1

CS-184: Computer Graphics

Lecture #8: Projection

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2

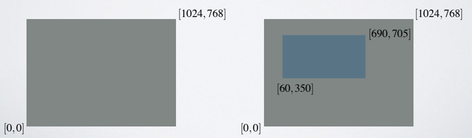
Today

- Windowing and Viewing Transformations
  - Windows and viewports
  - Orthographic projection
  - Perspective projection

3

Screen Space

- Monitor has some number of pixels
  - e.g. 1024 x 768
- Some sub-region used for given program
  - You call it a window
  - Let's call it a viewport instead



4

Screen Space

- May not really be a "screen"
  - Image file
  - Printer
  - Other
- Little pixel details
- Sometimes odd
  - Upside down
  - Hexagonal

From Shirley textbook.

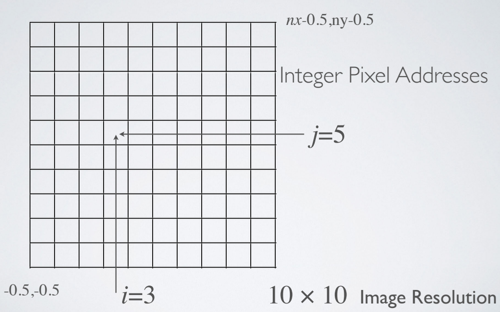
5

Screen Space

- Viewport is somewhere on screen
  - You probably don't care where
  - Window System likely manages this detail
  - Sometimes you care exactly where
- Viewport has a size in pixels
  - Sometimes you care (images, text, *etc.*)
  - Sometimes you don't (using high-level library)

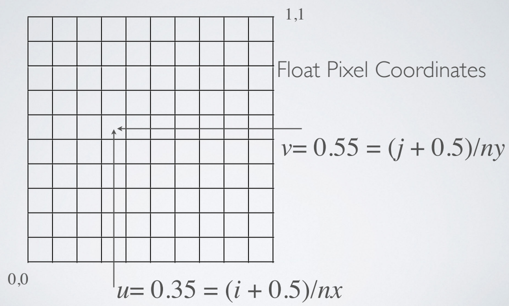
6

Screen Space



Screen Space

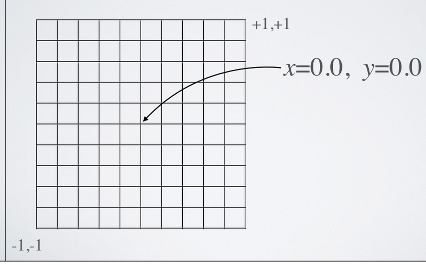
7



Canonical View Space

8

- Canonical view region
  - 2D: [-1,-1] to [+1,+1]

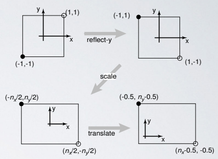


From Shirley textbook.

9

Canonical View Space

- Canonical view region
  - 2D: [-1,-1] to [+1,+1]



From Shirley textbook.
(Image coordinates are up-side-down.)

$$\begin{bmatrix} i \\ j \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & \frac{n_x-1}{2} \\ 0 & -\frac{n_y}{2} & \frac{n_y-1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Remove minus for right-side-up

9

10

Canonical View Space

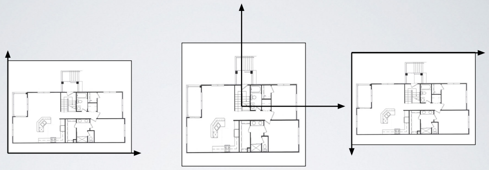
- Canonical view region
  - 2D: [-1,-1] to [+1,+1]
- Define arbitrary ***window*** and define objects
- Transform window to canonical region
- Do other things (we'll see clipping latter)
- Transform canonical to screen space
- Draw it.

From Shirley textbook.

10

11

Canonical View Space



World Coordinates (Meters) | Canonical | Screen Space (Pixels)

Note distortion issues...

12

Projection

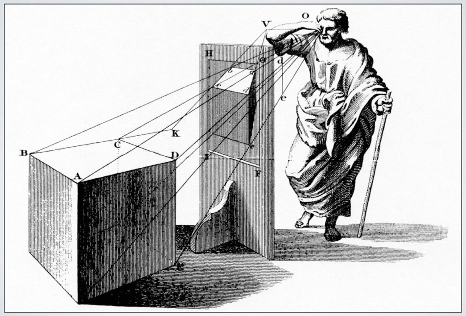
- Process of going from 3D to 2D
- Studies throughout history (*e.g.* painters)
- Different types of projection
  - Linear
    - Orthographic
    - Perspective
  - Nonlinear

Many special cases in books just one of these two...

Orthographic is special case of perspective...

