^2 \mathbf{a}^t$$



Physically Based Animation

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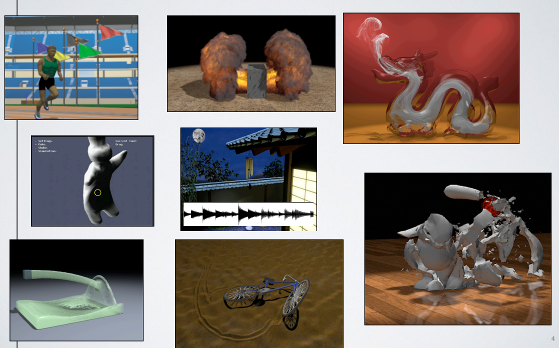

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Example: Cloth Simulation

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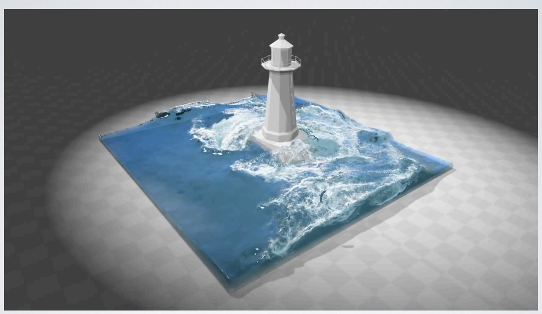
Example: Fluids



Macklin and Müller, Position Based Fluids

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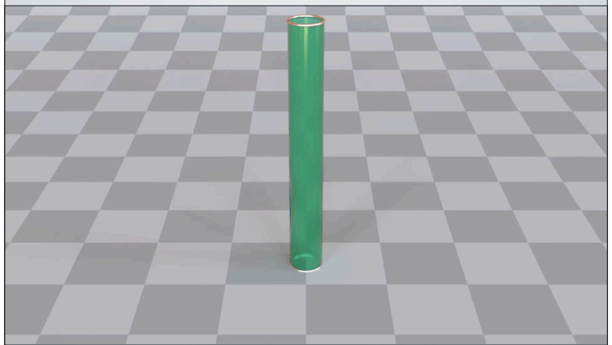
Example: Fracture



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Example: Fracture

Particle Systems

- Single particles are very simple
- Large groups can produce interesting effects
- Supplement basic ballistic rules
  - Collisions
  - Interactions
  - Force fields
  - Springs
  - Others...

Karl Sims, SIGGRAPH 1990

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Particle Systems

- Single particles are very simple
- Large groups can produce interesting effects
- Supplement basic ballistic rules
  - Collisions
  - Interactions
  - Force fields
  - Springs
  - Others...

Feldman, Klingner, O'Brien, SIGGRAPH 2005

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Basic Particles

- Basic governing equation
  - $\boldsymbol{f}$ is a sum of a number of things
    - Gravity: constant downward force proportional to mass
    - Simple drag: force proportional to negative velocity
    - Particle interactions: particles mutually attract and/or repell
      - Beware $O(n^2)$ complexity!
    - Force fields
    - Wind forces
    - User interaction

$$\ddot{\boldsymbol{x}} = \frac{1}{m}\boldsymbol{f}$$

Basic Particles

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- Properties other than position
  - Color
  - Temp
  - Age
- Differential equations also needed to govern these properties
- Collisions and other constrains directly modify position and/or velocity

Particle Rules

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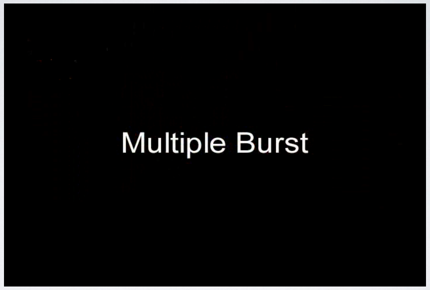

Bryan E. Feldman, James F. O'Brien, and Okan Arikan. "**Animating Suspended Particle Explosions**". In *Proceedings of ACM SIGGRAPH 2003*, pages 708–715, August 2003.

