PROBLEM B

TALES FROM DECRYPTION

Alice has designed a secret encryption method she wants Bob to use when he sends ASCII messages to her. The method is based on arithmetic in the integers mod 17.

1. Alice tells Bob to partition his ASCII message (8 bit bytes) into 4-bit nibbles \( m_1, m_2, m_3, \ldots \).
2. Alice asks Bob to add 1 to each nibble to get integers \( n_1, n_2, n_3, \ldots \) (so each is from the set \( \{1, 2, \ldots, 16\} \)).
3. Alice asks Bob to group these into threes, such as \( v = (n_1, n_2, n_3) = (14, 5, 7) \), augmented in the last triple by zeros, if necessary, so that each triple is complete.
4. Alice has secretly provided Bob with a sequence of 9 numbers, \( d_1, d_2, \ldots, d_9 \), each from the set \( \{0, 1, \ldots, 16\} \).
5. Alice asks Bob to group the nine numbers into a 3 by 3 matrix thus:

\[
A = \begin{pmatrix}
d_1 & d_2 & d_3 \\
d_4 & d_5 & d_6 \\
d_7 & d_8 & d_9
\end{pmatrix}
\]

6. Bob keeps \( A \) secret and follows Alice's instructions to use matrix multiplication to compute, for each triple \( v, \quad Av = w = (c_1, c_2, c_3) \) reducing mod 17 each entry of \( w \) so that these are in the range \( 0, 1, \ldots, 16 \). For example, for the matrix \( A \):

\[
\begin{pmatrix}
1 & 2 & 5 \\
3 & 4 & 1 \\
1 & 2 & 1
\end{pmatrix}
\]

and the triple \( v \) in step 3 the encrypted triple would be \( (8, 1, 14) \).

7. Alice knows the matrix \( A \) and knows how to decrypt a sequence of triples to get Bob's original ASCII message. (This is not a public key method, since \( A \), supplied by Alice, must be kept secret by Bob.)

Your job is to program this decryption, given the matrix \( A \) and the sequence of triples from Bob. (Alice's choice of \( A \) is guaranteed to enable a unique solution to the decryption process.) Bob's message starts as printable ASCII characters or spaces (this is guaranteed). You are required to recover and print Bob's ASCII message.

Input:
Input to your program will be a series of data sets. The first line of input will be an integer, denoting the total number of data sets to process. Following this line will be the data sets. Each data set contains the matrix "A" \( (d_1, d_2, \ldots, d_9) \), an integer, \( n \), (the number of triples in Bob's message), and Bob's set of triples. Each data set is constructed thusly:

\[
d_1 \quad d_2 \quad d_3 \\
d_4 \quad d_5 \quad d_6 \\
d_7 \quad d_8 \quad d_9 \\
n \\
t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \quad ?
\]

You are guaranteed at least 1 triple, and all triples in the data set will be on a single line.

Input will be from the text file B.in
Output:
Bob's original ASCII message, one line per data set. (The original message will be less than 50 characters.)

Sample I/O:

\[
\begin{array}{ccc}
\text{Input:} & \text{Output:} \\
2 & ME? \\
1 & ME?M \\
2 & \\
7 & \\
8 & \\
4 & \\
9 & \\
16 & \\
13 & \\
2 & \\
3 & \\
4 & \\
1 & \\
2 & \\
1 & \\
3 & \\
7 & \\
8 & \\
4 & \\
9 & \\
16 & \\
16 & \\
3 & \\
16 & \\
\end{array}
\]

Note that in this example, matrix "A" is the same for each data set, but this will not necessarily be the case!