13

Perspective Projections



14

Ray Generation vs. Projection

Viewing in ray tracing

- start with image point
- compute ray that projects to that point
- do this using geometry

Viewing by projection

- start with 3D point
- compute image point that it projects to
- do this using transforms

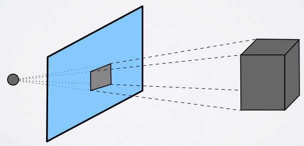
Inverse processes

- ray gen. computes the preimage of projection

15

Linear Projection

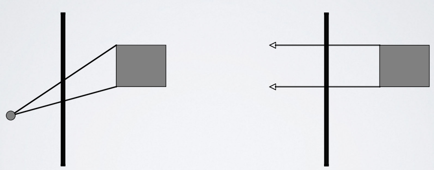
- Projection onto a planar surface
- Projection directions either
  - Converge to a point
  - Are parallel (converge at infinity)



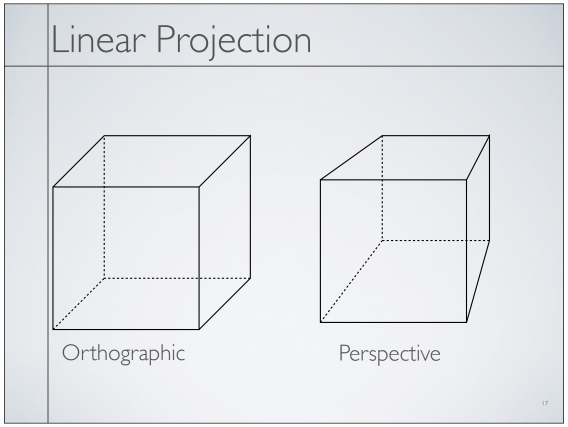
16

Linear Projection

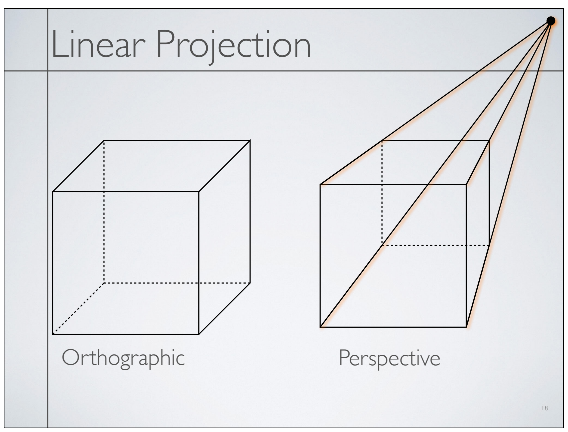
- A 2D view



17


Linear Projection
Orthographic
Perspective

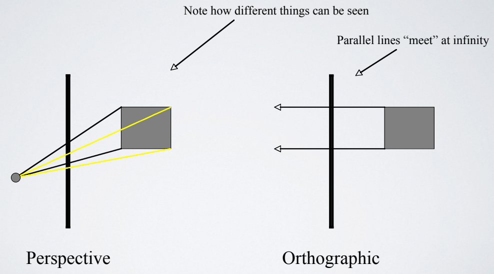
18


Linear Projection
Orthographic
Perspective

19

Linear Projection

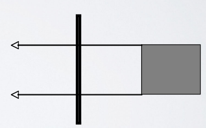
- A 2D view



20

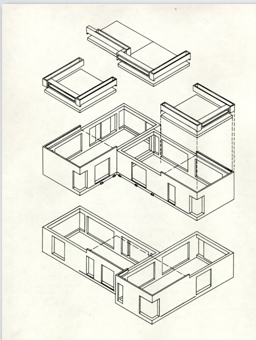
Orthographic Projection

- No foreshortening
- Parallel lines stay parallel
- Poor depth cues



Orthographic Projection

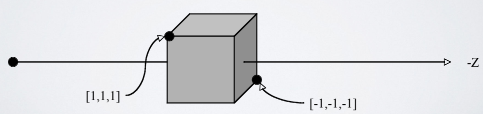
21



Canonical View Space

22

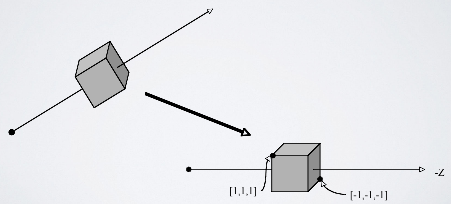
- Canonical view region
  - 3D: [-1,-1,-1] to [+1,+1,+1]
- Assume looking down -Z axis
  - Recall that "Z is in your face"