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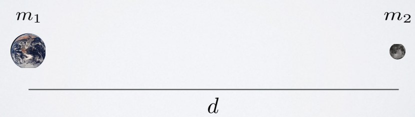
Gravitational Attraction

- Newton's universal law of gravitation
  - Gravitational pull between particles

$$F_g = G\frac{m_1 m_2}{d^2}$$
$$G = 6.67428 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$$



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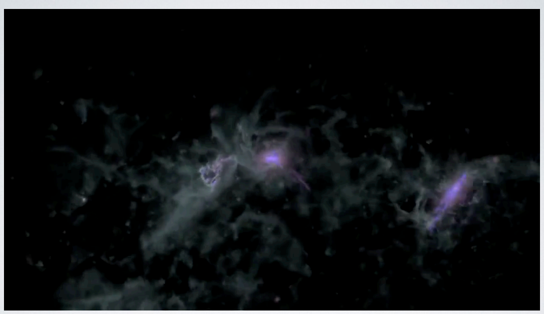
Example: Galaxy Simulation



Disk galaxy simulation, NASA Goddard

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Integration

- Euler's Method
  - Simple
  - Commonly used
  - Very inaccurate
  - Most often goes unstable

$$\mathbf{x}^{t+\Delta t} = \mathbf{x}^t + \Delta t\, \dot{\mathbf{x}}^t$$

$$\dot{\mathbf{x}}^{t+\Delta t} = \dot{\mathbf{x}}^t + \Delta t\, \ddot{\mathbf{x}}^t$$

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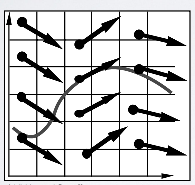
Integration

- For now let's pretend $\boldsymbol{f} = m\boldsymbol{v}$
  - *Velocity* (rather than acceleration) is a function of force



Witkin and Baraff

$$\dot{\boldsymbol{x}} = \mathsf{f}(\boldsymbol{x}, t)$$

Note: Second order ODEs can be turned into first order ODEs using extra variables.

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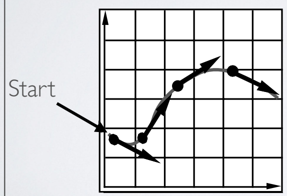
Integration

- For now let's pretend $\boldsymbol{f} = m\boldsymbol{v}$
  - *Velocity* (rather than acceleration) is a function of force



$$\dot{\boldsymbol{x}} = \mathsf{f}(\boldsymbol{x}, t)$$

Witkin and Baraff

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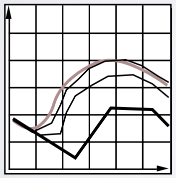
Integration

- With numerical integration, errors accumulate
- Euler integration is particularly bad



$$x := x + \Delta t\ \mathsf{f}(\boldsymbol{x}, t)$$

Witkin and Baraff

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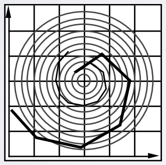
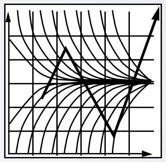
Integration

- Stability issues can also arise
  - Occurs when errors lead to larger errors
  - Often more serious than error issues





$\dot{\boldsymbol{x}} = [\,-\sin(\omega t)\ ,\ -\cos(\omega t)\,]$

Witkin and Baraff

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Integration

- Modified Euler

$$\boldsymbol{x}^{t+\Delta t} = \boldsymbol{x}^t + \frac{\Delta t}{2}\left(\dot{\boldsymbol{x}}^t + \dot{\boldsymbol{x}}^{t+\Delta t}\right)$$

$$\dot{\boldsymbol{x}}^{t+\Delta t} = \dot{\boldsymbol{x}}^t + \Delta t\ \ddot{\boldsymbol{x}}^t$$

$$\boldsymbol{x}^{t+\Delta t} = \boldsymbol{x}^t + \Delta t\ \dot{\boldsymbol{x}}^t + \frac{(\Delta t)^2}{2}\ \ddot{\boldsymbol{x}}^t$$

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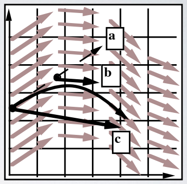
Integration

- Midpoint method
  a. Compute half Euler step
  b. Eval. derivative at halfway
  c. Retake step
- Other methods
  - Verlet
  - Runge-Kutta
  - And *many* others...



