CS184 Fall98 **Review Quiz** **Out: Aug 26, 1998**
J. Canny **Due: Aug 28, 1998**

This Quiz is due in class on Friday August 28th. It will be graded. Make sure you include your name and section number on your answer sheet.

1. A vector v with n components can be treated as an $n \times 1$ matrix, i.e. a single column. Then the dot product of two n-vectors u and v corresponds to the matrix product $u^T v$, where u^T is the matrix transpose of u. Now let A be an $n \times n$ matrix. Show that if $u^T A v = v^T A u$ for all u and v, then $A = A^T$. Hint: consider orthonormal basis vectors for u and v.

2. Let a, b, c be vectors in $\mathbb{R}^3$. Show that the 3×3 matrix whose columns are a, b, c, i.e. $A = [a, b, c]$, satisfies $\det(A) = a \cdot (b \times c)$ where $\times$ is vector cross product.

3. A *linear function* $f(v)$ of some vector v satisfies:

   (a) $f(u + v) = f(u) + f(v)$

   (b) $f(\lambda u) = \lambda f(u)$ where λ is a scalar.

   Any linear function of v can be written as the product of some matrix with v, i.e. as Av for some matrix A. Now suppose u and v are 3-vectors. Let $\times$ be the usual cross product of vectors in $\mathbb{R}^3$. Show that:

   (a) The cross product $f(v) = u \times v$ is a linear function of v.

   (b) Find the matrix A such that $u \times v = Av$ for all v. The matrix A will depend only on u, not on v. Aside: This matrix is often written as $u\times$, so that $u \times v = (u\times)v$.

4. Recall that if A is an $n \times n$ matrix, then v is an *eigenvector* of A if it satisfies $Av = \lambda v$ for some scalar λ. The scalar λ is the *eigenvalue* corresponding to the eigenvector v. Let A be any $n \times n$ matrix. Show that all the eigenvalues of $A^T A$ are non-negative.