23

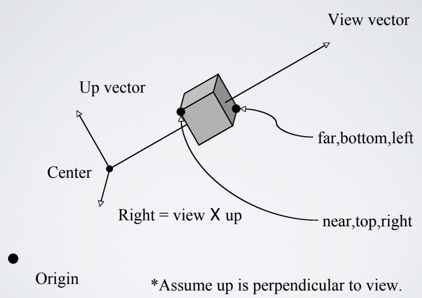
Orthographic Projection

• Convert arbitrary view volume to canonical



24

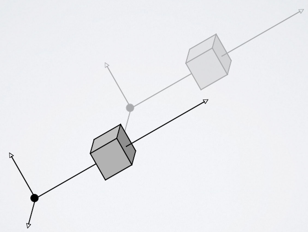
Orthographic Projection



25

Orthographic Projection

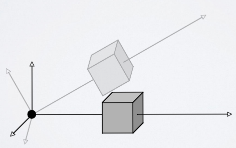
- Step 1: translate center to origin



26

Orthographic Projection

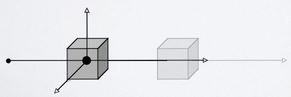
- Step 1: translate center to origin
- Step 2: rotate ***view*** to **-Z** and ***up*** to **+Y**



27

Orthographic Projection

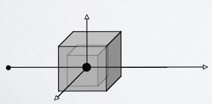
- Step 1: translate center to origin
- Step 2: rotate ***view*** to **-Z** and ***up*** to **+Y**
- Step 3: center view volume



28

Orthographic Projection

- Step 1: translate center to origin
- Step 2: rotate ***view*** to **-Z** and ***up*** to **+Y**
- Step 3: center view volume
- Step 4: scale to canonical size

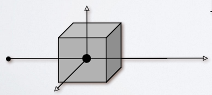


29

Orthographic Projection

- Step 1: translate center to origin
- Step 2: rotate *view* to **-Z** and *up* to **+Y**
- Step 3: center view volume
- Step 4: scale to canonical size

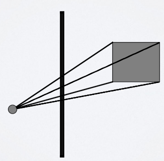
$$\mathbf{M} = \mathbf{S} \cdot \mathbf{T}_2 \cdot \mathbf{R} \cdot \mathbf{T}_1$$
$$\mathbf{M} = \mathbf{M}_o \cdot \mathbf{M}_v$$



30

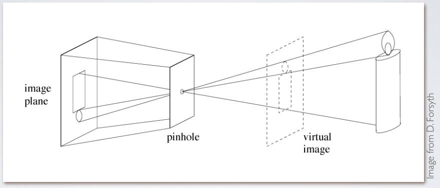
Perspective Projection

- Foreshortening: further objects appear smaller
- Some parallel line stay parallel, most don't
- Lines still look like lines
- **Z ordering preserved (where we care)**



31

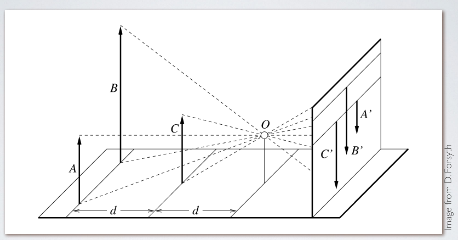
Perspective Projection



Pinhole ***a.k.a*** center of projection

32

Perspective Projection

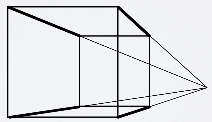


Foreshortening: distant objects appear smaller

33

Perspective Projection

- Vanishing points
  - Depend on the scene
  - Not intrinsic to camera



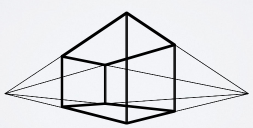
"One point perspective"

33

34

Perspective Projection

- Vanishing points
  - Depend on the scene
  - Nor intrinsic to camera



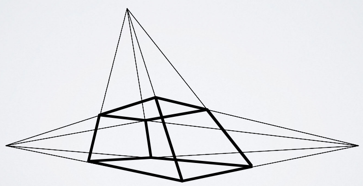
"Two point perspective"

34

35

Perspective Projection

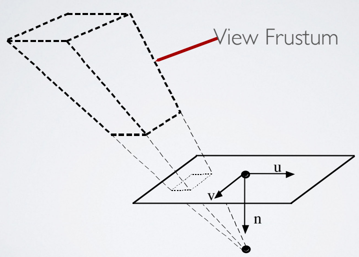
- Vanishing points
  - Depend on the scene
  - Not intrinsic to camera



"Three point perspective" 35

36

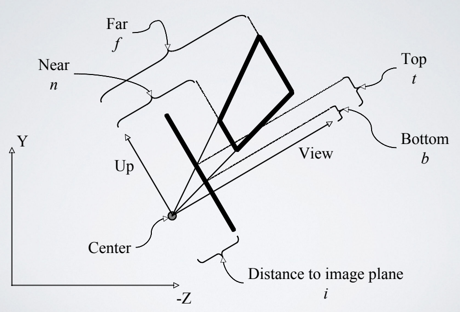
Perspective Projection



36

37

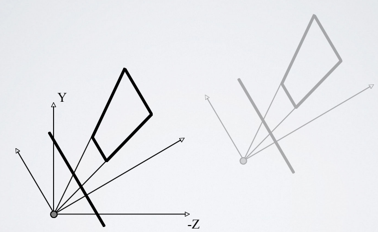
Perspective Projection



38

Perspective Projection

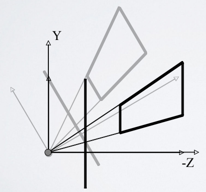
- Step 1: Translate ***center*** to origin



