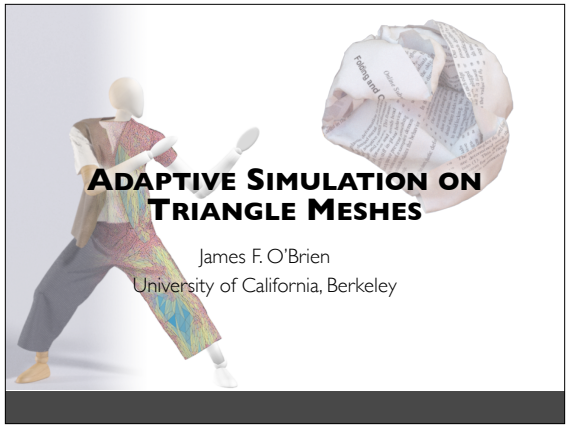


Adaptive Simulation on Triangle Meshes

James F. O'Brien
University of California, Berkeley

1

Collaborators

Adaptive Tearing and Cracking of Thin Sheets
with **Tobias Pfaff, Rahul Narain** and **Juan Miguel de Joya**
SIGGRAPH 2014

View-Dependent Adaptive Cloth Simulation
with **Woojong Koh** and **Rahul Narain**
SCA 2014

Folding and Crumpling Adaptive Sheets
with **Rahul Narain** and **Tobias Pfaff**
SIGGRAPH 2013

Adaptive Anisotropic Remeshing for Cloth Simulation
with **Rahul Narain** and **Armin Samii**
SIGGRAPH Asia 2012

2

2

Goal:
simulate realistic materials
at a reasonable speed with
no visible discretization artifacts

