An Initial Study of Low Rank Guided Motion Decomposition for Dynamic Magnetic Resonance Image Reconstruction
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**Introduction:** Low rank reconstruction methods\textsuperscript{1,2,3,4,5} are effective in capturing MR dynamics, including contrast enhancement, cardiac perfusion and parameter mapping. Yet large motion still poses problems to these methods, as these variations cannot be compactly represented with low rank matrices. A variety of methods\textsuperscript{5,7} were proposed to incorporate motion models to low rank methods, yet often involve intricate motion models and non-convex algorithms. In this work, we present initial results using a simple and alternative approach: we first augment our solution space to include the set of all translational motion states, and then reduce the problem to the usual low rank reconstruction problem in the augmented space. Unlike previous works, our method is convex and does not require explicit motion models.

**Theory:** Motion in dynamic MR can be approximated by local shifts. Yet solving for these shifts directly is non-convex, which translates to inconsistent performance in practice. Instead, we propose enumerating all possible spatial shifts, and consider the motion states \( \{X_s\} \) parameterized by each spatial shift \( s \). Under this model, a dynamic sequence with motion can be represented as a sum of these motion states. (See Figure 2 for visualization)

Our key observation is that the concatenation of these motion states \( \{X_1, \ldots, X_s\} \) has the same rank and column spaces as the motion-corrected dynamic sequence. Hence, the augmented matrix is as low rank as the motion-free image sequence. To recover the motion states, we simply solve the low rank reconstruction on the augmented matrix, shown in Fig. 1. Moreover, different scales of motion can potentially be captured by low rank matrices with different block sizes.

**Results:** Figure 2 shows the result of the globally low rank guided motion decomposition on a synthetically shifted brain images. The shifts considered were pre-determined as \([0, 5, 10]\) pixels vertically. The decomposition almost perfectly recovered each motion state, thereby compactly representing the augmented matrix as a rank-1 matrix. Figure 3 shows a slice of the preliminary reconstruction result on a DCE dataset with 18 contrast phases, matrix size 192x156x100, 1x1.4x2 mm\textsuperscript{3} resolution, and ~8 s temporal resolution. The acquisition was performed on a 3T GE MR750 scanner with a 32-channel cardiac array using an RF-spoiled gradient-echo sequence. The proposed method was combined with ESPiRiT\textsuperscript{8} and implemented in BART.


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**Figure 1.** Optimization model for low rank motion decomposition.

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\frac{1}{2} \| Y - A \sum_s \text{Shift}_s(X_s) \|^2 + \lambda \| \{X_1, \ldots, X_s\} \|_{\text{nuc}}
\]

\( Y \): Acquired dynamic data \( A \): Acquisition matrix \( X_s \): Dynamic sequence with shift \( s \) \( \| \cdot \|_{\text{nuc}} \): (Local/Global/Multi-scale) nuclear norm

**Figure 2.** Global low rank motion decomposition on a synthetically shifted brain images.

**Figure 3.** Preliminary result of a locally low rank reconstruction using the proposed model.