39

Perspective Projection

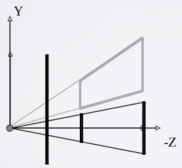
- Step 1: Translate ***center*** to origin
- Step 2: Rotate ***view*** to **-Z**, ***up*** to **+Y**



40

Perspective Projection

- Step 1: Translate ***center*** to origin
- Step 2: Rotate ***view*** to **-Z**, ***up*** to **+Y**
- Step 3: Shear center-line to **-Z** axis

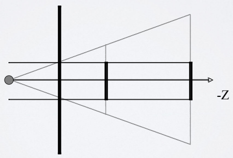


41

Perspective Projection

- Step 1: Translate ***center*** to origin
- Step 2: Rotate ***view*** to **-Z**, ***up*** to **+Y**
- Step 3: Shear center-line to **-Z** axis
- Step 4: Perspective

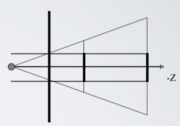
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{i+f}{i} & f \\ 0 & 0 & \frac{-1}{i} & 0 \end{bmatrix}$$



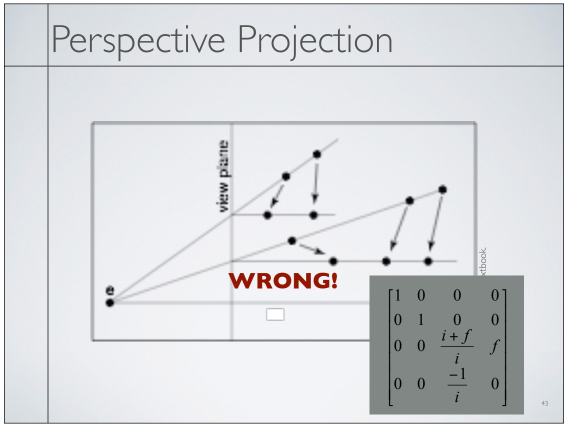
42

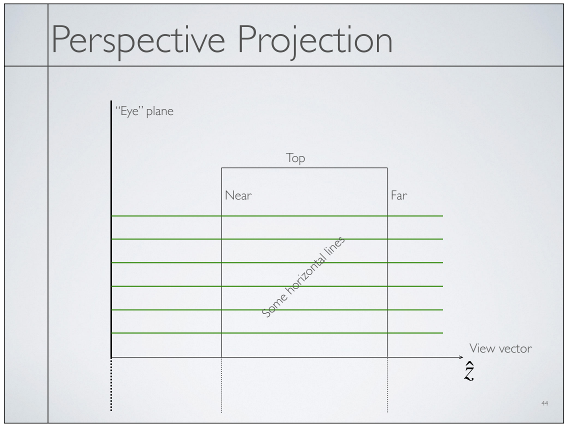
Perspective Projection

- Step 4: Perspective
  - Points at $z=-i$ stay at $z=-i$
  - Points at $z=-f$ stay at $z=-f$
  - Points at $z=0$ goto $z=\pm\infty$
  - Points at $z=-\infty$ goto $z=-(i+f)$
  - x and y values divided by $-z/i$
  - Straight lines stay straight
  - Depth ordering preserved in $[-i,-f]$
  - Movement along lines distorted



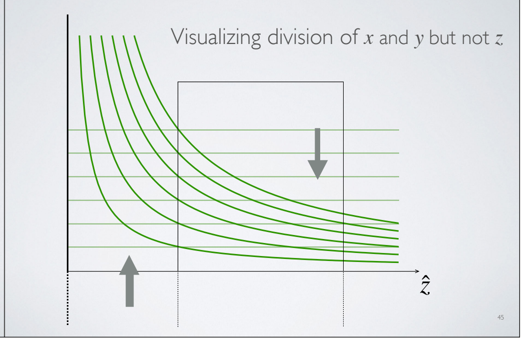
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{i+f}{i} & f \\ 0 & 0 & \frac{-1}{i} & 0 \end{bmatrix}$$


Perspective Projection
view plane
e
WRONG!
$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & \frac{i+f}{i} & f\\0 & 0 & \frac{-1}{i} & 0\end{bmatrix}$$


Perspective Projection
"Eye" plane
Top
Near
Far
Some horizontal lines
View vector
$\hat{z}$