Witkin and Baraff

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Integration

- Implicit methods
  - Informally (incorrectly) called backward methods
  - Use derivatives in the future for the current step

$$\boldsymbol{x}^{t+\Delta t} = \boldsymbol{x}^{t} + \Delta t\ \dot{\boldsymbol{x}}^{t+\Delta t}$$
$$\dot{\boldsymbol{x}}^{t+\Delta t} = \dot{\boldsymbol{x}}^{t} + \Delta t\ \ddot{\boldsymbol{x}}^{t+\Delta t}$$

$$\dot{\boldsymbol{x}}^{t+\Delta t} = \mathsf{V}(\boldsymbol{x}^{t+\Delta t}, \dot{\boldsymbol{x}}^{t+\Delta t}, t+\Delta t)$$
$$\ddot{\boldsymbol{x}}^{t+\Delta t} = \mathsf{A}(\boldsymbol{x}^{t+\Delta t}, \dot{\boldsymbol{x}}^{t+\Delta t}, t+\Delta t)$$

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Integration

- Implicit methods
  - Informally (incorrectly) called backward methods
  - Use derivatives in the future for the current step

$$\dot{\boldsymbol{x}}^{t+\Delta t} = \dot{\boldsymbol{x}}^{t} + \Delta t \ \mathsf{V}(\boldsymbol{x}^{t+\Delta t}, \dot{\boldsymbol{x}}^{t+\Delta t}, t+\Delta t)$$
$$\dot{\boldsymbol{x}}^{t+\Delta t} = \dot{\boldsymbol{x}}^{t} + \Delta t \ \mathsf{A}(\boldsymbol{x}^{t+\Delta t}, \dot{\boldsymbol{x}}^{t+\Delta t}, t+\Delta t)$$

  - Solve nonlinear problem for $\boldsymbol{x}^{t+\Delta t}$ and $\dot{\boldsymbol{x}}^{t+\Delta t}$
  - This is fully implicit backward Euler
  - Many other implicit methods exist...
  - Modified Euler is ***partially*** implicit as is Verlet

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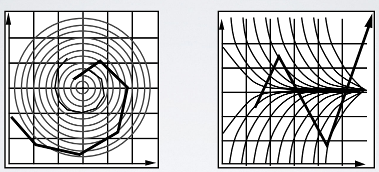
Temp Slide



Need to draw reverse diagrams....

Integration

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- Semi-Implicit
  - Approximate with linearized equations

$$\mathsf{V}(\boldsymbol{x}^{t+\Delta t}, \dot{\boldsymbol{x}}^{t+\Delta t}) \approx \mathsf{V}(\boldsymbol{x}^{t}, \dot{\boldsymbol{x}}^{t}) + \mathbf{A} \cdot (\Delta \boldsymbol{x}) + \mathbf{B} \cdot (\Delta \dot{\boldsymbol{x}})$$

$$\mathsf{A}(\boldsymbol{x}^{t+\Delta t}, \dot{\boldsymbol{x}}^{t+\Delta t}) \approx \mathsf{A}(\boldsymbol{x}^{t}, \dot{\boldsymbol{x}}^{t}) + \mathbf{C} \cdot (\Delta \boldsymbol{x}) + \mathbf{D} \cdot (\Delta \dot{\boldsymbol{x}})$$

$$\begin{bmatrix} \boldsymbol{x}^{t+\Delta t} \\ \dot{\boldsymbol{x}}^{t+\Delta t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}^{t} \\ \dot{\boldsymbol{x}}^{t} \end{bmatrix} + \Delta t \left(\begin{bmatrix} \dot{\boldsymbol{x}}^{t} \\ \ddot{\boldsymbol{x}}^{t} \end{bmatrix} + \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x} \\ \Delta \dot{\boldsymbol{x}} \end{bmatrix} \right)$$

