Marching cubes 1

Marching cubes

Input: 3D voxel data (e.g. medical scan)

$\boxed{\text{OR}}$

2D isosurface embedded in 3D

Output: Triangulation of object boundary or isosurface

Voxel = volume element (3D, usually cube)

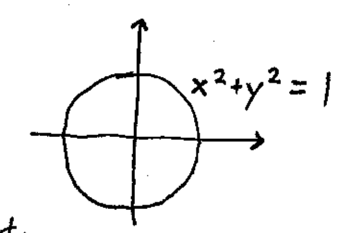
Pixel = picture element (2D, usually square)

Each voxel has a scalar value

Goal: triangulation separates space w/value $< \alpha$ from space w/value $> \alpha$

Define $f: \mathbb{R}^3 \to \mathbb{R}$

Isosurface: $\{p : f(p) = \alpha\}$ ← isovalue

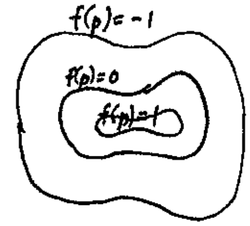


How defined? Implicit surface modeling, surface reconstruction, ...

2 possibilities: continuous f — e.g. signed distance function

$\boxed{\text{OR}}$

binary f (inside/outside) — e.g. solid modeler



Computing f is often dominant cost.

2 possibilities: f is known only at voxel centers (usual for voxel data)

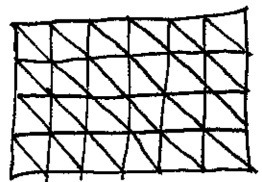
$\boxed{\text{OR}}$

f can be sampled at any point (usual for isosurfaces)

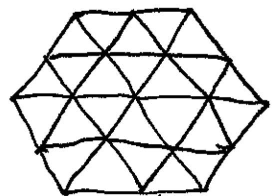
Marching cubes 2

Marching triangles

① Choose background grid.



(good for pixel data)



equilateral
(good for isocontours)

② Sample $f(p)$ at each grid vertex p.

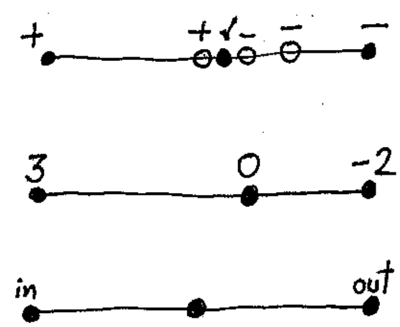
Label vertex "+" if $f(p) > \alpha$ (inside).

"−" if $f(p) < \alpha$ (outside).

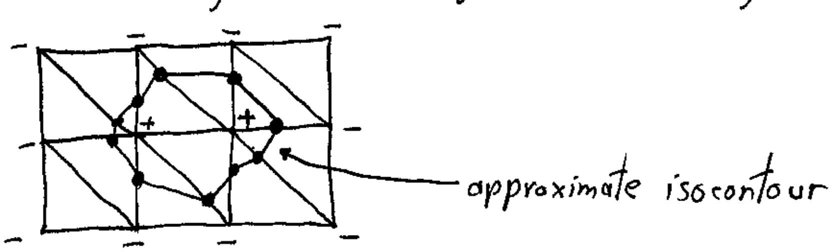
"+" if $f(p) = \alpha$ (perturb the value a bit).

③ For each edge with one + endpoint and one − endpoint, compute a <u>cut point</u> c where $f(c) \approx \alpha$.

If f can be sampled anywhere
 Use bisection.
else if f is continuously-valued
 Use inteprpolation.
else
 Use midpoint.
 (Consider smoothing the output later.)



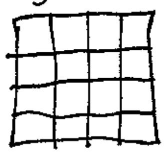
④ If a background triangle has differing signs, output a segment.



Marching cubes 3

Marching squares

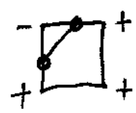
① Choose grid.

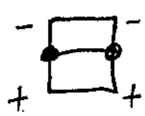


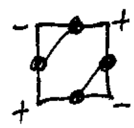
② Sample f at each vertex.

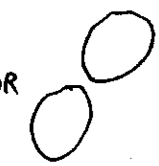
③ Compute cut points.

④ All corners the same sign:  No segment generated.

One corner differs:  One segment.

Two corners differ:  One segment.

 Two segments.

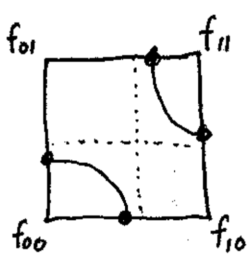
Ambiguity:  OR  ?  OR  ?