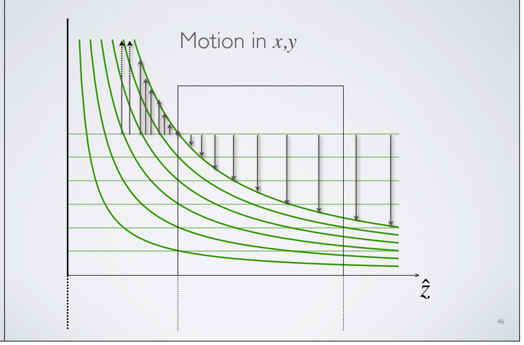
Perspective Projection

45



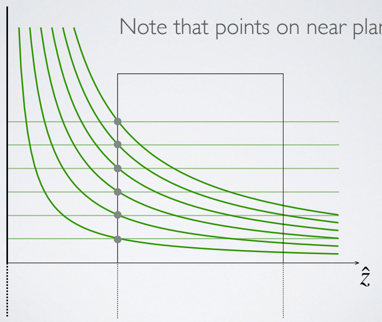
Perspective Projection

46



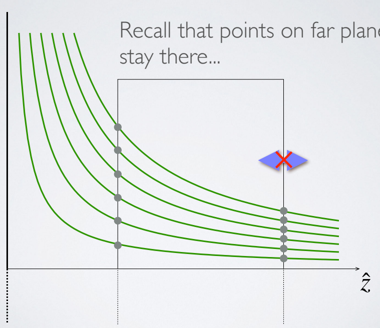
47

Perspective Projection

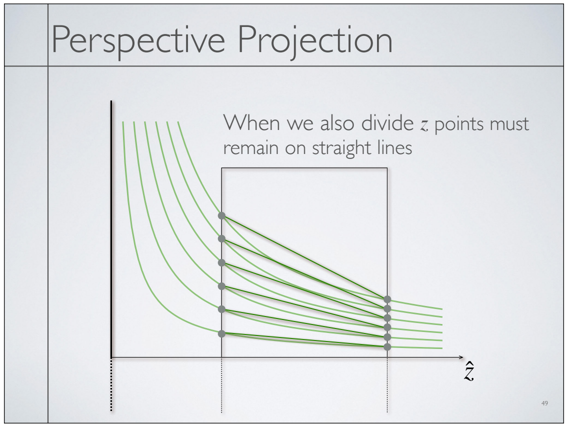


48

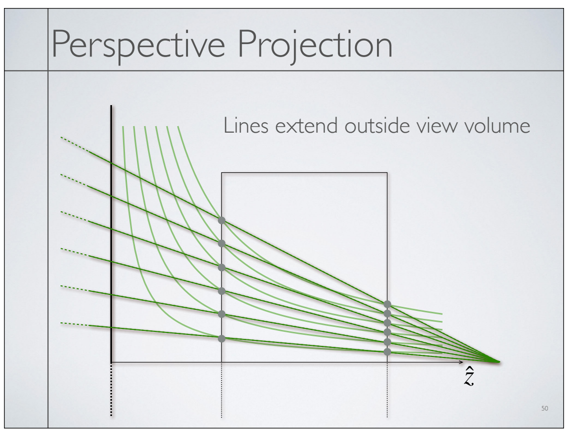
Perspective Projection



49


Perspective Projection
When we also divide z points must remain on straight lines
ẑ

50


Perspective Projection
Lines extend outside view volume
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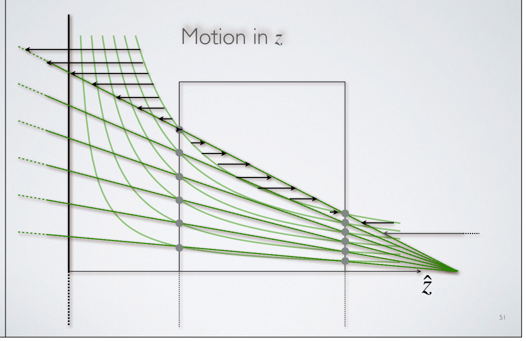
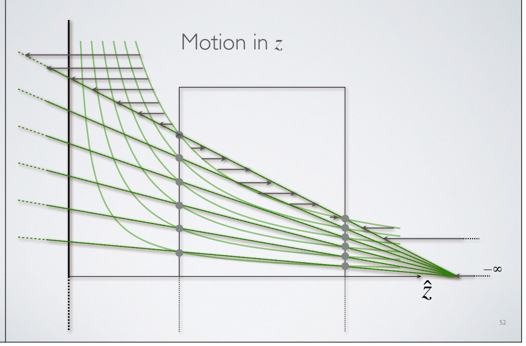
51

