

Analysis of a Web User Interface for Mathematics: TILU -- a Symbolic Integration Server

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I - What does TILU do?

- **A (simplified) interaction**
- **A brief defense of why it is interesting to look up integrals this way**

TILU Table of Integrals Look Up

Welcome to TILU, a web server that looks up integrals in a reference table.

If you have never used this page before, or if you need some review on the syntax for describing your integral, please be sure to check in the frame on the left. Once you have figured out how to ask your question, type or paste the integrand in the big box below:

Select language

Indefinite Integral

$x^2 * \sin[a x]$

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Lookup begun Jul 12, 1999 16:19:29, completed in 140 ms.
processor time, 171.0 ms. elapsed real time. Actual cpu time in
computing the answer: 0 ms.

The result of looking up this integral

$$\text{INTEGRATE}(X^2 \text{ SIN}(A X), X) =$$

$$\text{COS}(A X) A^{-3} (2 - A X^2) + 2 X \text{ SIN}(A X) A^{-2}$$

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Lookup begun Jul 12, 1999 16:24:40, completed in 120 ms. processor time, 139.0 ms. elapsed real time. Actual cpu time in computing the answer: 0 ms.

INTEGRATE($x^2 \operatorname{ERF}(x)$, x)

was not found.

We tried sending your integral request to Wolfram's Integrator and their answer was:

$(1/(3\sqrt{\pi}) + x^2/(3\sqrt{\pi}))/E^{x^2} + (x^3\operatorname{Erf}[x])/3$

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Why is a web lookup interesting?

- **Inferiority of algorithmically generated answers in some cases. Not just an integration issue but a simplification issue. Tables have neat answers when algorithms give poor or no answer; (of course algorithms sometimes do win...so the best approach is probably to combine).**
- **Once you get the integrand and limits into the system, we can further process them, including calling Mathematica, Maple, Macsyma, ... servers.**
- **Provides insight into storage and retrieval of mathematics generally on the web.**
 - **other reference material, textbooks.**
 - **An integral archive/knowledge base for all the net.**

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II- Math is Apparently Hard to Type

- *This is a summary report from June 1, 1999 --July 12, 1999, as we prepared these slides.*
- **519 user-connections to our site.**
 - About 75 of the inputs were so ill-formed the parser rejected them.
 - Of the remaining 445, we count only 300 distinct integrands
- **Of the 300 unique Mathematica expressions,**
 - 110 were illegal inputs, typically because of
 - the misuse of $\sin(x)$ for $\text{Sin}[x]$.
 - Operators missing: ax or $x\cosh x$ instead of $a*x$ or $x \text{Cosh}[x]$.
 - insertion of "dx" or "integrate."
 - Many more (156) we "legalized" by allowing $\sin[x]$ instead of $\text{Sin}[x]$, e instead of E etc.
- **Only 34 inputs were fully syntactically correct.**

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Speculation Regarding Users

Most don't know much about computer algebra or numerical mathematics programming languages

Most don't know about computer typesetting programs (TeX, Expressionist)

Of the dozens of people who meant $\int \sin x dx = -\cos x$

but got $\int \sin x dx = \sin * x^2/2$

Only one complained.

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III- Possibilities for Improving the User Interface

- **Very liberal “Mathese” grammar**
- **Off-the-shelf equation editor**
- **More Feedback to the typist**
 - Traditional “error detection”
 - Traditional “error correction”
- **Forced Choices**
 - The calculator model
 - The box-prompt model
 - typewriter input
 - hand-written input

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Mathese grammar

- **Not Context Free Grammar. Filled with ambiguity !**
- **Resolution based on semantics, heuristics**
 - $\cos x, \cos(x), \cos[x], \cos\{x\}$ $\cos x$ $x \cos x$
 - $a*x+b, a x+b, ax+b$
 - $d*x, d x, dx$
 - $1/2\pi i$??? $1/(2\pi i)$ or $.5*\pi i$?
- **Some of the same problems arise in parsing T_EX into math**

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Mathese grammar (continued)

– Texts and references are “informal” too, It’s not just casual hand-waving slobs!

- **G&R 3.884..** ... = $\cos a\sqrt{|b|} + \exp(-a\sqrt{|b|})$
 - Ps, this formula is wrong...
- **4.384.4** .. $\int \ln(\sin \pi x) \cos (2n + 1) \pi x \, dx$

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Math-oriented Off-the-Shelf Editors...

The idea here is to use a familiar editor or formalism to assemble the necessary pieces to define the math.

– **Cons (depending on choice):**

- **Not necessarily familiar or intuitive!**
- **Added expense**
- **Typesetting goals do not resolve ambiguous Math semantics (= mathese equivalent)**
- **Not standard or integrated**
- **Insufficient generality (overly constrained)**

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Existing software (continued)

– Pros (depending on choice)

- COTS support, platform independence, standard conformance (TeX, MathML..)
- Document-Centric view *may* be a **BIG WIN** if there is a CAS inside
 - Scientific Workplace (Maple, Mathematica)
 - Frame (Milo)
- Some are really pretty thoughtfully designed if you are willing to learn them!

