LETTER TO THE EDITOR

Dc electrical oxide thickness model for quantization of the inversion layer in MOSFETs

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Abstract. A simulator using the coupled Schrödinger equation, the Poisson equation and Fermi–Dirac statistics to analyse inversion-layer quantization has been shown to match the measured C–V data of thin-gate-oxide metal-oxide semiconductor (MOS) capacitors closely. This simulator is used to study in detail the effects of bias voltage, oxide thickness and doping concentration on the charge centroid and from this a simple empirical model for the dc charge centroid of the inversion layer is proposed. This model predicts the inversion charge density in terms of $T_{ox}$, $V_t$ and $V_g$ explicitly and can be used to estimate transistor current in device engineering and circuit simulation models.

In the very thin oxide regime, a better quantitative understanding of metal-oxide semiconductor (MOS) characteristics is needed. The effect of channel quantization cannot be ignored, because it determines the inversion-layer thickness, which is not negligible when compared with an oxide thickness less than 30 Å [1–4]. Quantization of electrons and holes and its impact on device characteristics have attracted a lot of attention due to the aggressive scaling of the gate oxide thickness [5–10]. The effect of channel quantization on the capacitance characteristics determined by the ac charge centroid of the charge layer has been discussed previously [11, 12]. In the prediction of the transistor current, however, the ac charge centroid is not directly relevant. To accurately explain the direct current in a transistor, a model of the dc charge centroid is needed. The classical model for solving the inversion charge distribution predicts that the charge peaks at the interface. The conventional method of ‘electrical oxide thickness’ characterization using capacitance–voltage (C–V) measurements in the accumulation region (to avoid polysilicon depletion) reflects the approximate value of the a.c. centroid in the inversion region but not the dc centroid. The approximate theoretical solution of the Schrödinger and Poisson equations for the average position suggests that the dc centroid $X_{dc} \propto (Q_b + 11 Q_{inv}/32)^{-1/3}$ [3, 13, 14]. In this work, we have developed a one-dimensional simulator using self-consistent method by solving the Schrödinger equation and the Poisson equation using Fermi–Dirac statistics iteratively and we have used it to find an empirical model for the dc charge centroid in terms of $V_g$, $V_t$ and $T_{ox}$ explicitly. Using this simple model, the $X_{dc}$ concept can be easily applied to give an estimation of the current in device engineering and circuit simulation models.

The simulator solves the Schrödinger equation, Poisson equation and Fermi–Dirac distribution self-consistently. The six equivalent band valleys are included for the calculation of the density of states. The wavefunction is assumed to be zero both at the oxide interface and deep in the substrate. Tunneling through the thin oxide is not included. (100) silicon is assumed for all cases presented here. The gate electrode is n$^+$ polysilicon and polygate depletion is included in the simulations. The substrate doping ranges from $2 \times 10^{16}$ to $2 \times 10^{18}$ cm$^{-3}$ and the oxide thickness from 30 to 150 Å. All the physical parameters are defined in table 1. The relation between gate voltage and inversion charge can be formulated in the following manner:

\[ V_g = V_{FB} + \varphi_s + \frac{Q_b + Q_{inv}}{C_{ox}} \]  \hspace{1cm} (1)

\[ V_T = V_{FB} + 2\varphi_B + \frac{Q_b}{C_{ox}}. \]  \hspace{1cm} (2)

The dc centroid of the inversion charge, $X_{dc}$, is defined as

\[ X_{dc} = \frac{\int \rho_{inv}(x) x \, dx}{\int \rho_{inv}(x) \, dx} \]  \hspace{1cm} (3)

where $\rho(x)$ is the charge density. The surface potential $\varphi_s$ can be expressed as

\[ \varphi_s = \frac{\int \int_0^L \rho_{inv}(x') \, dx' \, dx + \int \int_0^L \rho_d(x') \, dx'}{\varepsilon_s}. \]  \hspace{1cm} (3)
charge can be expressed as 

\[ Q_{inv} = \frac{\rho_{inv}(x)dx}{\varepsilon_s} \]

Tox oxide thickness and \( 3 \) is the ratio of \( \varepsilon_s/\varepsilon_{ox} \). The effective dc capacitance of the oxide is defined as

\[ C_{ox, eff} = \frac{X_{dc}}{X_{dc} + \frac{1}{3}X_{dc, elec}} \]

Equations (1), (2) and (3) can be rewritten as

\[ V_g - V_T = \frac{X_{dc}Q_{inv}}{\varepsilon_s} + \frac{Q_{inv}}{C_{ox, eff}} = \frac{Q_{inv}}{C_{ox, eff}} \]

where the effective dc capacitance of the oxide is defined as \( C_{ox, eff} = \varepsilon_{ox}/(T_{ox} + X_{dc}/3) \) where \( T_{ox} \) is the physical gate oxide thickness and \( 3 \) is the ratio of \( \varepsilon_s/\varepsilon_{ox} \). The inversion charge can be expressed as \( Q_{inv} = C_{ox, eff}(V_g - V_T) \). \( T_{ox} + X_{dc}/3 \) is the dc electrical oxide thickness. Using this new definition of dc electrical thickness, we can predict the inversion charge by finding a model for \( X_{dc} \).