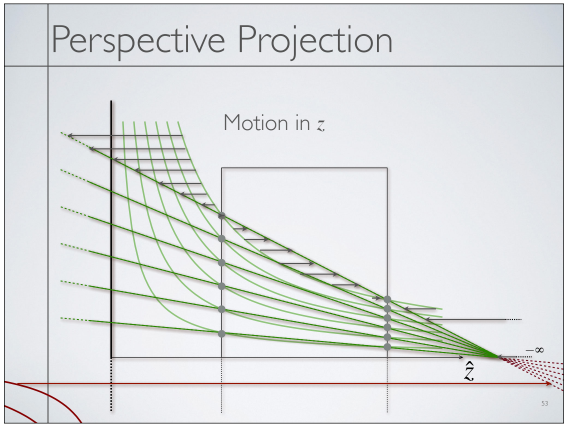
Perspective Projection



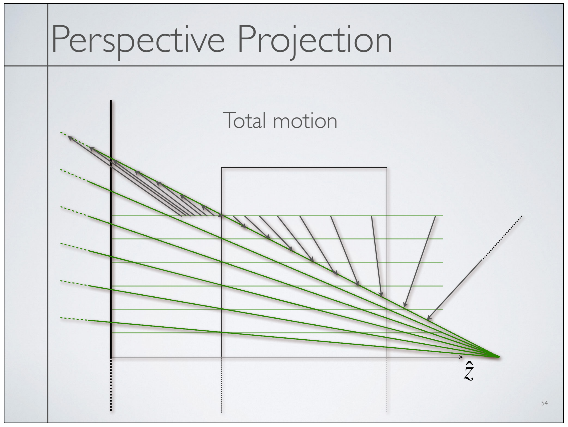
52

Perspective Projection




Perspective Projection
Motion in z
$-\infty$
$\hat{z}$

53

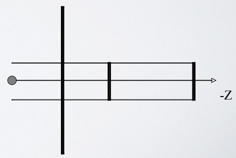

Perspective Projection
Total motion
$\hat{z}$

54

55

Perspective Projection

- Step 1: Translate ***center*** to orange
- Step 2: Rotate ***view*** to **-Z**, ***up*** to **+Y**
- Step 3: Shear center-line to **-Z** axis
- Step 4: Perspective
- Step 5: center view volume
- Step 6: scale to canonical size



56

Perspective Projection

- Step 1: Translate ***center*** to orange
- Step 2: Rotate ***view*** to **-Z**, ***up*** to **+Y**

} $\mathbf{M}_v$

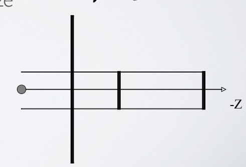
- Step 3: Shear center-line to **-Z** axis
- Step 4: Perspective

} $\mathbf{M}_p$

- Step 5: center view volume
- Step 6: scale to canonical size

} $\mathbf{M}_o$

$$\mathbf{M} = \mathbf{M}_o \cdot \mathbf{M}_p \cdot \mathbf{M}_v$$



57

Perspective Projection

- There are other ways to set up the projection matrix
  - View plane at $z=0$ zero
  - Looking down another axis
  - *etc...*
- Functionally equivalent

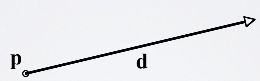
57

58

Vanishing Points

- Consider a ray:

$$\mathbf{r}(t) = \mathbf{p} + t\,\mathbf{d}$$



58

Vanishing Points

59

- Ignore **Z** part of matrix
- **X and Y will give location in image plane**
- Assume image plane at *z=-i*

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ & \text{whatever} & & \\ 0 & 0 & -1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} I_x \\ I_y \\ I_w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Vanishing Points

60

$$\begin{bmatrix} I_x \\ I_y \\ I_w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z \end{bmatrix}$$

$$\begin{bmatrix} I_x / I_w \\ I_y / I_w \end{bmatrix} = \begin{bmatrix} -x/z \\ -y/z \end{bmatrix}$$

Vanishing Points

61

- Assume

$$d_z = -1$$

$$\begin{bmatrix} I_x / I_w \\ I_y / I_w \end{bmatrix} = \begin{bmatrix} -x/z \\ -y/z \end{bmatrix} = \begin{bmatrix} \frac{p_x + td_x}{-p_z + t} \\ \frac{p_y + td_y}{-p_z + t} \end{bmatrix}$$

$$\lim_{t \to \pm\infty} = \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

Vanishing Points

62

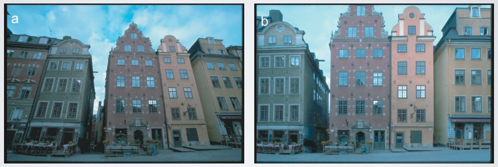
$$\lim_{t \to \pm\infty} = \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

- All lines in direction **d** converge to same point in the image plane -- the vanishing point
- Every point in plane is a v.p. for some set of lines
- Lines parallel to image plane ($d_z = 0$) vanish at infinity

What's a horizon?