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Integration

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- Explicit methods can be conditionally stable
  - Depends on time-step and ***stiffness*** of system
- Fully implicit can be **un**conditionally stable
  - May still have large errors
- Semi-implicit can be conditionally stable
  - Nonlinearities can cause instability
  - Generally more stable than explicit
  - Comparable errors as explicit
    - Often show up as excessive damping

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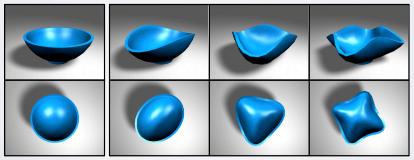
Integration

- Integrators can be analyzed in modal domain
- System have different component behaviors
- Integrators impact components differently



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Example: Mass Spring Rope

Credit: Elizabeth Labelle, https://youtu.be/Co8enp8CH34

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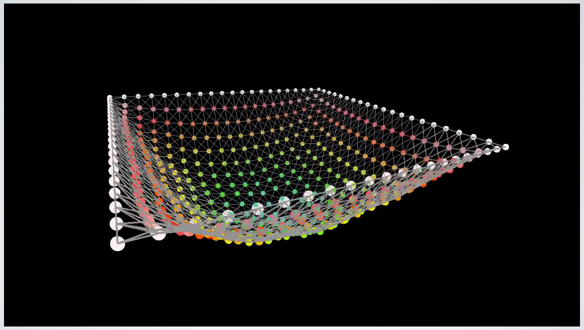
Example: Mass Spring Mesh



Slide from Ren Ng

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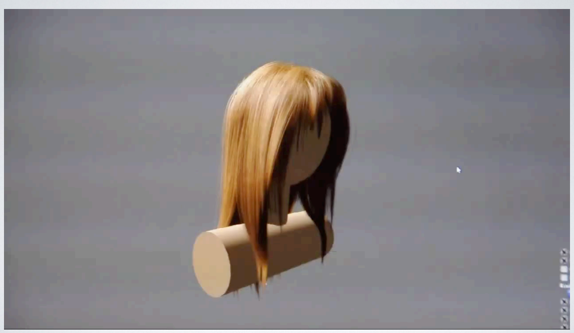
Example: Hair



Slide from Ren Ng

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Example: Cloth

Hanging Cloth

Huamin Wang, Ravi Ramamoorthi, and James F. O'Brien. "**Data-Driven Elastic Models for Cloth: Modeling and Measurement**". *ACM Transactions on Graphics*, 30(4):71:1–11, July 2011. Proceedings of ACM SIGGRAPH 2011, Vancouver, BC Canada.

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Example: Clothing on Character

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Strain Limiting

Bunny
Hollow Triangle Mesh
59K Elements

Huamin Wang, James F. O'Brien, and Ravi Ramamoorthi. "**Multi-Resolution Isotropic Strain Limiting**". In *Proceedings of ACM SIGGRAPH Asia 2010*, pages 160:1–10, December 2010.

