Robustness Of Cooperative Communication Schemes To Channel Models

Vasuki Narasimha Swamy*, Gireeja Ranade†, Anant Sahai*
vasuki@eecs.berkeley.edu, giranade@microsoft.com, sahai@eecs.berkeley.edu
*University of California, Berkeley, CA, USA
†Microsoft Research, Redmond, WA, USA

***Abstract*—Cooperative communication to extract multi-user diversity and network coding are two ideas for improving wireless protocols. These ideas can be exploited to design protocols for low-latency high-reliability communication for control. Given the high-performance constraints for this communication, it is critical, to understand how sensitive such protocols are to modeling assumptions. We examine the impact of channel reciprocity, quasi-static fading, and the spatial independence of channel fades in this paper.**

This paper uses simple models to explore the performance sensitivity to assumptions. It turns out that wireless network-coding is moderately sensitive to channel reciprocity and non-reciprocity costs about 2dB SNR. The loss of the quasi-static fading assumption has a similar cost for the network coding based protocol but has a negligible effect on the protocol that doesn't use network coding. The real sensitivity of cooperative communication protocols is to the spatial independence assumptions. Capping the amount of independence to a small number degrades performance but perhaps more surprisingly, a simple Gilbert-Elliott-inspired model shows that having a random amount of independence can also severely impact performance.

***Index Terms*—Cooperative communication, low-latency, high-reliability wireless, network coding, spatial diversity**

I. INTRODUCTION

How will real-world cooperative-communication systems behave when idealized assumptions break down? This paper studies the impact of common channel modeling assumptions on the performance of two cooperative communication schemes – one using network coding (the XOR-protocol) and the one without network coding (the CoW-protocol). These are described in further detail in section III.

These protocols combine descendants of Laneman et al.'s approaches to multiuser diversity [1] with wireless network coding [2], [3]. Multiuser diversity protocols use cooperation among distributed nodes to provide multi-antenna-style diversity without the need for physical arrays. Wireless network coding improves latency and throughput by effectively reducing the number of packet transmissions. The relays transmit linear combinations of messages and receivers use prior knowledge of one of the messages to decode the other one.

Such techniques are traditionally studied under a set of standard modeling assumptions: (a) channel reciprocity (b) quasi-static fading (c) spatially independent fading. To explore the impact of each of these assumptions, we choose a setting where cooperative-communication seems to offer some significant advantages: highly-reliable low-latency communication. In particular, we study protocols that we proposed in earlier work [4], [5] that target performance specifications inspired by wireless industrial control [6], [7] (10^{-9} probability of outage, 2 ms cycle time, and 0.25 bits/sec/Hz aggregate spectral efficiency). Diversity asymptotics might be practically important for high-reliability, hence this setting is of interest. Within this setting, this paper challenges the three main modeling assumptions to see which is truly important.

(a) No channel reciprocity: what happens when the fading coefficient of a link in one direction is independent of the fading coefficient of the link in the other direction? We find this diminishes the number of opportunities for useful network coding at the relays, costing about 2dB in SNR for the XOR-protocol. For the CoW-protocol, the effect of non-reciprocal channels is minimal.

(b) Non-quasi-static channels: we consider a setup in which channel fading coefficients can change anytime during a single cycle of the protocol but only one change is permitted per channel. For the CoW-protocol this has a negligible effect. For the XOR-protocol, the effect of this is like losing reciprocity.

(c) Non-i.i.d. channels: what if k potential relays do not automatically give us k independently faded links? Since this multiuser diversity is critical to achieving a probability of outage of 10^{-9}, limiting independence turns out to be very significant and so we consider two different models. First, we consider a simple "diversity-cap" model where we limit the number of independent paths. This shows that capping diversity at 5 independent paths has a significant effect and raises the required SNR by close to 10dB. Second, we try a model for spatial fading that is loosely inspired by Gilbert-Elliott models for temporal fading: in this each "additional relay" has a probability q to bring a fresh independently faded channel but a probability $1-q$ of just giving us the same fade again. Here, we see that unless q is large, the SNR penalty could be quite large (in the tens of dB) for moderate-sized networks. This final observation tells us that the details of how multi-path fading is spatially independent really do matter if cooperative-communication protocols are going to be practical, but the other standard assumptions are probably safe to make.

