CS-184: Computer Graphics

Lecture #23: Rigid Body Dynamics

Prof. James O'Brien
University of California, Berkeley

V2011-F-23-1.0

Announcement

- Final Project Poster Session
  - Thursday, Friday December 8th, 2:30-5:00 pm
  - Poster stands and tables provided
  - Laptop videos or demos are highly recommended
  - Limited AC outlets
- Final project reports
  - **Hardcopy** due to me by December 15th 5pm.
  - *No time for late submission!*
- Final exam
  - Tuesday, December 13th, 8:00 - 11:00 am
  - 10 Evans

Today

- Rigid-body dynamics
- Articulated systems

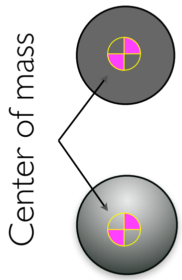
A Rigid Body

- A solid object that does not deform
  - Consists of infinite number of infinitesimal mass points...
  - ...that share a single RB transformation
    - Rotation + Translation (no shear or scale)

$$\boldsymbol{x}^W = \boldsymbol{R} \cdot \boldsymbol{x}^L + \boldsymbol{t}$$

    - Rotation and translation vary over time
  - Limit of deformable object as $k_S \rightarrow \infty$

A Rigid Body



In 2D:

- Translation 2 "directions"
- Rotation 1 "direction"
- 3 DOF Total

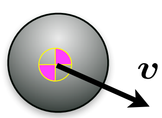
In 3D:

- Translation 3 "directions"
- Rotation 3 "direction"
- 6 DOF Total

Translation and rotation are ***decoupled***

2D is boring... we'll stick to 3D from now on...

Translational Motion

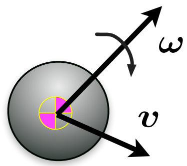


Just like a point mass:

$$\dot{\boldsymbol{p}} = \boldsymbol{v}$$
$$\dot{\boldsymbol{v}} = \boldsymbol{a} = \boldsymbol{f}/m$$

Note: Recall discussion on integration...

Rotational Motion

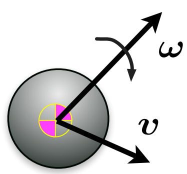


Rotation gets a bit odd, as well see...

Rotational "position" $\boldsymbol{R}$
- Rotation matrix
- Exponential map
- Quaternions

Rotational velocity $\boldsymbol{\omega}$
- Stored as a vector
- (Also called angular velocity...)
- Measured in radians / second

Rotational Motion



Kinetic energy due to rotation:

"Sum energy (from rotation) over all points in the object"

$$E = \int_\Omega \frac{1}{2} \rho \, \dot{\boldsymbol{x}} \cdot \dot{\boldsymbol{x}} \, du$$

$$E = \int_\Omega \frac{1}{2} \rho ([\omega \times] \boldsymbol{x}) \cdot ([\omega \times] \boldsymbol{x}) \, du$$

Rotational Motion

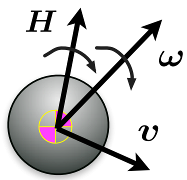


Figure is a lie if this really is a sphere...

Angular momentum
- Similar to linear momentum
- Can be derived from rotational energy

$$\boldsymbol{H} = \int_\Omega \rho\, \boldsymbol{x} \times \dot{\boldsymbol{x}}\, du$$

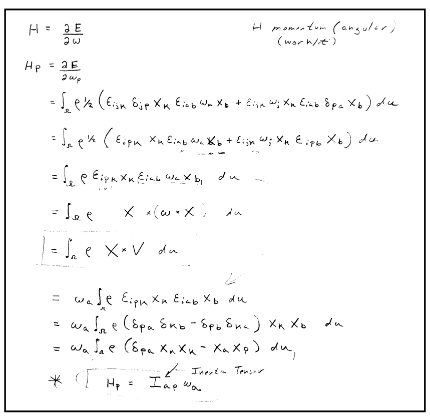
$$\boldsymbol{H} = \int_\Omega \rho\, \boldsymbol{x} \times (\omega \times \boldsymbol{x})\, du$$

$$\boldsymbol{H} = \left(\int_\Omega \cdots\, du\right) \boldsymbol{\omega}$$

$$\boldsymbol{H} = \boldsymbol{I}\boldsymbol{\omega}$$

"Inertia Tensor" not identity matrix...

Rotational Motion


H momentum (angular) (work/dt)

$$H = \frac{\partial E}{\partial \omega}$$

$$H_p = \frac{\partial E}{\partial \omega_p}$$

$$= \int_\Omega \rho \tfrac{1}{2} \left(\varepsilon_{ijk}\, \delta_{jp}\, x_k\, \varepsilon_{iab}\, \omega_a\, x_b + \varepsilon_{ijk}\, \omega_j\, x_k\, \varepsilon_{iab}\, \delta_{pa}\, x_b \right) du$$

$$= \int_\Omega \rho \tfrac{1}{2} \left(\varepsilon_{ipk}\, x_k\, \varepsilon_{iab}\, \omega_a\, x_b + \varepsilon_{ijk}\, \omega_j\, x_k\, \varepsilon_{ipb}\, x_b \right) du$$

$$= \int_\Omega \rho\, \varepsilon_{ipk}\, x_k\, \varepsilon_{iab}\, \omega_a\, x_b\, du$$

$$= \int_\Omega \rho\, X \times (\omega \times X)\, du$$

$$= \int_\Omega \rho\, X \times V\, du$$

$$= \omega_a \int_\Omega \rho\, \varepsilon_{ipk}\, x_k\, \varepsilon_{iab}\, x_b\, du$$

$$= \omega_a \int_\Omega \rho \left(\delta_{pa}\, \delta_{kb} - \delta_{pb}\, \delta_{ka} \right) x_k\, x_b\, du$$

$$= \omega_a \int_\Omega \rho \left(\delta_{pa}\, x_k x_k - x_a x_p \right) du$$

$$* \quad H_p = I_{ap}\, \omega_a$$

Inertia Tensor

Angular momentum
- Similar to linear momentum
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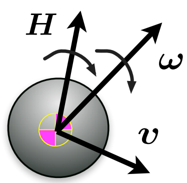
"Inertia Tensor" not identity matrix...