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A Simple Spring

- Ideal ***zero***-length spring

$$\boldsymbol{f}_{a\to b} = k_s(\boldsymbol{b} - \boldsymbol{a})$$

$$\boldsymbol{f}_{b\to a} = -\boldsymbol{f}_{a\to b}$$

- Force pulls points together
- Strength proportional to distance

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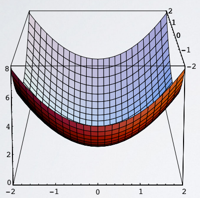
A Simple Spring

- Energy potential



$$E = 1/2\ k_S(\boldsymbol{b} - \boldsymbol{a}) \cdot (\boldsymbol{b} - \boldsymbol{a})$$

$$\boldsymbol{f}_{a \to b} = k_S(\boldsymbol{b} - \boldsymbol{a})$$

$$\boldsymbol{f}_{b \to a} = -\boldsymbol{f}_{a \to b}$$

$$\boldsymbol{f}_a = -\nabla_a E = -\left[\frac{\partial E}{\partial a_x}, \frac{\partial E}{\partial a_y}, \frac{\partial E}{\partial a_z}\right]$$

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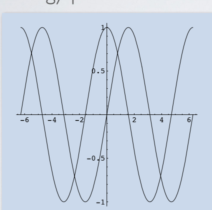
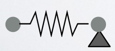
A Simple Spring

- Energy potential: kinetic *vs* elastic



$$E = 1/2\ k_S(\boldsymbol{b} - \boldsymbol{a}) \cdot (\boldsymbol{b} - \boldsymbol{a})$$

$$E = 1/2\ m(\dot{\boldsymbol{b}} - \dot{\boldsymbol{a}}) \cdot (\dot{\boldsymbol{b}} - \dot{\boldsymbol{a}})$$



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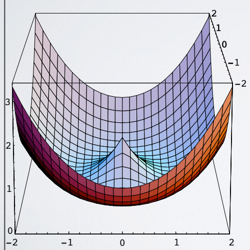
Non-Zero Length Springs

$$\boldsymbol{f}_{a\rightarrow b} = k_s \frac{\boldsymbol{b}-\boldsymbol{a}}{||\boldsymbol{b}-\boldsymbol{a}||}\left(||\boldsymbol{b}-\boldsymbol{a}|| - l\right)$$

Rest length



$$E = k_s\left(||\boldsymbol{b}-\boldsymbol{a}|| - l\right)^2$$

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Comments on Springs

- Springs with zero rest length are linear
- Springs with non-zero rest length are nonliner
  - Force ***magnitude*** linear w/ discplacement (from rest length)
  - Force direction is non-linear
  - Singularity at

$$||\boldsymbol{b}-\boldsymbol{a}|| = 0$$

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Damping

- "Mass proportional" damping

$$\boldsymbol{f} = -k_d \dot{\boldsymbol{a}}$$

  - Behaves like viscous drag on all motion
  - Consider a pair of masses connected by a spring
    - How to model rusty *vs* oiled spring
    - Should internal damping slow group motion of the pair?
  - Can help stability... up to a point

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Damping

- "Stiffness proportional" damping

$$\boldsymbol{f}_a = -k_d \frac{\boldsymbol{b}-\boldsymbol{a}}{||\boldsymbol{b}-\boldsymbol{a}||^2}(\boldsymbol{b}-\boldsymbol{a}) \cdot (\dot{\boldsymbol{b}}-\dot{\boldsymbol{a}})$$

  - Behaves viscous drag on change in spring length
  - Consider a pair of masses connected by a spring
    - How to model rusty *vs* oiled spring
    - Should internal damping slow group motion of the pair?

Spring Constants

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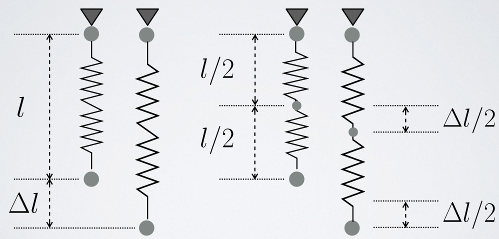
- Two ways to model a single spring



Spring Constants

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- Constant k_s gives inconsistent results with different discretizations
- Change in length is not what we want to measure
- Strain: change in length as fraction of original length

$$\epsilon = \frac{\Delta l}{l_0}$$

Nice and simple for 1D...