3

3

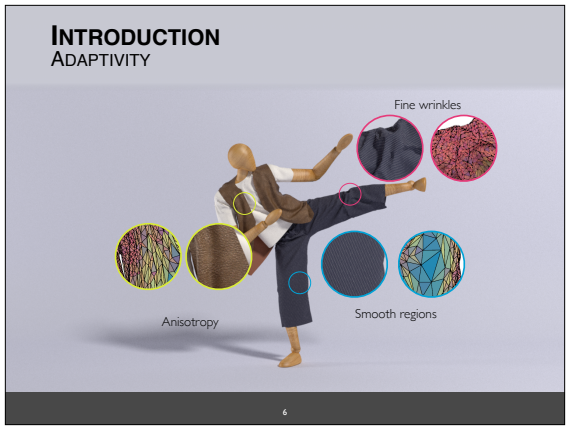
INTRODUCTION
ADAPTIVITY

4

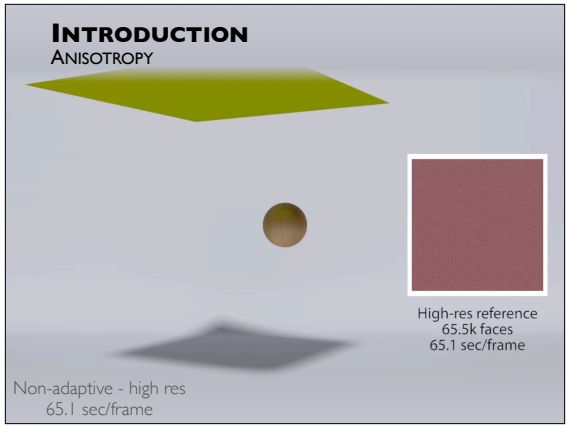
4


INTRODUCTION
ADAPTIVITY
5

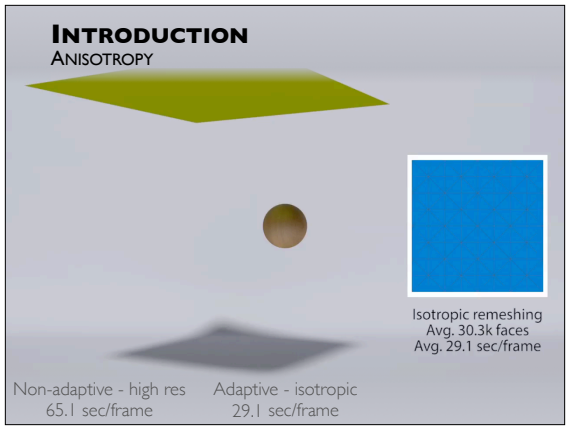
5


INTRODUCTION
ADAPTIVITY
Fine wrinkles
Anisotropy
Smooth regions
6

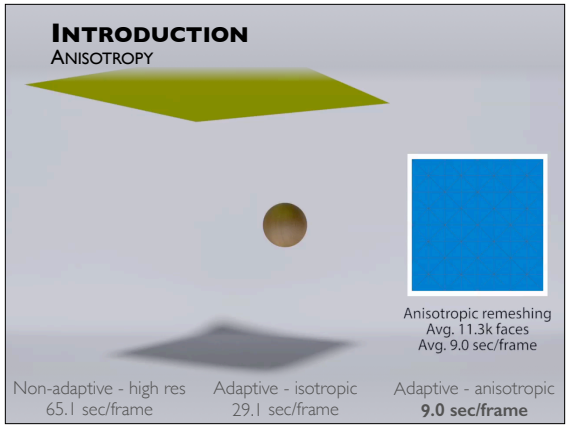
6


INTRODUCTION
ANISOTROPY
High-res reference
65.5k faces
65.1 sec/frame
Non-adaptive - high res
65.1 sec/frame

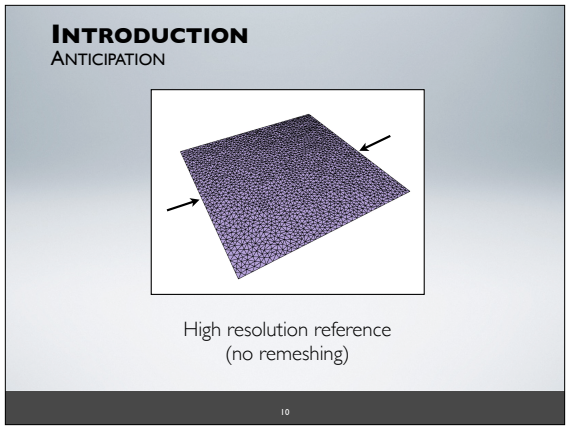
7


INTRODUCTION
ANISOTROPY
Isotropic remeshing
Avg. 30.3k faces
Avg. 29.1 sec/frame
Non-adaptive - high res
65.1 sec/frame
Adaptive - isotropic
29.1 sec/frame

8

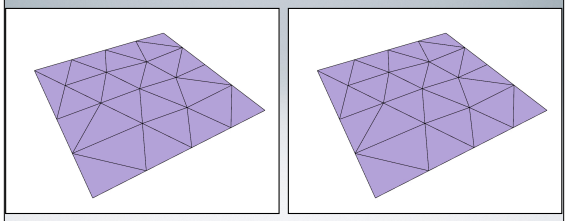

INTRODUCTION
ANISOTROPY
Anisotropic remeshing
Avg. 11.3k faces
Avg. 9.0 sec/frame
Non-adaptive - high res
65.1 sec/frame
Adaptive - isotropic
29.1 sec/frame
Adaptive - anisotropic
9.0 sec/frame

9


INTRODUCTION
ANTICIPATION
High resolution reference
(no remeshing)
10

10

Introduction
Anticipation



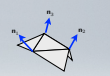
Without buckling anticipation | With buckling anticipation

11

11

Remeshing
Remeshing Criteria

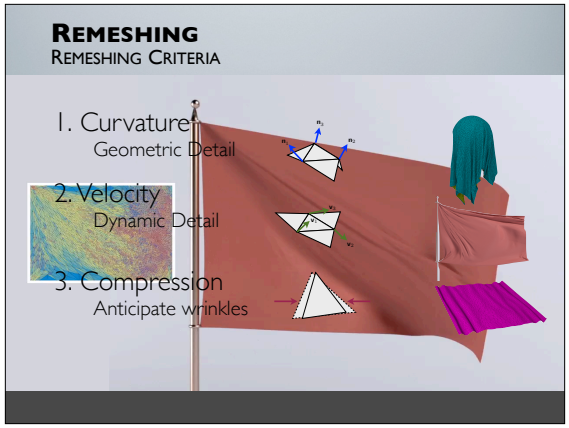
1. Curvature
   Geometric Detail
2. Velocity
   Dynamic Detail
3. Compression
   Anticipate wrinkles



