1. $\mathbb{Z}_N^*$ is a multiplicative group is cyclic iff $n$ is either: 1, 2, 4, $p^k$, or $2p^k$ where $p$ is an odd prime.

   $\phi(2) = 1$. If $n = 4$, $\phi(n) = 2$. If $n = p^k$, $\phi(n) = p^{k-1}(p-1)$, so set $p^{k-1}(p-1) = 2^m$, we get $p$ needs to have the form $2^m + 1$ and $k = 1$. If $n = 2p^k$, $\phi(n) = p^{k-1}(p-1)$ which is the same as for a prime power. So $n$ needs to be either 1, 4, or $2^m + 1$ or 2($2^m + 1$) for some $m$ where $2^m + 1$ is an odd prime.

2. $\phi(\phi(5^k)) = \phi(5^{k-1}2^2) = 2 \times 5^{k-2} \times 4 = 5^{k-2} \times 8$. So the fraction of generators is $8/25$.

3. Given gcd$(e_1, e_2) = 1$, we can find $x_1$ and $x_2$ using Euclid’s algorithm such that $e_1 x_1 + e_2 x_2 = 1$. So $C_1^{x_1} C_2^{x_2} = M \pmod n$.

4. $\phi(\phi(p)) = q - 1$. So the fraction of generators is $(q - 1)/(2q + 1)$.