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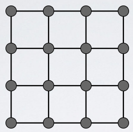
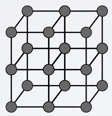
Structures from Springs

- Sheets



- Blocks



- Others

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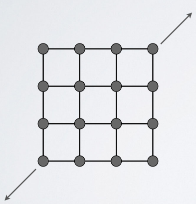
Structures from Springs

- They behave like what they are (obviously!)



This structure will not resist shearing

This structure will not resist out-of-plane bending either...

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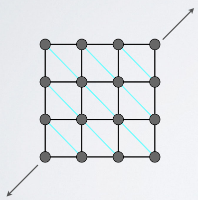
Structures from Springs

- They behave like what they are (obviously!)



This structure will resist shearing but has anisotopic bias

This structure still will not resist out-of-plane bending

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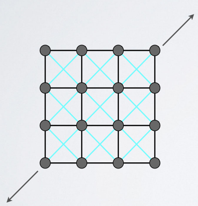
Structures from Springs

- They behave like what they are (obviously!)



This structure will resist shearing
Less bias
Interference between spring sets

This structure still will not resist out-of-plane bending

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Structures from Springs

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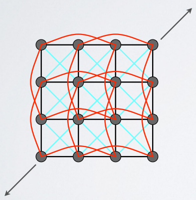
- They behave like what they are (obviously!)



This structure will resist shearing
Less bias
Interference between spring sets

This structure will resist out-of-plane bending
Interference between spring sets
Odd behavior

How do we set spring constants?

Edge Springs

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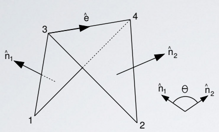

$$u_1 = |E|\frac{N_1}{|N_1|^2} \qquad u_2 = |E|\frac{N_2}{|N_2|^2}$$

$$u_3 = \frac{(x_1 - x_4)\cdot E}{|E|}\frac{N_1}{|N_1|^2} + \frac{(x_2 - x_4)\cdot E}{|E|}\frac{N_2}{|N_2|^2}$$

$$u_4 = -\frac{(x_1 - x_3)\cdot E}{|E|}\frac{N_1}{|N_1|^2} - \frac{(x_2 - x_3)\cdot E}{|E|}\frac{N_2}{|N_2|^2}$$

$$F_i^e = k^e \frac{|E|^2}{|N_1| + |N_2|}\sin(\theta/2)\, u_i$$

From Bridson ***et al.***, 2003, also see Grinspun ***et al.***, 2003

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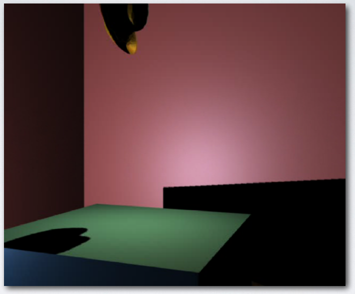
Example: Thin Material



Discrete Shells
SCA 2003
Eitan Grinspun, Anil Hirani, Mathieu Desbrun and Peter Schröder

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Sharp Creases

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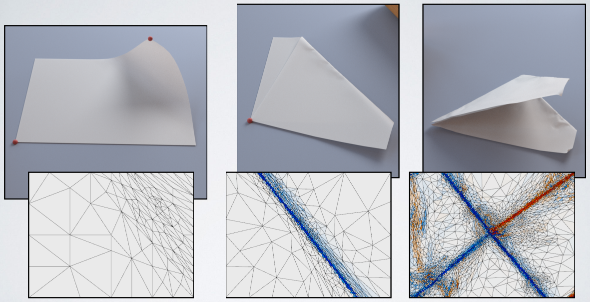
Sharp Creases



Anisotropic remeshing avoids locking.

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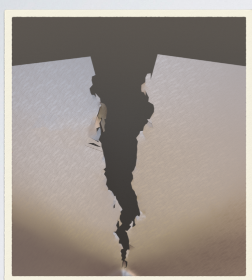
Fracture



Dominated by discretization artifacts

Natural appearance

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Variety of Materials