12

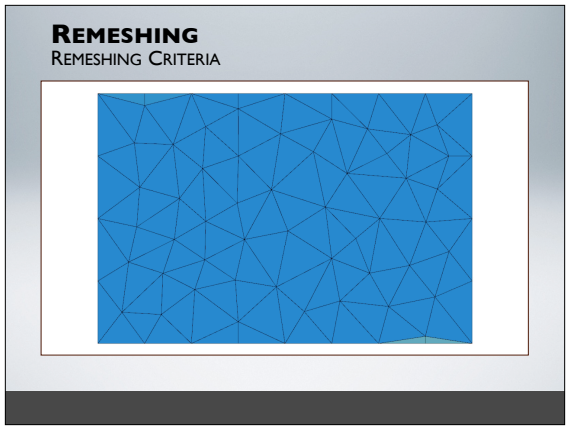
12

Remeshing
Remeshing Criteria



13

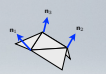
Remeshing
Remeshing Criteria



14

Remeshing
Remeshing Criteria

1. Curvature
   Geometric Detail
2. Velocity
   Dynamic Detail
3. Compression
   Anticipate wrinkles

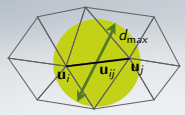



15

15

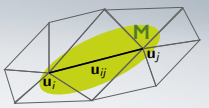
Remeshing
Remeshing Criteria

How long should each edge be?



$\mathbf{u}_{ij}^T \mathbf{u}_{ij} \leq d_{max}^2$

Isotropic remeshing



$\mathbf{u}_{ij}^T \mathbf{M} \mathbf{u}_{ij} \leq 1$

Anisotropic remeshing

16

16

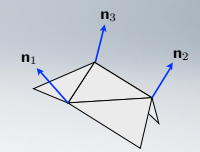
Remeshing
Remeshing Criteria

1. Curvature

We want $\|\mathbf{n}_i - \mathbf{n}_j\|^2 \leq \Delta n_{\max}^2$

$\mathbf{n}_i - \mathbf{n}_j = \nabla \mathbf{n} \cdot \mathbf{u}_{ij}$

So $\mathbf{u}_{ij}^T \left(\frac{\nabla \mathbf{n}^T \nabla \mathbf{n}}{\Delta n_{\max}^2} \right) \mathbf{u}_{ij} \leq 1$ → $\mathbf{M}_{crv}$



17

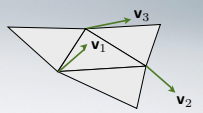
17

Remeshing
Remeshing Criteria

2. Velocity

We want $\|\mathbf{v}_i - \mathbf{v}_j\|^2 \leq \Delta v_{\max}^2$

$\mathbf{u}_{ij}^T \left(\frac{\nabla \mathbf{v}^T \nabla \mathbf{v}}{\Delta v_{\max}^2} \right) \mathbf{u}_{ij} \leq 1$ → $\mathbf{M}_{vel}$



18

18

Remeshing
Remeshing Criteria

3. Compression

Expected size of wrinkles $\propto 1/\sqrt{\text{strain}}$

Green strain tensor: $\mathbf{G} = \nabla\mathbf{x}^T\nabla\mathbf{x} - \mathbf{1}$

$$\varepsilon_c = (\mathbf{1} - \nabla\mathbf{x}^T\nabla\mathbf{x})^+$$

Rib-Stiffening $\mathbf{M}_{\text{cmp}} = \frac{1}{2}\left(\varepsilon_c + \sqrt[+]{\varepsilon_c^2 - \frac{k_b}{4k_s}(\mathbf{S}^\perp)^2}\right)$

19

19

Remeshing
Remeshing Criteria

Combining metric terms

Summation ends up overly conservative

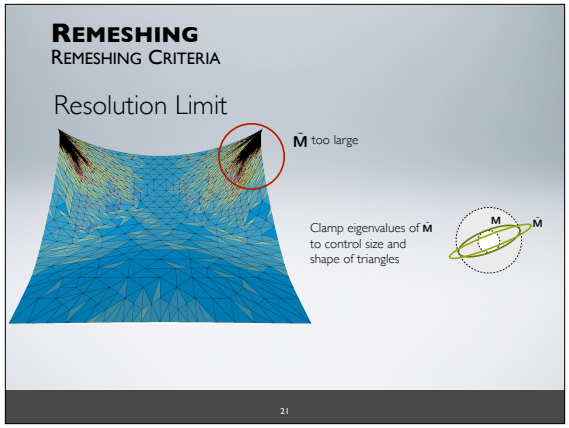
$$\tilde{\mathbf{M}} = \frac{\mathbf{M}_{\text{crv}}}{\Delta p_{\max}^2} + \frac{\mathbf{M}_{\text{vel}}}{\Delta v_{\max}^2} + \frac{\mathbf{M}_{\text{cmp}}}{c_{\max}^2}$$

