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CS-184: Computer Graphics

Lecture #16: Global Illumination

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University of California, Berkeley

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Today

- The Rendering Equation
- Radiosity Method
- Photon Mapping
- Ambient Occlusion

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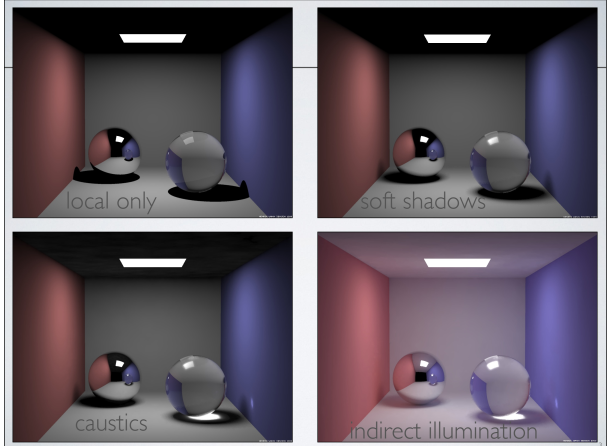

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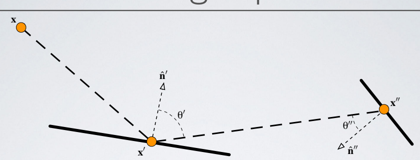
The Rendering Equation



The light shining on x from x' is equal to:
- the emitted light from x' toward x, plus
- for each bit of surface in the scene, how much light shines from that bit onto x' and is reflected toward x, scaled appropriately

$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E(\mathbf{x},\mathbf{x}') + \int_S \rho_{x'}(\mathbf{x},\mathbf{x}'')L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}d\mathbf{x}''\right]$$

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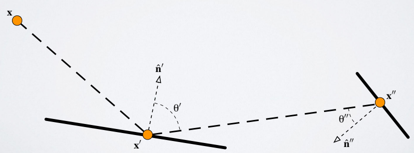
The Rendering Equation

$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E(\mathbf{x},\mathbf{x}') + \int_S \rho_{x'}(\mathbf{x},\mathbf{x}'')L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}\mathrm{d}\mathbf{x}''\right]$$

sum over every bit of surface in the scene

Light energy hitting x from x'

orientation ("form factor")



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Radiosity

- Assume all materials are perfectly Lambertian (diffuse only, no specularities)
  - Removes all dependance on directions
  - Reduces dimensionality of lightfield
  - Allows a FEM solution (break up into chunks)
- Can also relax assumption slightly...

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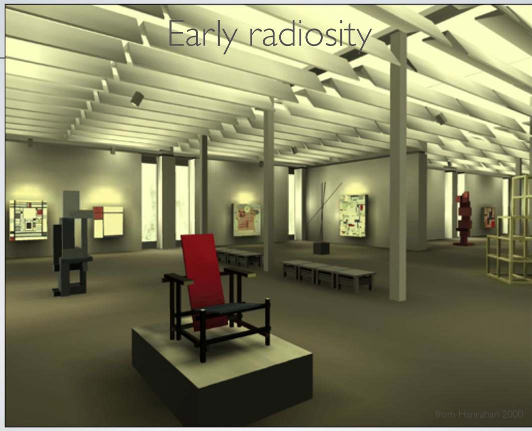
Early radiosity



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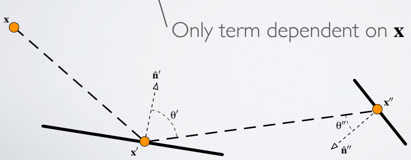
Assume Lambertian

$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E(\mathbf{x},\mathbf{x}') + \int_S \rho_{x'}(\mathbf{x},\mathbf{x}'')L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}d\mathbf{x}''\right]$$

$$L_s(\mathbf{x},\mathbf{x}') = \underline{\delta(\mathbf{x},\mathbf{x}')}\left[E_{x'} + \int_S \rho_{x'}L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}d\mathbf{x}''\right]$$