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A list of a few prospects

MS Equation, Frame, Mathematica, TeX, Milo, LiveMath/MathView/Theorist/Expressionist, Mathtype, EzMath, Scientific Word, WebEq, Swift. Macintosh Graphing Calculator...

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Examples of error detection/correction

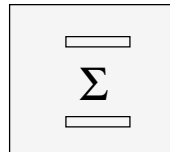
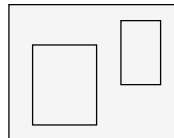
- **Expression has no “x” in it. $x \cosh x$ gets fixed to $x \cosh x$**
- **Expression has “dx” or “integrate” in it: remove.**
- **Improbable though correct Mathematica: $\sin(x)$ should be $\sin [x]$**
- **Insufficient closing brackets repaired.**
- **Misspellings corrected (uncommon).**
- **Diagnostic aids: Better rephrasing of the question.**

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Familiar System Paradigms

- **Menu (extensible) + pre/post selection**
- **Fortranish [Bonadio, Theorist]**
- **Copy/Paste of completed subexpressions**
- **Calculator (TI, HP, ?)**
- **Direct Manip. (Pacific Tech, Avitzur’s NuCalc)**
- **Mathematica (Soiffer, Mathscribe)**

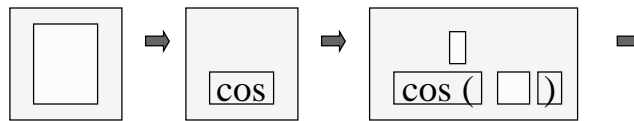


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Idiotproof/ Forced-Choice/ Structured Input for the Web

- More nearly free-form, yet forbids incorrect syntax; disambiguates murky forms (based on semantic associations)
- Menus/templates are used primarily for information.
- Input could be keyboard only, stylus only.
- Here's the general idea: insert in the box.

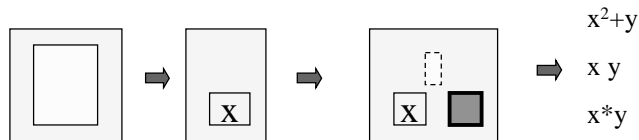


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Details: Typed Boxes Clarify Choices

- "Infix" Verbs (+, *, "space", -, /)
- Nouns or Noun Phrases (x, y, z, x+y*cos(z))
- Function Name (cos sin tan {f, g, ...}, derivative)
- Lists "a,b,c" ...



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Selection of previous expressions

➡ Select box by click or backtab, arrow
 ➡ Insert brackets in pairs only

- Insert/Delete brackets, Show/Hide boxes, Name?
- Appeal to menu if necessary for more structures: user-templates?

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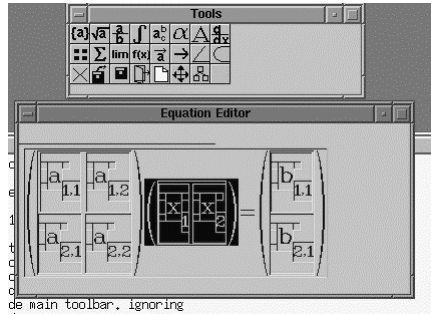
Following our earlier example

Tab, 2, +

- direct manipulation design intuition is tricky... how does the user know to hit <tab> or click with mouse (where)?
- time will tell if this can be made to work

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If you keep looking, you will find almost anything
... Swift (free Java application)



While we haven't gotten Swift to work, we have corresponded with its authors. It looks interesting; we'd like to have handwritten input as an option, too.

Biscotti, a Java applet

- Undergrad project (E. Heien, Gifford Cheung)
- Based on Macintosh Graphing Calc, but written in Java as an applet (cf. NuCalc from Pacific Tech.)
- Fortranish + "calculator keyboard"
- Additional features: "send to TILU" key
 - Uses Lisp syntax and TCP/IP socket
- Extensions in progress or contemplated
 - Can send "any" message, by agreement to server (subject to Java restrictions, unfortunately)
 - Could be used as base for other calculations, e.g. Potential front-end for CAS more generally
 - Simple environment (local values)
 - Other output forms (XML)
 - Other communication styles (Java inspired)

Future Prospects for Tilu

- **More ambitious web server processing**
 - larger lookup tables, pseudo-CAS, more tasks (ODEs?) Sums, Products, Transforms
 - Put the Mathese parser on-line
- **Hooking up to XML based front-ends (via a Lisp XML reader)**
- **Detailed design and implementation for MathSE, keyboard/ stylus-handwritten input**
- **Lisp mini-plugin front end replacing Java?**
- **Installation on CD or web site for Academic Press/ Gradshteyn & Rhyzik**

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