CS-184: Computer Graphics

Lecture 22: Radiometry

James O'Brien
University of California, Berkeley

V2013-F-22-1.0

1

Today

Radiometry: measuring light

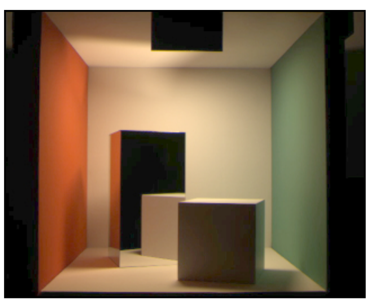
- Local Illumination and Raytracing were discussed in an ***ad hoc*** fashion
- Proper discussion requires proper units
- Not just pretty pictures... but correct pictures

2

Matching Reality

Unknown

Matching Reality



Photo

Rendered

Cornell Box Comparison
Cornell Program of Computer Graphics

Units

Light energy

- Really power not energy is what we measure
- Joules / second (J/s) = Watts (W)

Spectral energy density

- Power per unit spectrum interval
- Watts / nano-meter (W/nm)
- Properly done as function over spectrum
- Often just sampled for RGB

Often we assume people know we're talking about S.E.D. and just say E...

5

5

Irradiance

Total light striking surface from all directions

- Only meaningful w.r.t. a surface
- Power per square meter (W/m^2)
- Really S.E.D. per square meter ($\mathrm{W}/\mathrm{m}^2\ /\mathrm{nm}$)
- Not all directions sum the same because of foreshortening

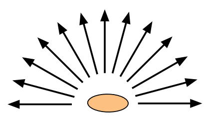
6

6

Radiant Exitance

Total light ***leaving*** surface over all directions

- Only meaningful w.r.t. a surface
- Power per square meter $(\mathrm{W/m^2})$
- Really S.E.D. per square meter $(\mathrm{W/m^2\ /nm})$
- Also called Radiosity
- Sum over all directions $\nRightarrow$ same in all directions



7

7

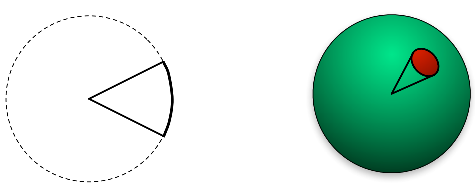
Solid Angles

Regular angles measured in ***radians*** $[0..2\pi]$

- Measured by arc-length on unit circle

Solid angles measured in ***steradians*** $[0..4\pi]$

- Measured by area on unit sphere
- Not necessarily little round pieces...