In this paper we focus on the main results and discuss the implications. The details of the protocol analysis is presented in an appendix to this paper in [8].

II. PRIOR WORK

Laneman et al. showed that cooperation amongst distributed antennas can provide full diversity without the need for physical arrays [1]. Multi-user cooperation increases capacity even with noisy inter-user channels and that it is robust to channel variations [9]. This field saw an explosion in research, see [10]–[14] for just a smattering of the hundreds of papers in this field. Multiple-antennas are the main reason for the underlying boost in performance [15].

The work of Ahlswede et al. [3] made it apparent that network coding can improve throughput in wireless protocols (see [2] for a tutorial). Polynomial time algorithms to make network coding suitable for real-time applications have been studied in [16], [17]. Katti et al. implemented a coding layer in between the IP and MAC layers to improve throughput showing network coding's practicality [18].

Many of the protocols studied above were developed under idealized channel assumptions and the effect of some of these assumptions have also been studied. For instance, the physical limitations on radios in cooperative communication settings and non-ergodic and delay-constrained channels were explored in [1]. Non-reciprocity of channels was explored in [19] where it was found that the rate-regions of two-way relay networks shrinks when the channels are non-reciprocal. However, non-reciprocity of inter-user channels was found to have no effect on the diversity order of cooperative schemes [20]. The effect of channel estimation errors and non-quasi-static channels on behavior of space-time codes was studied in [21] where they showed that the diversity order achieved by space-time codes is unchanged under a variety of mobility conditions.

The physical limits on MIMO gains, especially multiplexing gains, have been explored with sophisticated modeling. These models challenge the very optimistic linear scaling with the number of antennas that were in very early work [22]–[24]. The nature of the propagation environment really matters. The limits to earlier optimistic scaling analyses in the finite-SNR regime [25] as well as in the context of practical systems [26] have also begun to be explored. Recent works have studied the effect of transceiver limitations where they show that MIMP channels with transciever limitations have a finite upper capacity limit, for any channel distribution and SNR [27].

Measurements have confirmed theoretical scepticism of aggressive independence models — multipaths indeed seem to be limited [28]. Molisch et al. [29] experimentally studied MIMO systems in different physical settings and extracted a radio channel parameterization that can determine the CDF of the capacity. They also provide a great survey of how different fading assumptions have been studied in the literature. Performance of OFDM under deviations from the standard flat fading channel model has been studied: delay spread channels may provide advantages over flat fading channels both in terms of outage capacity and ergodic capacity [30]. Failure of assumptions of independence and scattering for networks employing cooperative communication has also been studied [31]–[34]. These represent just a small selection of the extensive work on robustness. However, these sensitivity results are in the asymptotic regime of diversity order or high SNR, not finite error probability and moderate SNRs of practical interest.

III. PROTOCOLS

We consider a network with a central controller (C) that wishes to send and receive separate messages to and from each node in a set of N nodes, denoted by the set $\mathcal{S}$ (see Fig. 1). Distinct messages (for our plots here each of size $m = 160$bits, i.e. 20B) flow in a star topology from the central controller to individual nodes, and in the reverse direction from the nodes to the controller within a "cycle" of length T (here $T = 2$ms). This cycle of communication must be achieved with a very small outage probability (on order of 10^{-9}). The available bandwidth is 20MHz for our plots. We assume that all channels are Rayleigh faded.

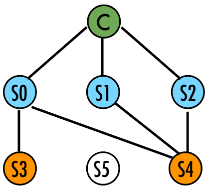

Fig. 1: Topology schematic for the protocols.

A. Occupy CoW

The Occupy CoW protocol (which we abbreviate as the CoW-protocol) uses multi-user diversity to overcome bad fading events. The basic idea is to use a flooding strategy where the controller broadcasts a packet that includes messages for all nodes. As this is a broadcast message, the intended set of recipients are all nodes. Nodes with good channels act as relays for other nodes. This is done in a carefully scheduled, phased manner with dedicated time slots for each node to talk. Details of the protocol are discussed in [4] — here we consider a two-hop scheme with four phases: two rounds of downlink-uplink each.