Find smallest enclosing disk

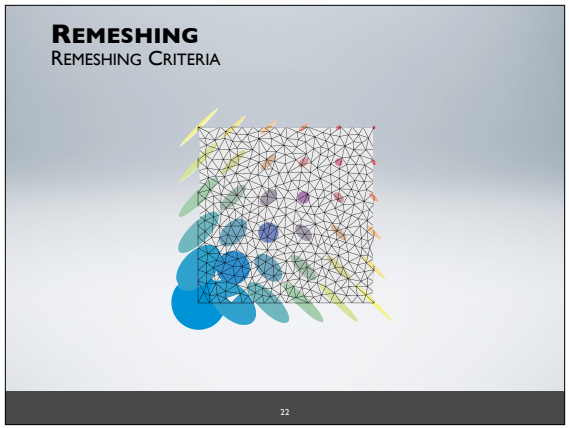
$\tilde{\mathbf{M}} =$ Welzl's algorithm [1991]
- generalize points to disks
- Base case is Apollonius' problem

20

20

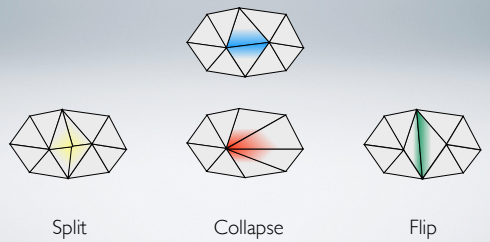

REMESHING
REMESHING CRITERIA
Resolution Limit
$\tilde{\mathbf{M}}$ too large
Clamp eigenvalues of $\tilde{\mathbf{M}}$ to control size and shape of triangles
M
$\tilde{\mathbf{M}}$
21

21


REMESHING
REMESHING CRITERIA
22

22

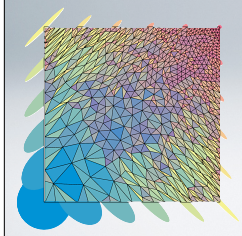
Remeshing
Remeshing Operations



23

23

Remeshing
Algorithm: Adaptive Anisotropic Remeshing



1. Split
   Until all edges are valid
   Flip until anisotropically Delaunay
2. Collapse
   Without creating invalid edges
   Flip until anisotropically Delaunay