8

8

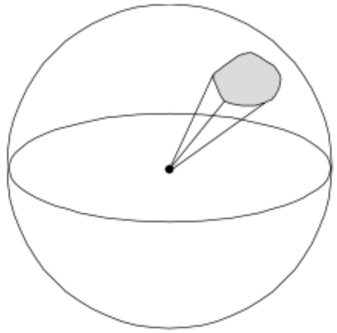
Angles and Solid Angles

- **Angle** $\theta = \frac{l}{r}$

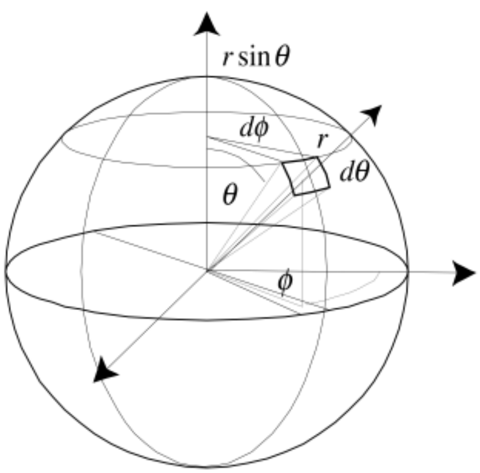
  $\Rightarrow$ circle has 2π radians

- **Solid angle** $\Omega = \frac{A}{R^2}$

  $\Rightarrow$ sphere has 4π steradians

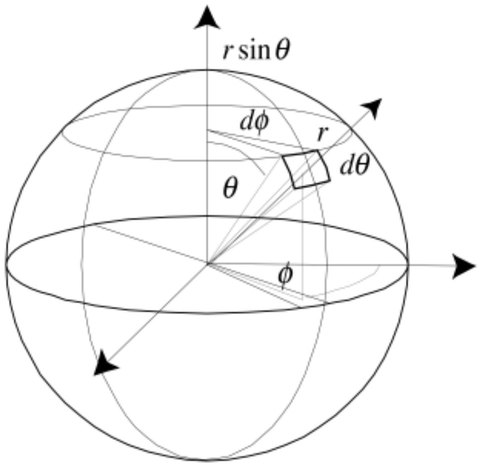


Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

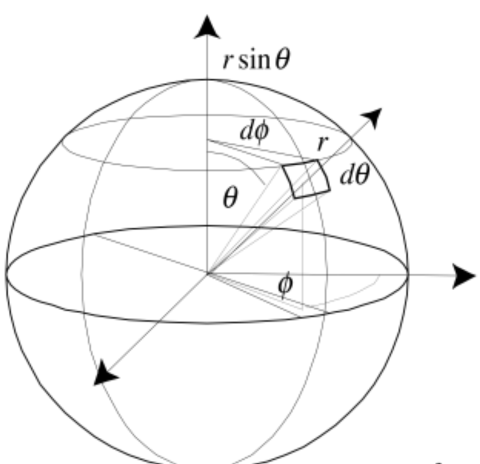
Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

Differential Solid Angles



Sphere S^2

$$d\omega = \sin\theta\,d\theta\,d\phi$$

$$\Omega = \int_{S^2} d\omega$$
$$= \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi$$
$$= \int_{-1}^{1}\int_0^{2\pi} d\cos\theta\;d\phi$$
$$= 4\pi$$

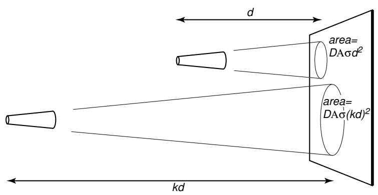
Radiance

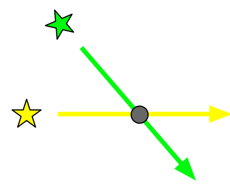
Light energy passing though a point in space within a given solid angle

- Energy per steradian per square meter $(\mathrm{W/m^2\ /sr})$
- S.E.D. per steradian per square meter $(\mathrm{W/m^2\ /sr\ /nm})$

Constant along straight lines in free space

- Area of surface being sampled is proportional to distance and light inversely proportional to squared distance





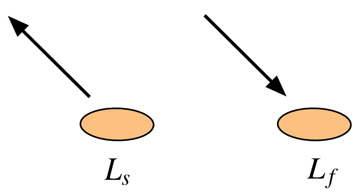
13

13

Radiance

Near surfaces, differentiate between

- Radiance from the surface (surface radiance)
- Radiance from other things (field radiance)



14

14

Light Fields

Radiance at every point in space, direction, and frequency: 6D function

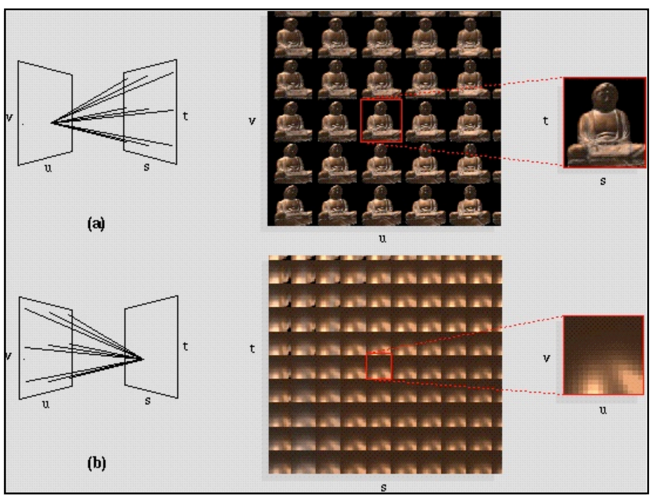
Collapse frequency to RGB, and assume free space: 4D function

Sample and record it over some volume

15

15

Light Fields

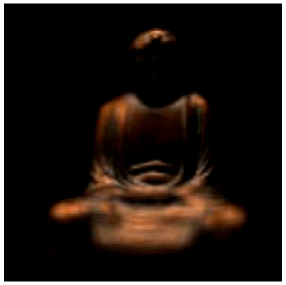


Levoy and Hanrahan, SIGGRAPH 1996

16

16

Light Fields



Levoy and Hanrahan, SIGGRAPH 1996

17

17

Light Fields

Michelangelo's ***Statue of Night***
From the Digital Michelangelo Project

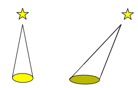
18

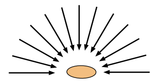
18

Computing Irradiance

Integrate incoming radiance (field radiance) over all direction

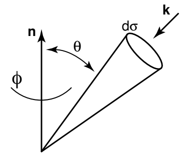
- Take into account foreshortening





$$H = \int_{\Omega} L_f(\mathbf{k}) \cos(\theta) d\sigma$$

$$H = \int_0^{2\pi} \int_0^{\pi/2} L_f(\theta, \phi) \cos(\theta) \sin(\theta)\, d\theta\, d\phi$$