Denote the set of nodes with direct controller links by $\mathcal{A}$. Other nodes may connect to the controller through these nodes in a two-hop fashion. At its essence, our simple analysis fundamentally examines how the size of this set $\mathcal{A}$ (which is denoted by A) changes under different channel conditions. Uplink transmissions are similar. Here, the transmission rate in each phase is $R_O = \frac{mN}{T/4}$ and hence the probability of a single link outage due to fading (assuming Rayleigh fading) is $p_O = 1 - \exp(-\frac{2^{R_O}-1}{SNR})$. Then A follows a Binomial distribution.

Under the ideal conditions that the channels do not change during a cycle and they are reciprocal, the probability of cycle failure is the probability that at least one of the nodes in the set $\mathcal{S} \backslash \mathcal{A}$ does not connect to $\mathcal{A}$. Then we can write:

$$P(\text{fail}|A = a) = 1 - (1 - p_O^a)^{N-a}$$

Thus, the probability of cycle failure is given by:

$$P(\text{fail}) = \sum_{a=0}^{N-1} P(A = a) \cdot P(\text{fail}|A = a) \tag{1}$$

This gives the fundamental flavor of the analyses in this work.

B. *XOR-CoW*

The XOR-CoW protocol [5] (which we abbreviate as the XOR-protocol) follows the same key ideas as the CoW-protocol described above, except that it also uses network coding inspired by works such as [14], [18]. The protocol increases efficiency by dividing the cycle length into only three phases. The first downlink phase and the second uplink phase follow the same broadcast ideas as the previous protocol. In the third "XOR" phase, strong nodes that have both uplink and downlink messages for other nodes broadcast the XOR of the two messages, thus simultaneously serving as an uplink relay for the controller and as a downlink relay for other nodes.

Here, $R_X = \frac{mN}{T/3}$, and this immediately gives the probability of link failure $p_X = 1 - \exp(-\frac{2^{R_X}-1}{SNR})$ just as in the previous case. This reduction of rate is one of the key differences between the CoW and the XOR protocols, and is one of the factors that leads to the significant gains using this network coding based scheme [5].

Let $\mathcal{A}_{du}$ be the set of nodes which succeeded in the downlink and uplink phases to the controller. It is the size of this set that now matters to the failure probability of the protocol, and the expression for system outage is essentially the same as Eq. (1) with p_O replaced by p_X.

IV. MAIN RESULTS

We analyze the effects of channel reciprocity, quasi-static fading and total spatial independence considering one effect at a time. The results presented here are based on the exact calculations in the appendix to this paper in [8].

A. *Non-reciprocal channels*

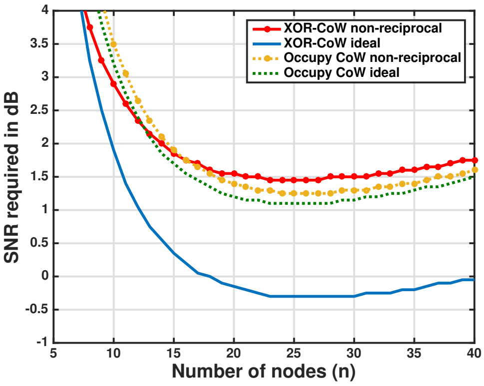

Fig. 2: Performance of the CoW and XOR protocols with non-reciprocal channels

Non-reciprocity of channel fades breaks a key assumption in the protocols — now, the nodes that received downlink packets need not be able to act as relays for uplink packets. We expect that non-reciprocity will slightly degrade the performance of both of the protocols — effectively we need two coins to turn up heads instead of just one. As we know from [5], the XOR-protocol saves us almost 1.5 dB (compare the solid blue line for XOR to the dotted green line for CoW in Fig. 2). The network coding aspects of the XOR-protocol help it outperform the CoW-protocol.