24

24

Remeshing
Summary

1. Compute sizing field

Compute tensors
- Curvature
- Velocity
- Compression

Sum to get $\bar{\mathcal{M}}$

Clamp eigenvalues to get $\mathcal{M}$

2. Adaptively remesh

Refine by splitting

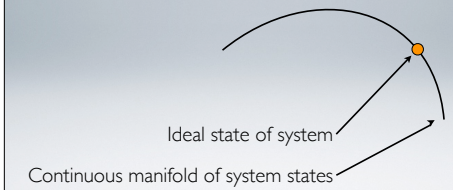
Coarsen by collapsing

Flip edges if necessary

25

25

Post-Remeshing Projection
Minimizing state space discretization error

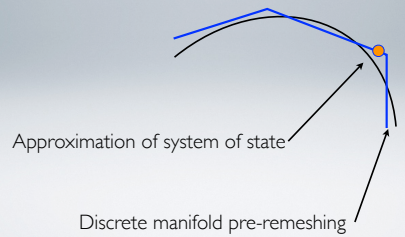


26

26

Post-Remeshing Projection

Minimizing state space discretization error

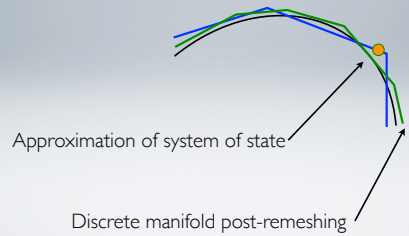


27

27

Post-Remeshing Projection

Minimizing state space discretization error



28

28

Post-Remeshing Projection

Minimizing state space discretization error

Ideal projection matches instantaneous derivatives

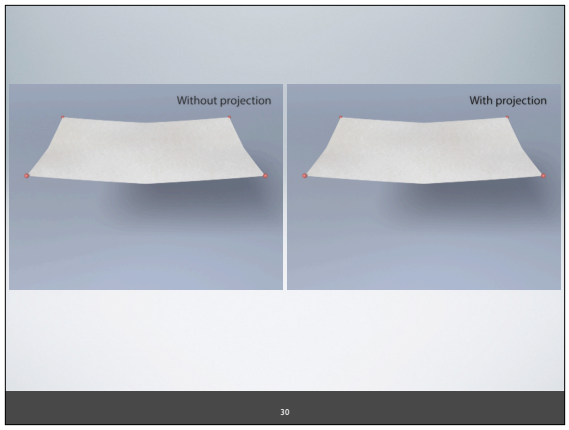
$$\frac{\partial E}{\partial \mathbf{x}_i} = -m_i \mathbf{a}_i$$

Solve by minimizing

$$\min_{\mathbf{x}} \left(E + \sum m_i \mathbf{a}_i \cdot (\mathbf{x}_i - \mathbf{x}_i^0) + \sum \frac{\mu}{2} \|\mathbf{x}_i - \mathbf{x}_i^0\|^2 \right)$$

29

29



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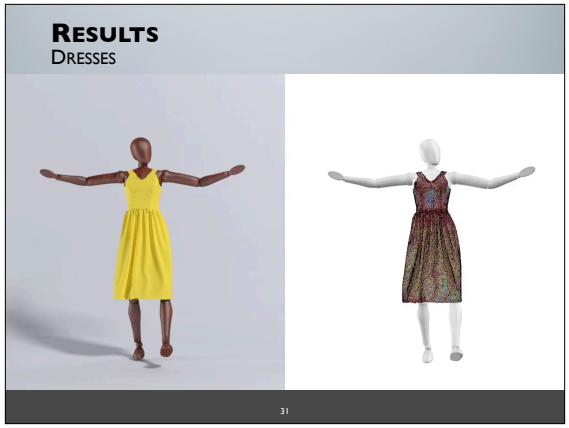
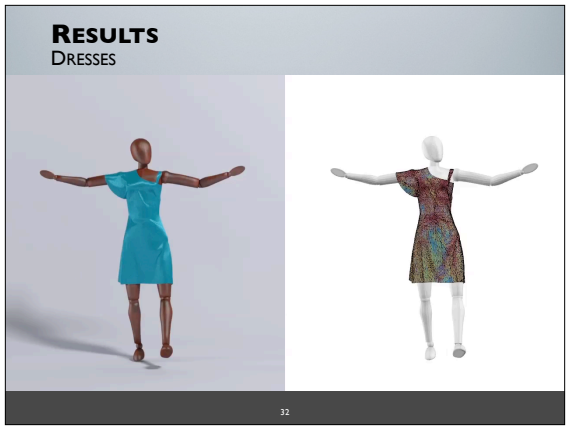
30



31



32

Results
Dresses

33

33

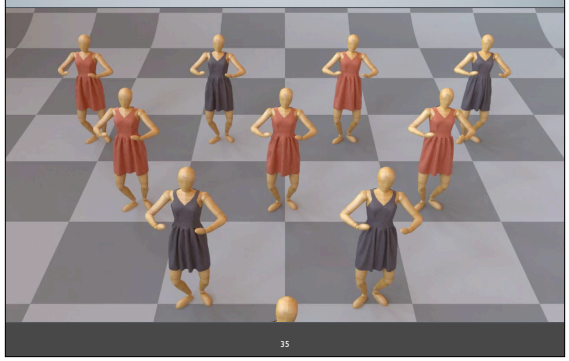
Timing

Name	Faces		Time per frame (seconds)				
	Input	Mean	Integration	Strain lim.	Collision	Remeshing	Total
Kick	884	37,627	13.055	14.075	22.300	0.551	49.980
Sphere	4	11,313	4.077	3.557	1.222	0.177	9.033
Flag	97	11,710	3.103	3.559	1.353	0.428	8.443
Cylinders	32	5,267	0.684	0.625	0.455	0.050	1.814
Blue dress	112	13,149	4.028	3.656	3.811	0.196	11.691
Yellow dress	428	22,785	10.075	6.997	5.396	0.340	22.809