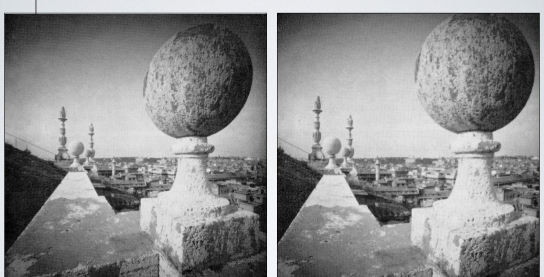
Perspective Tricks

63



Right Looks Wrong (Sometimes)

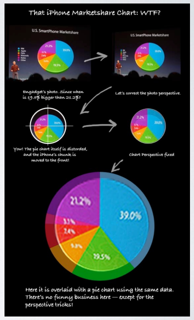
64



From *Correction of Geometric Perceptual Distortions in Pictures*, Zorin and Barr SIGGRAPH 1995

65

Right Looks Wrong (Sometimes)



From WIRED Magazine

65

66

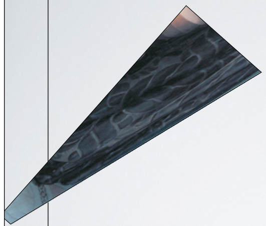
Strangeness

The Ambassadors
by Hans Holbein the Younger

66

67

Strangeness



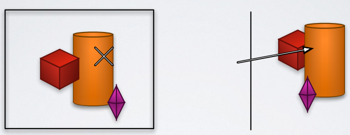
The Ambassadors
by Hans Holbein the Younger

67

68

Ray Picking

- Pick object by picking point on screen



- Compute ray from pixel coordinates.

68

69

Ray Picking

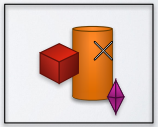
- Transform from World to Screen is:

- Inverse:

$$\begin{bmatrix} I_x \\ I_y \\ I_z \\ I_w \end{bmatrix} = \mathbf{M} \begin{bmatrix} W_x \\ W_y \\ W_z \\ W_w \end{bmatrix}$$

- What $\mathbf{Z}$ value?

$$\begin{bmatrix} W_x \\ W_y \\ W_z \\ W_w \end{bmatrix} = \mathbf{M}^{-1} \begin{bmatrix} I_x \\ I_y \\ I_z \\ I_w \end{bmatrix}$$



69

70

Ray Picking

Depends on screen details, YMMV
General idea should translate...

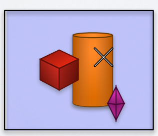
- Recall that:
  - Points at z=-i stay at z=-i
  - Points at z=-f stay at z=-f

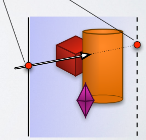
$$\mathbf{a}_s = [s_x, s_y, -i]$$

$$\mathbf{r}(t) = \mathbf{p} + t\,\mathbf{d}$$

$$\mathbf{b}_s = [s_x, s_y, -f]$$

$$\mathbf{r}(t) = \mathbf{a}_w + t(\mathbf{b}_w - \mathbf{a}_w)$$



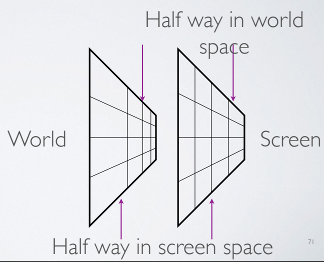


70

71

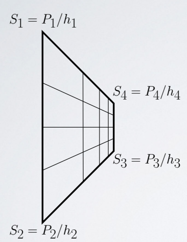
Depth Distortion

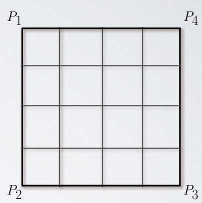
- Recall depth distortion from perspective
  - Interpolating in screen space different than in world
  - Ok, for shading (mostly)
  - Bad for texture



72

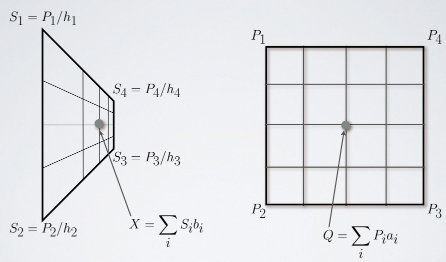
Depth Distortion





73

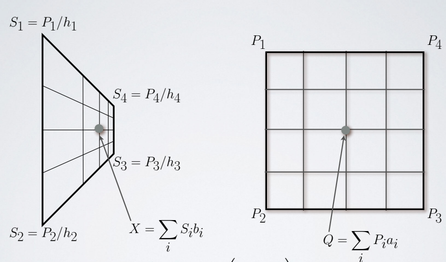
Depth Distortion



We know the S_i , P_i , and b_i , but not the a_i .