Only term dependent on $\mathbf{x}$



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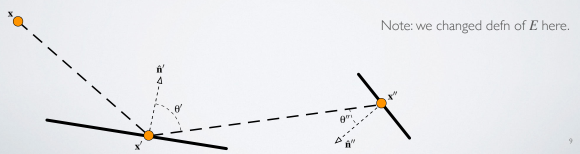
Rewrite in Terms of Radiosity

$$L_s(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x},\mathbf{x}')\left[E_{x'} + \int_S \rho_{x'} L_s(\mathbf{x}',\mathbf{x}'')\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}d\mathbf{x}''\right]$$

$$H_{x'} = E_{x'} + \rho_{x'}\int_S \delta(\mathbf{x}',\mathbf{x}'')\frac{H_{x''}}{2\pi}\frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}'-\mathbf{x}''||^2}d\mathbf{x}''$$

Note: we changed defn of E here.



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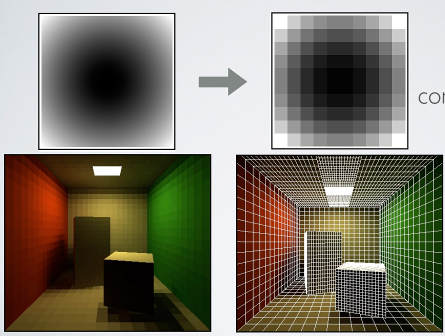
Discretize into Patches



Piece-wise constant patches

Example mesh for Cornell Box
by Mark Schmelzenbach

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Discretize into Patches

The Candlestick Theater,
Mark Mack Architects.

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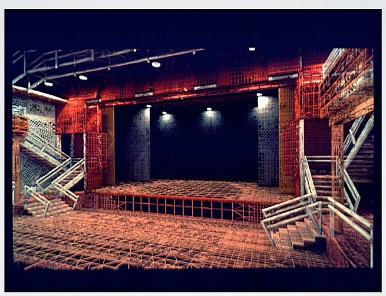
Discretize into Patches



The Candlestick Theater,
Mark Mack Architects.

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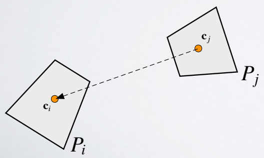
Rewrite in Terms of Patches

$$H_{x'} = E_{x'} + \rho_{x'} \int_S \delta(\mathbf{x}', \mathbf{x}'') \frac{H_{x''}}{2\pi} \frac{\cos(\theta')\cos(\theta'')}{||\mathbf{x}' - \mathbf{x}''||^2} d\mathbf{x}''$$

$$H_i = E_i + \rho_i \sum_j H_j \int_{S_j} \delta_{ij} \frac{\cos(\theta_i)\cos(\theta_j)}{2\pi||\mathbf{c}_i - \mathbf{x}||^2} d\mathbf{x}$$

Form factor from j to i, F_{ij}



Example of a rough approximation:

$$F_{ij} \approx \delta_{ij} \frac{\cos(\theta_i)\cos(\theta_j)}{2\pi||\mathbf{c}_i - \mathbf{c}_j||^2} A_j$$

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Radiosity Method

- Given the E_i and ρ_i
- First compute F_{ij}
- Then solve $H_i = E_i + \rho_i \sum_j H_j F_{ij}$

$$\mathbf{h} = \mathbf{e} + \mathbf{A}\mathbf{h}$$
$$(\mathbf{I} - \mathbf{A})\mathbf{h} = \mathbf{e}$$

- Comments:
  - The matrix $\mathbf{A}$ is typically very large
  - It is also sparse (why?)
  - Should be solved with an iterative method
    - e.g.: Jacobi or Gauss-Seidel
  - **Solution is view independent**

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Radiosity Method

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- Given the light emitted and surface properties
- First compute F_{ij} , form factors between patches
- Then **solve a linear system to balance energy between all patches**
- Comments:
  - The system is very large
  - It is also sparse (why?)
  - Should be solved with an iterative method
    - e.g.: Jacobi or Gauss-Seidel
  - **Solution is view independent**

