1. List all possible values of \( n \) such that the multiplicative group \( \mathbb{Z}_n^* \) is cyclic and has order which is a power of 2, i.e. \( \phi(n) = 2^k \).

2. If \( n = 5^k \) so that \( \mathbb{Z}_n^* \) is cyclic, what fraction of the elements of \( \mathbb{Z}_n^* \) (the multiplicative group) are generators?

3. Suppose that a message \( M \) is encrypted using RSA twice, as \( C_1 = M^{e_1} \mod n \) and \( C_2 = M^{e_2} \mod n \). Note that the encryption keys are different, but the modulus \( n \) is the same in both cases. Show that \( M \) can be recovered from \( C_1 \) and \( C_2 \) in polynomial time.

4. If \( q \) is a prime, then \( p = 2q + 1 \) is also prime in some cases. Assuming \( q \) is large and \( p \) is prime, what fraction of the elements in \( \mathbb{Z}_p^* \) are generators? Combinations like this are important in discrete-log crypto-systems.