There are three reasons why using the effective gate oxide thickness extracted from \( C-V \) measurements to predict current gives results that are not accurate. First, the carrier distributions of electrons and holes in the inversion layer are different from each other. As shown in figure 1, the hole wavefunction extends away from the interface further than does the electron wavefunction due to different effective masses. From an analytical approximation, we know that \( X_{dc} \sim m_{eff}^{1/3} \) [3]. Therefore the ratio between the dc centroid of a hole and that of an electron is expected to be about 1.3. Second, figure 2 shows that the ac charge centroid measured from accumulation \( C-V \) measurements is not equal to the dc charge centroid. From our simulation results, the ac charge centroid is 5–7 Å less than the dc centroid at strong inversion. Third, the carrier distributions in inversion and accumulation are also slightly different from each other according to our simulation results. As summarized in figure 2, the electrical oxide thickness characterized by the accumulation \( C-V \) measurements is not the same as the dc effective thickness for the current model. In the NMOS p-type substrate case, the hole accumulation charge ac centroid reflected in the \( C-V \) measurements is close to the dc thickness of the inversion electron charge. However, in the case of the PMOS device with a \( p^+ \) polysilicon gate, it is expected that the electron accumulation-layer ac thickness is much smaller than the hole inversion-layer dc thickness. This difference can be as large as 15 Å in the silicon thickness (see figure 2), which corresponds to 5 Å in the equivalent oxide thickness. Figure 3 shows that the doping concentration dependences can be made the same by plotting \( X_{dc} \) calculated from the self-consistent solution against \( (V_g + 3V_T) \). The dc centroid of the hole is about 1.2 times larger than that of the electron, which is similar to what is expected from the theoretical approximation \( X_{dc} \sim m_{eff}^{1/3} \). A plot including 12 curves of \( X_{dc} \) versus \( (V_g + 3V_T)/T_{ox} \) for three different doping densities and four different oxide thicknesses is shown in figure 4. One universal model fits all 12 curves. The semi-empirical model for \( X_{dc} \) is

\[ X_{dc}(NMOS) = \frac{\beta}{\alpha + [(V_g + 3V_T)/2 \times 10^6T_{ox}]^{0.7}} \]

where \( \alpha = 1 \) (MV cm\(^{-1}\))\(^{0.7} \) and \( \beta = 5 \times 10^{-7} \) cm
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Table 1. Descriptions and units of the physical symbols used in the analysis.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>$T_{ox}$</td>
<td>Gate oxide thickness</td>
<td>cm</td>
</tr>
<tr>
<td>$V_g$</td>
<td>Gate voltage</td>
<td>V</td>
</tr>
<tr>
<td>$V_t$</td>
<td>Threshold voltage</td>
<td>V</td>
</tr>
<tr>
<td>$X_{dc}$</td>
<td>DC inversion charge centroid</td>
<td>cm</td>
</tr>
<tr>
<td>$Q_b$</td>
<td>Depletion charge in the substrate</td>
<td>C cm$^{-2}$</td>
</tr>
<tr>
<td>$Q_{inv}$</td>
<td>Inversion charge in the substrate</td>
<td>C cm$^{-2}$</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Surface potential at the substrate/oxide interface</td>
<td>V</td>
</tr>
<tr>
<td>$\phi_B$</td>
<td>Potential difference between the Fermi level of the substrate and the intrinsic Fermi level</td>
<td>V</td>
</tr>
<tr>
<td>$\rho_{inv}$</td>
<td>Inversion charge density in the substrate</td>
<td>C cm$^{-3}$</td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>Depletion charge density in the substrate</td>
<td>C cm$^{-3}$</td>
</tr>
<tr>
<td>$C_{ox}$</td>
<td>Gate oxide capacitance</td>
<td>F cm$^{-2}$</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Silicon dioxide permittivity</td>
<td>F cm$^{-1}$</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Silicon permittivity</td>
<td>F cm$^{-1}$</td>
</tr>
<tr>
<td>$C_{ox,eff}$</td>
<td>Effective gate oxide thickness</td>
<td>F cm$^{-2}$</td>
</tr>
<tr>
<td>$m_{eff}$</td>
<td>Effective mass</td>
<td>g</td>
</tr>
</tbody>
</table>

Figure 4. The complete set of $X_{dc}$ fitting curves for 12 different combinations of substrate doping and $T_{ox}$. This plot shows how curves with different $T_{ox}$ reduce to give one universal curve when $(V_g + 3V_t)/T_{ox}$ is used as the x-axis variable.

$(MV cm^{-1})^{0.7}$ are fitting parameters

$$X_{dc} = 1.2X_{dc} - PMOS \cdot \text{only}$$

This $X_{dc}$ model can be applied to the estimation of the inversion charge density in the channel. Figure 5 shows that using the $X_{dc}$ model, the predicted $Q_{inv}$ agrees well with the $Q_{inv}$ data from integrating the $C_{gc}$ data with respect to $V_g$ [15].

A universal expression for the dc charge centroid in terms of $T_{ox}$, $V_t$ and $V_g$ is found for the quantization effect on the inversion layer of both n- and p-MOSFETs. This model is suitable for MOSFET device engineering and circuit simulation models. The inversion layer adds the equivalent of 6 or 7 Å to the electrical oxide thickness which determines the direct current in strong inversion for MOSFETs. The equivalent thickness for p-MOSFETs is about 1.2 times larger than that of n-MOSFETs.

Figure 5. Inversion-charge prediction using the developed dc centroid model. By including the dc centroid in calculating the inversion charge, $Q_{inv}$ can be predicted much more accurately.

References

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