Without reciprocity, the nominal SNR required for 20 nodes increases by only approximately .2dB under the CoW-protocol and on the other hand for XOR-protocol it increases by more than 1.5dB at 20 nodes (see Fig. 2). This is because the probability of a node being able to help as a relay in the XOR phase is suddenly lower — a good channel on downlink does not guarantee a good channel for uplink (and vice versa). Effectively four good links per node need to be realized instead of just two, leading to the large performance degradation.

On the other hand since the uplink and downlink paths of success are completely independent in the CoW-protocol, it provides a sort of robustness against bad fading realizations in the case where uplink and downlink are independently realized. Around 16 nodes, the CoW-protocol starts to outperform the XOR-protocol when the channels are non-reciprocal but at higher aggregate rates XOR-protocol outperforms CoW-protocol even when the channels are non-reciprocal.

B. *Non-quasi-static channels*

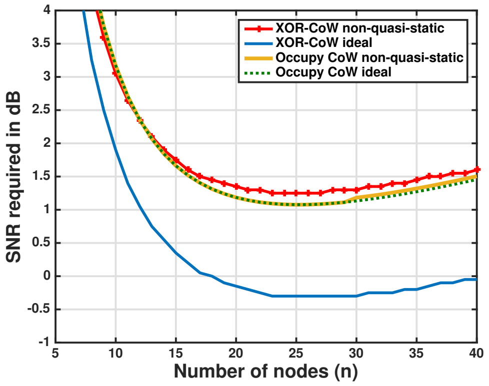

Fig. 3: Performance of the CoW and XOR protocols with non-quasi-static channels

Given that the round-trip latency for one cycle of the CoW or XOR protocols is 2ms and we are considering coherence times in the indoor environments around 30ms, it is unlikely that channels will change significantly in the middle of a protocol cycle. What is the worst that would happen if they do? Here, we analyzed protocol performance when channels change at phase boundaries of the protocol: for instance, it might change between the first downlink and uplink phases for the CoW-protocol. However, we assumed that the channel was static while a transmission was actually happening and once realized, channels were reciprocal.

Just as with non-reciprocal channels, non-quasi-static channels introduce more randomness into the system. In the two phase CoW-protocol, this extra randomness might actually give some nodes two chances to directly establish a link to the controller (before and after a mid-cycle channel change). This means that downlink-only performance of a protocol can improve due to the extra diversity introduced by a channel change. The same is true for uplink connections.

However, the combined performance of uplink & downlink take a small hit with non-quasi-static channels. Why? In the quasi-static case, a path that worked for two-hop downlink to a node was guaranteed to work for two-hop uplink for the same node. However, this is no longer true in the presence of changing channel fades. Each node must find two independent paths to the controller — one for uplink & one for downlink. This effect is similar to the effect of non-reciprocal channels, except its impact is less pronounced because of the advantage of possible extra "channels" to the controller. Fig. 3 shows the yellow line hugging the dotted green line.

The XOR-protocol faces a hit that is similar to the hit it faces in the non-reciprocity case (Fig. 3). Again, the primary performance hit is due to the potential decoupling between uplink and downlink — this can lead to a smaller set of nodes that have both uplink and downlink information for any given node — and thus a smaller set of nodes that can help anyone who does not have a direct link to the controller. A careful analysis shows that certain new paths can show up when channels change which help out compared to the case where channels are non-reciprocal.

C. Dependence in fade realizations

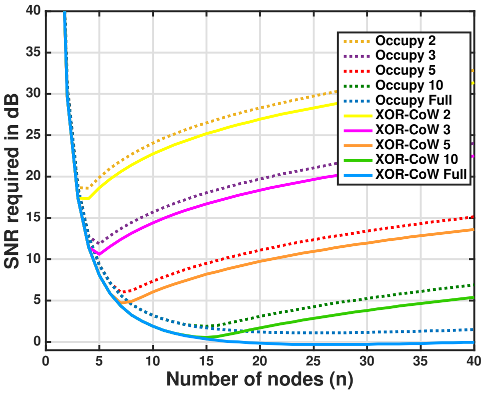

Fig. 4: Performance of the CoW and XOR protocols with a cap on the maximum diversity that can be harvested by any node. The legend indicates the protocol being used and the imposed cap on diversity.