Rotational Motion



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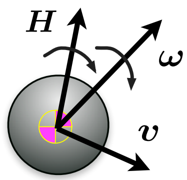
Inertia Tensor

$$\mathbf{I} = \int_\Omega \rho \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -xy & z^2+x^2 & -yz \\ -xz & -yz & x^2+y^2 \end{bmatrix} \mathrm{d}u$$

See example for simple shapes at
http://scienceworld.wolfram.com/physics/MomentofInertia.html

Can also be computed from polygon models by transforming volume integral to a surface one.
See paper/code by Brian Mirtich.

Rotational Motion



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Conservation or momentum:

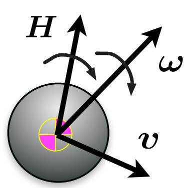
$$\boldsymbol{H}^W = \boldsymbol{I}^W \boldsymbol{\omega}^W$$
$$\boldsymbol{H}^W = \boldsymbol{R}\boldsymbol{I}^L\boldsymbol{R}^\mathsf{T}\boldsymbol{\omega}^W$$
$$\dot{\boldsymbol{H}}^W = \dot{\boldsymbol{R}}\boldsymbol{I}^L\boldsymbol{R}^\mathsf{T}\boldsymbol{\omega}^W + \cancel{\boldsymbol{R}\boldsymbol{I}^L\dot{\boldsymbol{R}}^\mathsf{T}}\boldsymbol{\omega}^W + \boldsymbol{R}\boldsymbol{I}^L\boldsymbol{R}^\mathsf{T}\boldsymbol{\alpha}^W$$

$$\dot{\boldsymbol{H}}^W = 0$$
$$\dot{\boldsymbol{R}} = \boldsymbol{\omega} \times \boldsymbol{R}$$

$$\boldsymbol{\alpha}^W = (\boldsymbol{R}\boldsymbol{I}^L\boldsymbol{R}^\mathsf{T})^{-1}(-\boldsymbol{\omega}^W \times \boldsymbol{H}^W)$$

In other words, things wobble when they rotate.

Rotational Motion



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$$\dot{\boldsymbol{R}} = [\omega \times]\boldsymbol{R}$$
$$\dot{\boldsymbol{\omega}} = \boldsymbol{\alpha}$$

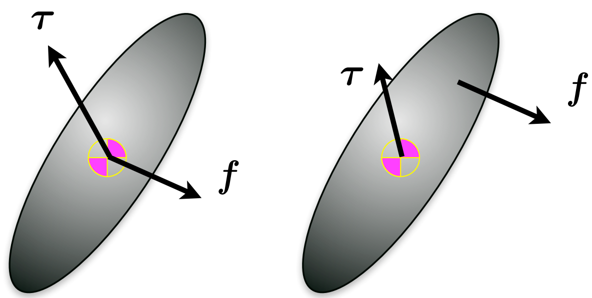
$$\boldsymbol{\alpha}^W = (\boldsymbol{R}\boldsymbol{I}^L\boldsymbol{R}^\mathsf{T})^{-1}\left((-\boldsymbol{\omega}^W \times \boldsymbol{H}^W) + \boldsymbol{\tau}\right)$$

$$\boldsymbol{\tau} = \boldsymbol{f} \times \boldsymbol{x}$$

Take care when integrating rotations, they need to stay rotations.

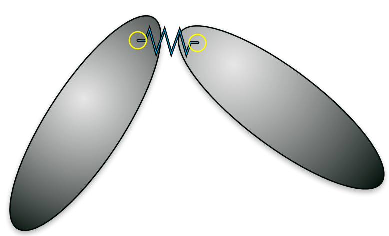
Couples

- A force / torque pair is a couple
  - Also a wrench
- Many couples are equivalent



Constraints

- Simples method is to use spring attachments
  - Basically a penalty method



  - Spring strength required to get good results may be unreasonably high
    - There are ways to cheat in some contexts...

Constraints

- Articulation constraints
  - Spring trick is an example of a full coordinate method
    - Better constraint methods exist
  - Reduced coordinate methods use DOFs in kinematic skeleton for simulation
    - Much more complex to explain
- Collisions
  - Penalty methods can also be used for collisions
  - Again, better constraint methods exist

Suggested Reading

•Brian Mirtich, ``Fast and Accurate Computation of Polyhedral Mass Properties,'' Journal of Graphics Tools, volume 1, number 2, 1996. http://www.cs.berkeley.edu/~jfc/mirtich/papers/volInt.ps

•Brian Mirtich and John Canny, ``Impulse-based Simulation of Rigid Bodies,'' in Proceedings of 1995 Symposium on Interactive 3D Graphics, April 1995. http://www.cs.berkeley.edu/~jfc/mirtich/papers/ibsrb.ps

•D. Baraff. Linear-time dynamics using Lagrange multipliers. Computer Graphics Proceedings, Annual Conference Series: 137-146, 1996. http://www.pixar.com/companyinfo/research/deb/sig96.pdf

•D. Baraff. Fast contact force computation for nonpenetrating rigid bodies. Computer Graphics Proceedings, Annual Conference Series: 23-34, 1994. http://www.pixar.com/companyinfo/research/deb/sig94.pdf