"Doesn't matter" vs. "Redo w/finer grid" vs. "Try to guess better choice".

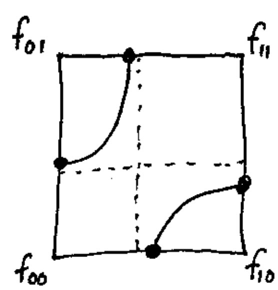
Heuristic [Nielson & Hamann]: Assume f is bilinear function.

$$f(x,y) = Axy + Bx + Cy + D$$

Then $f(x,y) = \alpha$ is hyperbola w/axis-aligned asymptotes.



$$(f_{01}-\alpha)(f_{10}-\alpha) > (f_{00}-\alpha)(f_{11}-\alpha)$$

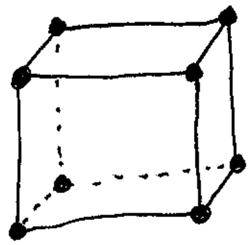


$$(f_{01}-\alpha)(f_{10}-\alpha) < (f_{00}-\alpha)(f_{11}-\alpha)$$

Marching cubes 4

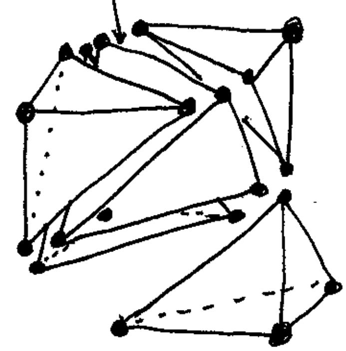
Marching tetrahedra

① Choose grid. Can't be equilateral!



• Even vertices
• Odd vertices

center tet (equilateral, not identical to other 4)

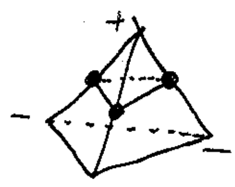


Better alternative for isosurfaces: body centered cubic lattice.

② Sample f at vertices.

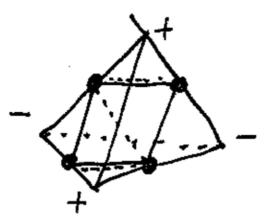
③ Cut points.

④ One corner differs:

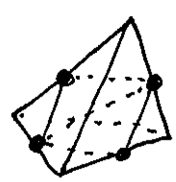


Generate one triangle.

Two corners differ:



OR



? Generate two.

"Doesn't matter" vs. "choose dihedral angle nearest 180°"
(dot product of unit normal vectors is closest to one)

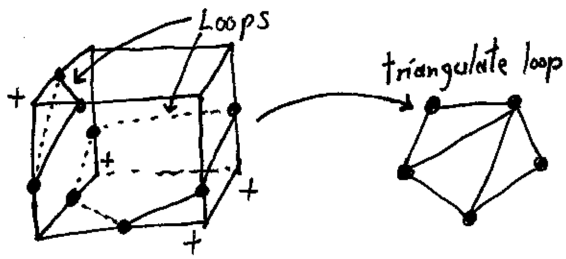
Marching cubes (Lorensen Cline '87; Wyvill McPheeters Wyvill '86)

① Choose grid.

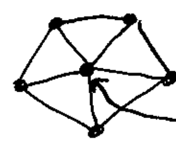
② Sample f.

③ Cut points.

④ Stencils.



Important: No triangle should lie on a cube face!
[Surface could fold over on self.]

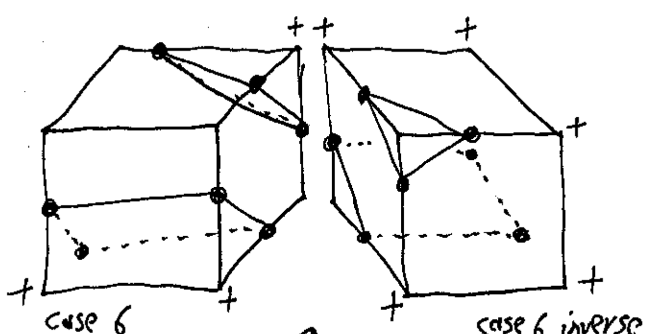
Wyvill: 

Look-up table: each vertex is + or −, giving $2^8 = 128$ entries.

Each entry is list of triangles indexing cut points (12 edges of cube).

Symmetries: rotation, reflection, swapping + & −. ⟶ 15 distinct cases.

Problem:



"Hole" in surface... not watertight, not manifold.

Solution [Nielson Hamann '91]: Use bilinear function to disambiguate  vs.  on faces.

⟶ 34 distinct cases.
(Some of which introduce vertex inside cube, like Wyvill.)