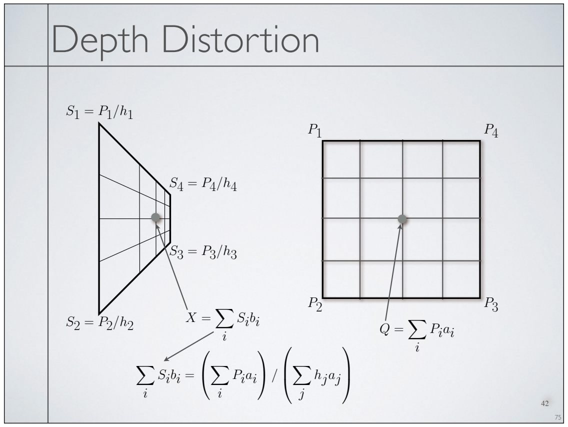
74

Depth Distortion

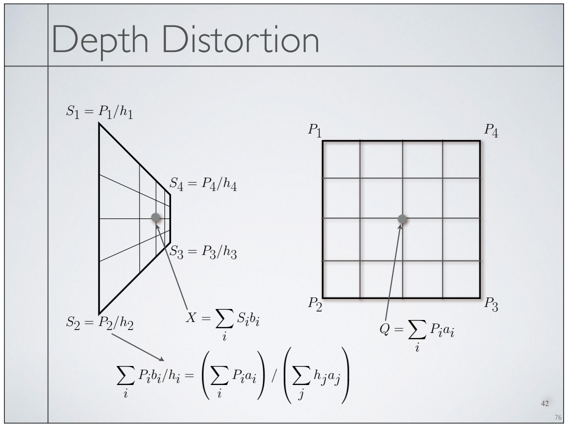


$$X = Q/h = \left(\sum_i P_i a_i\right) / \left(\sum_j h_j a_j\right)$$

75

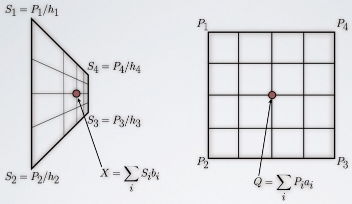

Depth Distortion
$S_1 = P_1/h_1$
$S_4 = P_4/h_4$
$S_3 = P_3/h_3$
$S_2 = P_2/h_2$
$X = \sum_i S_i b_i$
P_1
P_4
P_2
P_3
$Q = \sum_i P_i a_i$
$\sum_i S_i b_i = \left(\sum_i P_i a_i\right) / \left(\sum_j h_j a_j\right)$
42

76


Depth Distortion
$S_1 = P_1/h_1$
$S_4 = P_4/h_4$
$S_3 = P_3/h_3$
$S_2 = P_2/h_2$
$X = \sum_i S_i b_i$
P_1
P_4
P_2
P_3
$Q = \sum_i P_i a_i$
$\sum_i P_i b_i / h_i = \left(\sum_i P_i a_i\right) / \left(\sum_j h_j a_j\right)$
42

77

Depth Distortion



$$\sum_i P_i b_i / h_i = \left(\sum_i P_i a_i \right) / \left(\sum_j h_j a_j \right)$$

Independent of given vertex locations.

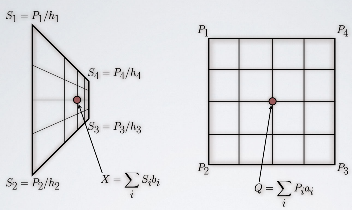
$$b_i / h_i = a_i / \left(\sum_j h_j a_j \right) \quad \forall i$$

42

77

78

Depth Distortion



$$b_i / h_i = a_i / \left(\sum_j h_j a_j \right) \quad \forall i$$

Linear equations in the a_i .

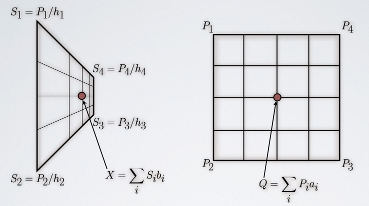
$$\left(\sum_j h_j a_j \right) b_i / h_i - a_i = 0 \quad \forall i$$

42

78

79

Depth Distortion



Linear equations in the a_i .

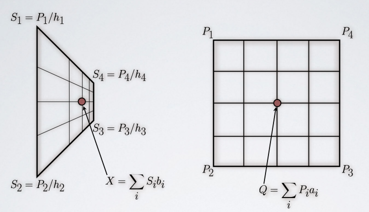
$$\left(\sum_j h_j a_j\right) b_i/h_i - a_i = 0 \quad \forall i$$

Not invertible so add some extra constraints.

$$\sum_i a_i = \sum_i b_i = 1$$

80

Depth Distortion



For a line: $a_1 = h_2 b_i/(b_1 h_2 + h_1 b_2)$

For a triangle: $a_1 = h_2 h_3 b_1/(h_2 h_3 b_1 + h_1 h_3 b_2 + h_1 h_2 b_3)$

Obvious Permutations for other coefficients.