19

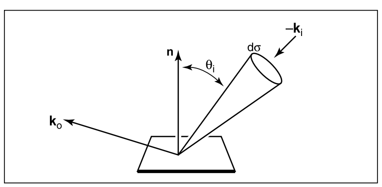
19

Revisiting The BRDF

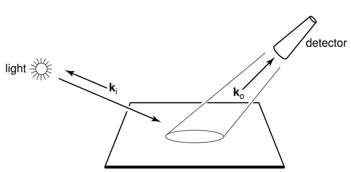
How much light from direction k_i goes out in direction k_o

Now we can talk about units:

- BRDF is ratio of surface radiance to the foreshortened field radiance



$$\rho(\mathbf{k}_i, \mathbf{k}_o) = \frac{L_s(\mathbf{k}_o)}{L_f(\mathbf{k}_i) \cos(\theta_i)}$$



We left out frequency dependance here...

Also note for perfect Lambertian reflector with constant BRDF $\rho = 1/\pi$

20

20

The Rendering Equation

Total light going out in some direction is given by an integral over all incoming directions:

$$L_s(\mathbf{k}_o) = \int_\Omega \rho(\mathbf{k}_i, \mathbf{k}_o) L_f(\mathbf{k}_i) \cos(\theta_i) \mathrm{d}\sigma_i$$

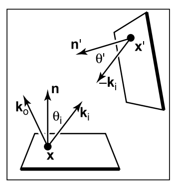
- Note, this is recursive (my L_f is another's L_s)

The Rendering Equation

$$L_s(\mathbf{k}_o) = \int_\Omega \rho(\mathbf{k}_i, \mathbf{k}_o) L_f(\mathbf{k}_i) \cos(\theta_i) \mathrm{d}\sigma_i$$

Rewrite explicitly in terms of surface radiances only

$$L_f(\mathbf{k}_i) = L_s(-\mathbf{k}_i) \qquad \Delta\sigma_i = \frac{\Delta A' \cos(\theta')}{||\mathbf{x} - \mathbf{x}'||^2}$$



$$L_s(\mathbf{x}, \mathbf{k}_o) = \int_{x' \text{ visible to } x} \frac{\rho(\mathbf{k}_i, \mathbf{k}_o) L_s(\mathbf{x}', \mathbf{x} - \mathbf{x}') \cos(\theta_i) \cos(\theta')}{||\mathbf{x} - \mathbf{x}'||^2} \mathrm{d}\mathbf{A}'$$

$$L_s(\mathbf{x}, \mathbf{k}_o) = \int_{\text{all } x'} \frac{\rho(\mathbf{k}_i, \mathbf{k}_o) L_s(\mathbf{x}', \mathbf{x} - \mathbf{x}') \delta(\mathbf{x}, \mathbf{x}') \cos(\theta_i) \cos(\theta')}{||\mathbf{x} - \mathbf{x}'||^2} \mathrm{d}\mathbf{A}'$$

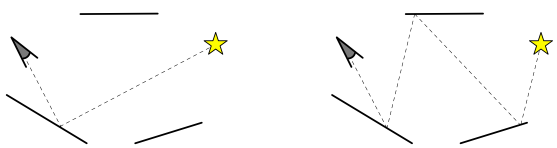
$$\delta(\mathbf{x}, \mathbf{x}') = \begin{cases} 1 \text{ if } \mathbf{x} \text{ and } \mathbf{x}' \text{ are mutually visible} \\ 0 \text{ otherwise} \end{cases}$$

Light Paths

Many paths from light to eye

Characterize by the types of bounces

- Begin at light
- End at eye
- "Specular" bounces
- "Diffuse" bounces



Light Paths

Describe paths using strings

- LDE, LDSE, LSE, ***etc.***

Describe types of paths with regular expressions

- L{D|S}*E ← Visible paths
- L{D|S}S*E ← Standard raytracing
- L{D|S}E ← Local illumination
- LD*E ← Radiosity method (have not talked about yet)