34

34

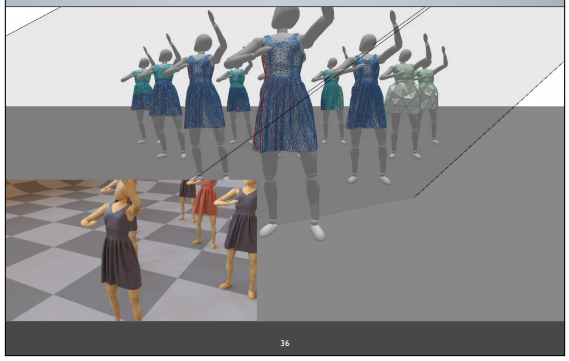
View-Dependent Refinement



35

35

View-Dependent Refinement



36

36

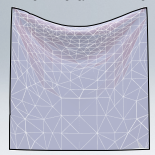
View-Dependent Refinement

37

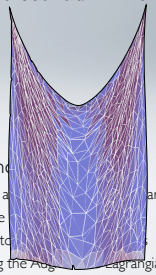
37

Strain Limiting
For irregular meshes

With Strain Limiting



Without Strain Limiting



Existing methods

- Jacobi or Gauss-Seidel iterations
- Slow to converge
- Fails on irregular meshes

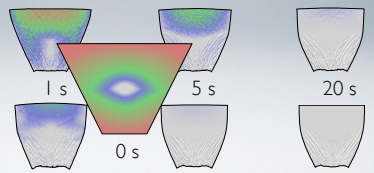
Our metho

- Formulate a … ar optimization
- Minimize …
- Subject to …
- Solve using the Aug… Lagrangian method

38

38

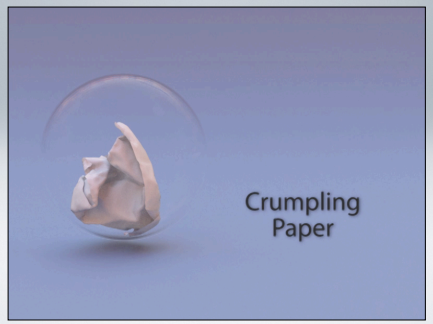
Strain Limiting
For irregular meshes



39

39

Bending Plasticity



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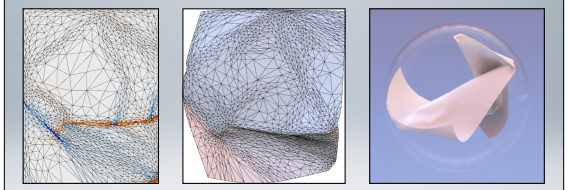
Bending Plasticity



Parameter Space | Plastic Embedding | World Space

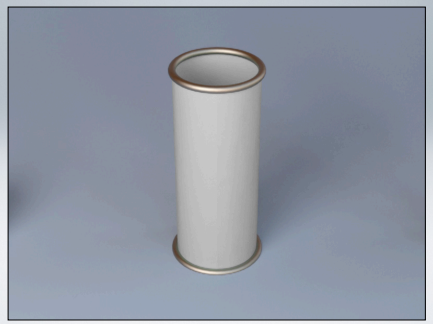
$$\mathbf{S}_p \leftarrow \mathbf{S}_p + \frac{\mathbf{S}_e}{\|\mathbf{S}_e\|}(\|\mathbf{S}_e\| - \kappa)$$

Optimize *plastic embedding* to adsorb $\mathbf{S}_p$ into shape.

