1. From the lecture notes, \( E[\sum H_{ij}] \leq n/2 \). If the probability of a given packet is delayed more than \( T(n) \) steps is bounded by \( 2^{-2n} \), then we can guarantee that all packets reach their destination in time \( T(n) \) with probability \( 1 - 2^{-n} \).

If we assume \( \delta > 2\epsilon - 1 \), \( \Pr[\sum H_{ij} > (1 + \delta)\mu] \leq 2^{-\mu\delta} \). If we set \( \mu \delta = 2n \), we get \( \delta = 4 < 2\epsilon - 1 \). So we should use the other formula, where when \( \delta \leq 2\epsilon - 1 \), \( \Pr[\sum H_{ij} > (1 + \delta)\mu] \leq \exp\left(-\mu\delta^2/4\right) \). We set \( \exp\left(-\mu\delta^2/4\right) = 2^{-2n} \), so \( \delta = \sqrt{16\ln 2} \approx 3.3 \). So \( T(n) \) should be \( (3.3 + 1)n/2 = 2.15n \).

2. This is equivalent to check whether \( AB = I \). Hence we can simply use the matrix multiplication checker program in the lecture notes to check whether \( AB = I \). The running time of the checker program is \( O(n^2) \).

3. \( p(a_{11}, \ldots, a_{ij}, \ldots, a_{mn}) \) is a multivariate polynomial total degree \( n \). So the program checker simply picks \( a_{11}, \ldots, a_{ij}, \ldots, a_{mn} \) uniformly at random from \( \{0, \ldots, M - 1\} \). We can compute such the determinant of the corresponding matrix \( A \), denoted as \( x = |A| \). We then compare the \( x \) with the output of the polynomial \( p \). Due to Schwartz-Zippel theorem, the error bound is \( n/M \). The running time to compute a determinant is \( O(n^3) \). So the running time of the program checker is \( O(n^3) \).

4. Let \( b_i \) denote the substring \( a_{i+1} \cdots a_{i+k} \). Let \( f(a, x) = \sum_i a_i x^i \mod p \) denote the fingerprint function. So \( f(b, x) = (f(b_{i-1}, x) - a_i)x^{-1} + a_{i+k}x^{k-1} \mod p \). Thus we can compute \( f(b, x) \) recursively in \( O(n) \) time independent of \( k \).