Progressive Radiosity

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- If magnitude of eigenvalues of $\mathbf{A}<1$

$$(\mathbf{I}-\mathbf{A})^{-1}=\mathbf{I}+\mathbf{A}+\mathbf{A}^2+\mathbf{A}^3+\cdots$$

  - True for form-factor matrices
  - Use Gauss-Seidel-like iteration but reorder by priority

$$\mathbf{h}^{k+1}=\mathbf{h}^k+\mathbf{u}^{k+1}$$

$$\mathbf{u}^{k+1}=\mathbf{A}\,\mathbf{u}^k$$

$$\mathbf{h}^0=0 \quad \mathbf{u}^0=\mathbf{e}$$

Idea: let important sources of light energy emit first, maybe don't even bother with dark things

Southwell Relaxation

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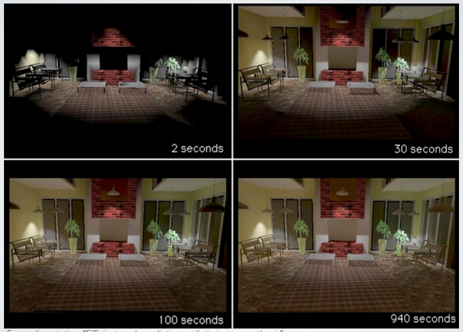
Progressive Radiosity



From dissertation "Efficient and predictive realistic image synthesis"
by Karol Myszkowski

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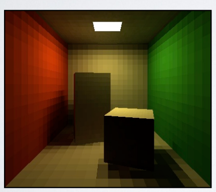
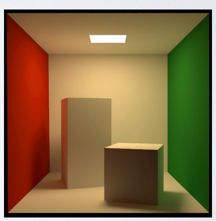
Touchup

- Each patch will have a constant color
  - Smooth solution (e.g. average to vertices)



Example mesh for Cornell Box
by Mark Schmelzenbach



Does not match but you get the idea...

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Other Things

- Each patch will have a constant color
  - Smooth solution (*e.g.* average to vertices)
- No specular reflection
  - Add Phong specular term or raytraced specular reflection
- Grid artifacts
  - Be clever with grid...

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White Light by Paul Cocksedge

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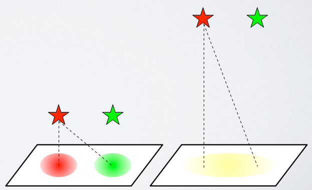
Hierarchical Radiosity

- Light smoothes with distance
  - Compare $1/h^2$ with $1/(h^2+d^2)$ as h gets large



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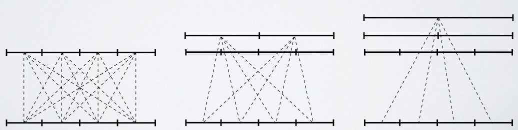
Hierarchical Radiosity

- Light smoothes with distance
  - Compare $1/h^2$ with $1/(h^2+d^2)$ as h gets large
- Group patches into hierarchy
  - Far interactions use lower-res form factors



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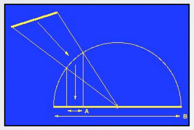
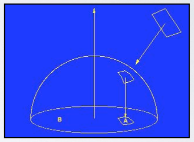
Computing Form Factors

- Form factors have a geometric meaning





Images from
SIGGRAPH 93 Education Slide Set
by Stephen Spencer

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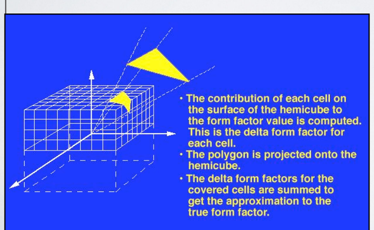
Computing Form Factors

- Form factors have a geometric meaning
- "Hemicube" algorithm uses regular scan conversion