The first model for studying correlations and dependence between the fading realization in the system caps the total diversity that any node can harvest. Fig. 4 shows the performance of both protocols when the maximum diversity of each node is capped to be $2, 3, 5$ or 10 respectively. The impact is significant. Even when the number of diverse paths maxes out at 10, we see that the minimum required SNR starts to rise when the network has more than 15 nodes. The additional nodes in the system stop being useful as "new" relays at this point. With a diversity cap of 5 independent paths per node, the nominal SNR required to hit the target of $p_e = 10^{-9}$ is always over 5dB. And it grows with the number of nodes in the network as the aggregate data rate rises but there is no offsetting increase in diversity to compensate.

The second approach uses a more nuanced probability model for diversity. Every new channel has a probability q of coming from an independent fading distribution. However, with probability $1-q$ the channel is identical to a channel that has already been realized. Fig. 5 demonstrates how this affects both our protocols. First it is worth noting that the modeling choice makes a significant difference to the qualitative nature of the results. The first notable point is that the SNR curves in Fig. 5 decay as the number of nodes increases. This contrasts with Fig. 4 where after a threshold, the required SNR increased as the number of nodes increased. This is because the first model capped the total diversity independent of the number of paths available.

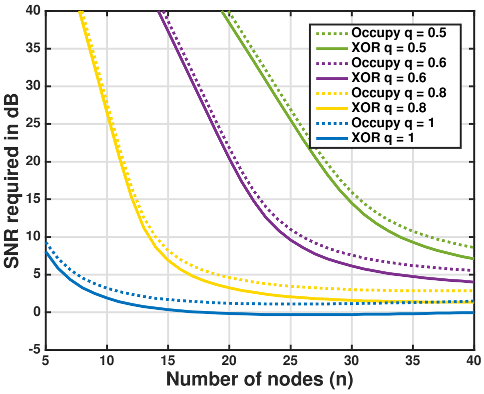

Fig. 5: Performance of CoW and XOR protocols with the Gilbert-Elliott inspired model for independence of channel fades. q represents the probability of an independent fade on a channel.

Second, a low probability of independence has a severe impact on both CoW-protocol and XOR-protocol. With $q = .5$ at 40 nodes the XOR-protocol requires SNR 7dB to succeed and the CoW-protocol required slightly more than 8dB. The more troubling behavior is when the network is smaller, around 20 nodes. There, SNR penalty is about 40dB. Why is this happening? This is happening because the desired probability of outage is very small, and so the dominant failure event can just become not getting enough diversity rather than getting an unlucky sequence of fades. After all, $\left(\frac{1}{2}\right)^{20} \approx 10^{-6}$ by itself — and so it is possible to draw 20 heads in a row and get a 10^{-3} fading event on the one independent fading realization and be in outage too often. And there are 20 different nodes this can happen to. So, we need about $40dB$ of SNR to punch through this fade (see Fig. 5).

V. CONCLUSIONS AND FUTURE WORK

In this paper, we find that while the penalty to be paid for non-reciprocal and time-varying channels is small (about 2db), the effect of possible non-independence of fading coefficients of channels is substantial ($>$ 10dB in some cases).

At first glance, this seems like a pessimistic preliminary result for the practicality of cooperation-based wireless protocols and further investigation is required. However, these effects can be minimized through various ways. There is an intuitive reason to believe that diversity behaves very differently than multiplexing (and rank) when it comes to real-world multiple antennas. Consider the extreme of empty-space with line-of-sight. In this case, it is true that the geometry of space will force the rank of a MIMO channel matrix to be small. However, the chance of a bad fade is zero. So the diversity will go to infinity, not to 1. If physically there are only a few scatterers, then once again, the fact that analytic functions cannot be zero in positive-measure sets tells us that the actual spatial regions where fades are bad must indeed be thin and not everywhere. The relevant source of independence here is just the movement of nodes not deliberately following channel nulls. This was studied in [35] where they actually showed that spatial correlation exhibits a spatially oscillatory structure which supports our claim that unless nodes deliberately move in a fashion which ensures that they are mostly in nulls or heavily correlated environments (which would lead to the $q = 0.6$ curves in Fig. 5), we should be able to get sufficiently high amount of independent fades (corresponding to the $q = 0.8$ & $q = 1$ curves in Fig. 5).