41

41

Sharp Creases



42

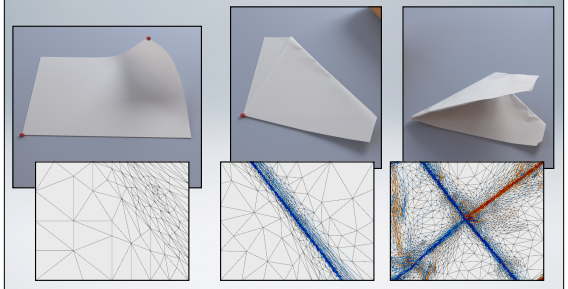
42

Sharp Creases

43

43

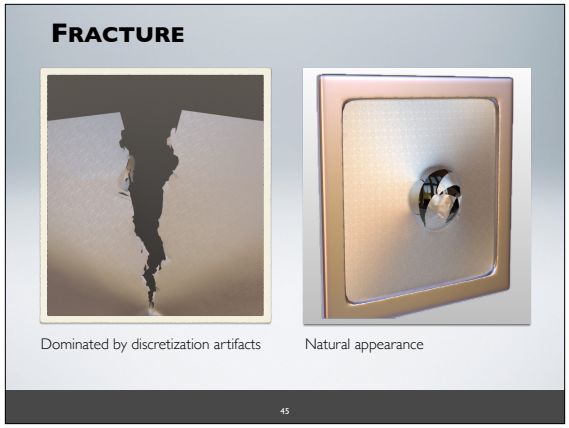
Sharp Creases



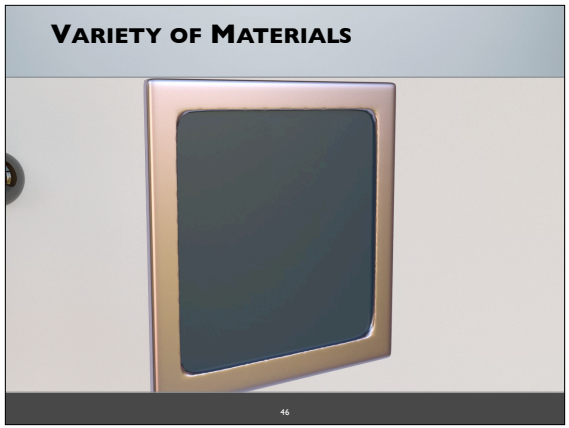
Anisotropic remeshing avoids locking.

44

44


FRACTURE
Dominated by discretization artifacts
Natural appearance
45

45

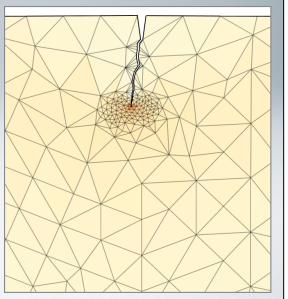

VARIETY OF MATERIALS
46

46

Crack-Tip Refinement

47

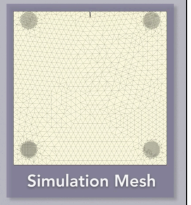
- High-resolution crack paths
  Refine during crack formation
- Low resolution in other parts
  Coarsen after crack has formed



47

Crack-Tip Refinement

48



Simulation Mesh

48

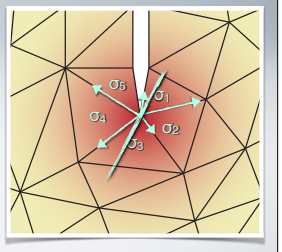
Failure Criterion

49

Maximum principal stress $\sigma_p > \tau$

But: Discretization error and noise often ruin maximality

1. Averaged stress
   not maximal at crack tip
2. Separation tensor
   [O'Brien and Hodgins 1999]
   depends on triangle size



49

Failure Criterion

50

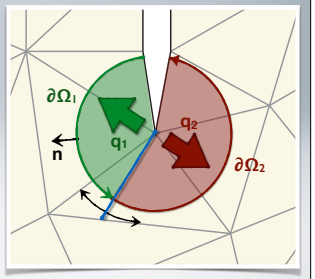
Find maximal separating traction

$$\mathbf{q_1} = \int_{\partial\Omega_1} \sigma^+ \cdot \mathbf{n}\, dS$$
$$\mathbf{q_2} = \int_{\partial\Omega_2} \sigma^+ \cdot \mathbf{n}\, dS$$

Separation criterion

$$\min(q_1, q_2) > \tau$$

→ Find split-plane maximizing separation



50

Failure Criterion

51

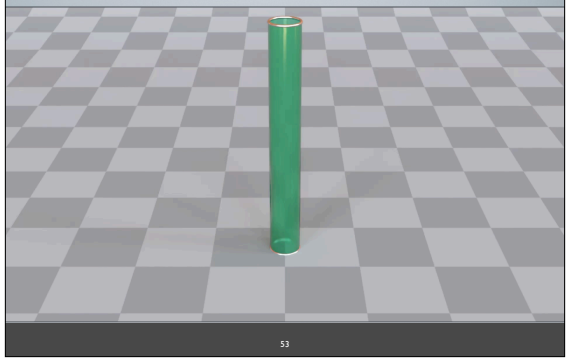
51

Editorial Control

52

52

Bending Effects



53

53

Bending Effects

54

54

Code and Data Distribution

ARCSim: Adaptively Refining and Coarsening Simulator

http://graphics.berkeley.edu/resources/ARCSim

Berkeley Garment Library

http://graphics.berkeley.edu/resources/GarmentLibrary

55