Images from
SIGGRAPH 93 Education Slide Set
by Stephen Spencer

Computing Form Factors

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- Form factors have a geometric meaning
- "Hemicube" algorithm uses regular scan conversion
- Also computed by ray-based sampling
- **In practice, computing form factors is the bottleneck**

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Photon Mapping

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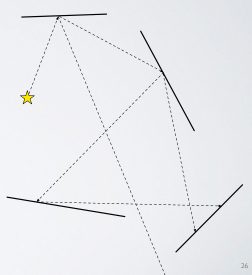
- Lights cast "photons" into environment
  - Cast in random directions
  - Trace into environment
  - Store records at intersections



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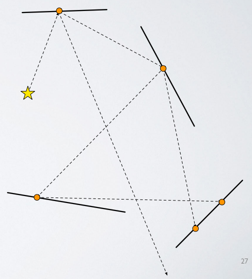
Photon Mapping

- Lights cast "photons" into environment
  - Cast in random directions
  - Trace into environment
  - Store records at intersections
    - With KD-Trees...



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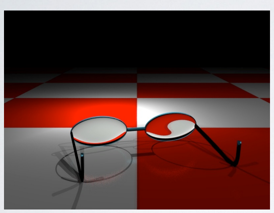
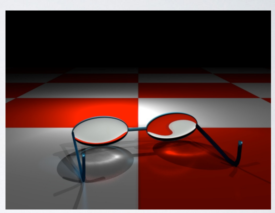
Comparison



Ray Tracing



Ray Tracing w/ Photon Map

Catherine Bendebury and Jonathan Michaels
CS 184 Spring 2005

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Photon Mapping

Image by Per Christensen

A ray traced image

Note:
- Dark shadows
- Unlit corners
- Nice reflections

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Photon Mapping

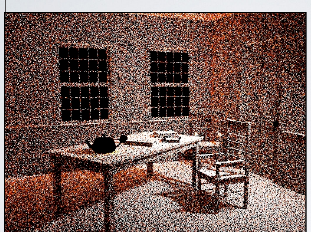

Image by Per Christensen

Raw photons

Note:
- Noisy
- Sparse

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Photon Mapping

Interpolated Photons

Note:
- Still noisy
- Biased

Image by Per Christensen

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Photon Mapping

Interpolated Photons (multiplied by diffuse)

Note:
- Still noisy
- Biased

Image by Per Christensen

Photon Mapping

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- Final Gather
  - Ray trace scene
  - Direct and specular rays as normal
  - Diffuse rays traced into photon map
  - *Diffuse reflection smoothes noise*

Photon Mapping

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Image by Per Christensen

Final Image

Note:
Not noisy
Nice lighting
Reflections
May still be biased

Final gather often bottleneck...

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Ambient Occlusion

- A "hack" to create more realistic ambient illumination cheaply
- Assume light from everywhere is partially blocked by local objects
  - At a point on the surface cast rays at random
  - Ambient term is proportional to percent of rays that hit nothing
  - Weight average by cosine of angle with normal
  - Take into account how far before occluded

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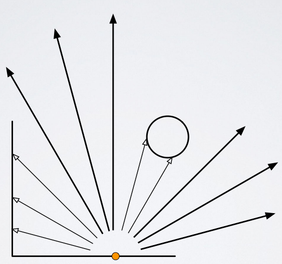
Ambient Occlusion



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Ambient Occlusion

Diffuse Only   Ambient Occlusion   Combined

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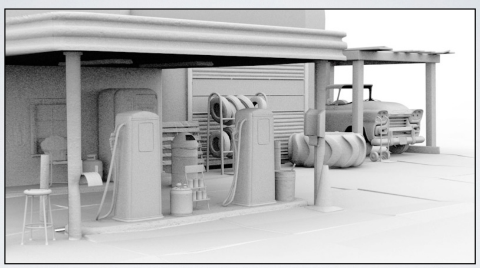
Ambient Occlusion



nVidia Gelato Demo Image

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