Recent developments in mmWave communications has shown the regions of bad fade are 'much' narrower and the probability of two of more nodes being in the same small region is presumably very small. Also because the wavelengths under consideration are much smaller than traditional WiFi-like bands (mm vs cm), channel fades vary at a much higher rate spatially [36]. Plus with an antenna array one has control over where the multipath is going which should allow us to avoid bad fades in a better way or perhaps purposely decorrelate everyone. This suggests that a large amount of spatial diversity is going to be reliably achieved, even if fades aren't strictly independent. A good proxy for independence needs to be found so such a result can be made precise.

ACKNOWLEDGEMENTS

This work grew out of an ongoing project with Borivoje Nikolić, Venkat Anantharam, and Paul Rigge who we thank for many discussions leading up to this paper. We thank the NSF for grants CNS-0932410, CNS-1321155, and ECCS-1343398.

REFERENCES

[1] J. N. Laneman *et al.*, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, 2004.

[2] S. Deb *et al.*, "Network coding for wireless applications: A brief tutorial." IWWAN, 2005.

[3] R. Ahlswede *et al.*, "Network information flow," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1204–1216, 2000.

[4] V. Narasimha Swamy *et al.*, "Cooperative communication for high-reliability low-latency wireless control," in *2015 IEEE International Conference on Communications*, June 2015, pp. 4380–4386.

[5] ——, "Network coding for high-reliability low-latency wireless control," in *2016 IEEE Wireless Communications and Networking Conference Workshop on The Tactile Internet: Enabling Technologies and Applications (IEEEWCNC2016-TACNET)*, Doha, Qatar, Apr. 2016.

[6] G. Fettweis, "The Tactile Internet: Applications and Challenges," *Vehicular Technology Magazine, IEEE*, vol. 9, no. 1, pp. 64–70, March 2014.

[7] M. Weiner *et al.*, "Design of a low-latency, high-reliability wireless communication system for control applications," in *IEEE International Conference on Communications, Sydney, Australia*, 2014, pp. 3829–3835.

[8] V. Narasimha Swamy *et al.*, "Robustness of cooperative communication schemes to channel models." [Online]. Available: https://www.eecs.berkeley.edu/~vasuki/papers/ISIT2016_fullversion.pdf

[9] A. Sendonaris *et al.*, "User cooperation diversity. Part I. System description," *IEEE Transactions on Communications*, vol. 51, no. 11, pp. 1927–1938, Nov 2003.

[10] A. Nosratinia *et al.*, "Cooperative communication in wireless networks," *IEEE Communications Magazine*, vol. 42, no. 10, pp. 74–80, 2004.

[11] G. Kramer *et al.*, "Cooperative strategies and capacity theorems for relay networks," *IEEE Transactions on Information Theory*, vol. 51, no. 9, pp. 3037–3063, 2005.

[12] A. Bletsas *et al.*, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Transactions on Wireless Communications*, vol. 6, no. 9, pp. 3450–3460, 2007.

[13] B. Sirkeci-Mergen *et al.*, "Randomized space-time coding for distributed cooperative communication," *IEEE Transactions on Signal Processing*, vol. 55, no. 10, pp. 5003–5017, 2007.

[14] S. Bagheri *et al.*, "Randomized decode-and-forward strategies for two-way relay networks," *IEEE Transactions on Wireless Communications*, vol. 10, no. 12, pp. 4214–4225, 2011.

[15] A. Goldsmith *et al.*, "Capacity limits of MIMO channels," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 5, pp. 684–702, 2003.

[16] R. Koetter *et al.*, "An algebraic approach to network coding," *IEEE/ACM Transactions on Networking (TON)*, vol. 11, no. 5, pp. 782–795, 2003.

[17] A. Douik *et al.*, "Instantly decodable network coding for real-time device-to-device communications," *EURASIP Journal on Advances in Signal Processing*, vol. 2016, no. 1, pp. 1–14, 2016.

[18] S. Katti *et al.*, "XORs in the air: practical wireless network coding," *IEEE/ACM Transactions on Networking*, vol. 16, no. 3, pp. 497–510, 2008.

[19] M. Zeng *et al.*, "On design of collaborative beamforming for two-way relay networks," *IEEE Transactions on Signal Processing*, vol. 59, no. 5, pp. 2284–2295, 2011.

[20] M. Xiao *et al.*, "Multiple-user cooperative communications based on linear network coding," *IEEE Transactions on Communications*, vol. 58, no. 12, pp. 3345–3351, 2010.

[21] V. Tarokh *et al.*, "Space-time codes for high data rate wireless communication: performance criteria in the presence of channel estimation errors, mobility, and multiple paths," *IEEE Transactions on Communications*, vol. 47, no. 2, pp. 199–207, 1999.

[22] A. Poon *et al.*, "Degrees of freedom in multiple-antenna channels: a signal space approach," *IEEE Transactions on Information Theory*, vol. 51, no. 2, pp. 523–536, Feb 2005.

[23] A. S. Poon *et al.*, "Does superdirectivity increase the degrees of freedom in wireless channels?" in *2015 IEEE International Symposium on Information Theory*. IEEE, 2015, pp. 1232–1236.

[24] W. Jeon *et al.*, "Improving degrees of freedom of wireless channels using superdirectivity," in *2015 IEEE International Symposium on Information Theory*. IEEE, 2015, pp. 2999–3003.

[25] R. Narasimhan, "Finite-SNR Diversity-Multiplexing Tradeoff for Correlated Rayleigh and Rician MIMO Channels," *IEEE Transactions on Information Theory*, vol. 52, no. 9, pp. 3965–3979, Sept 2006.

[26] A. Lozano *et al.*, "Fundamental limits of cooperation," *IEEE Transactions on Information Theory*, vol. 59, no. 9, pp. 5213–5226, Sept 2013.

[27] E. Bjornson *et al.*, "Capacity limits and multiplexing gains of MIMO channels with transceiver impairments," *IEEE Communications Letters*, vol. 17, no. 1, pp. 91–94, 2013.

[28] A. A. Saleh *et al.*, "A statistical model for indoor multipath propagation," *IEEE Journal on Selected Areas in Communications*, vol. 5, no. 2, pp. 128–137, 1987.

[29] A. F. Molisch *et al.*, "Capacity of MIMO systems based on measured wireless channels," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 3, pp. 561–569, 2002.

[30] H. Bölcskei *et al.*, "On the capacity of OFDM-based spatial multiplexing systems," *IEEE Transactions on Communications*, vol. 50, no. 2, pp. 225–234, 2002.

[31] A. G. Burr, "Channel capacity evaluation of multi-element antenna systems using a spatial channel model," in *Millennium Conference on Antennas and Propagation (AP2000), Davos*, 2000.

[32] P. F. Driessen *et al.*, "On the capacity formula for multiple input-multiple output wireless channels: A geometric interpretation," *IEEE Transactions on Communications*, vol. 47, no. 2, pp. 173–176, 1999.

[33] D.-S. Shiu *et al.*, "Fading correlation and its effect on the capacity of multielement antenna systems," *IEEE Transactions on Communications*, vol. 48, no. 3, pp. 502–513, 2000.

[34] W. C. Lee, "Effects on correlation between two mobile radio base-station antennas," *IEEE Transactions on Communications*, vol. 21, no. 11, pp. 1214–1224, 1973.

[35] M. Denis *et al.*, "Characterizing spatial correlation in indoor channels," in *IEEE Wireless Communications and Networking Conference*, vol. 3, 2004, pp. 1850–1855.

[36] D. Tse *et al.*, *Fundamentals of Wireless Communication*. New York, NY, USA: Cambridge University Press, 2005.