Assignment 2: More lisp practice

Due: Sept. 18, 2006

Reading Chapter 4, Norvig.

General guidelines: You should provide brief but suitable documentation, stating any assumptions or limitations you make on the input. (Naturally you must have some taste in not assuming away a significant part of the problem. Ask if you have any questions about what you can assume.) Normally you should include a test run or two to exercise the programs demonstrating they work at least sometimes.

1. Norvig, problems 3.10 and 3.12 on page 103. (Explain the answer for 3.12 on page 106).

2. Be prepared to discuss exercise 4.3 (page 148) in Norvig.

3. The point of this exercise is to get you to think carefully about recursion, data structures, and Lisp programming. And not get confused by a little math. This is not a numerical analysis problem, and is not intended to draw upon such prior experience. Although it is likely you’ve encountered the notion of a determinant of a square matrix before, you probably have not seen the algorithm expressed this way, and so your prior experience does not give you much help.

We use an algorithm taken from W. M. Gentleman and S. C. Johnson, ACM Trans. Math. Soft. 2 no 3. (1976), (which you need not reference at all) for computing the determinant of an \( n \) by \( n \) matrix over the integers by Gaussian elimination.

If we ignore pivoting (but see the optional problem below), the algorithm consist of \( n - 1 \) steps, indexed by a variable \( k \) running from 1 to \( n - 1 \). The \( k \)th step requires the computation of an \( n - k + 1 \) by \( n - k + 1 \) matrix which we shall call \( A^{(k+1)} \); the entries will be denoted \( a^{(k+1)}_{ij} \), with \( k \leq i, j \leq n \).

The original matrix is \( A^{(1)} \). For each \( k \), the entry \( a^{(k+1)}_{ij} \) is computed by the formula

\[
a^{(k+1)}_{ij} = \frac{a^{(k)}_{ii} a^{(k)}_{jj} - a^{(k)}_{ik} a^{(k)}_{kj}}{a^{(k-1)}_{i-1,j-1}}
\]

for \( k + 1 \leq i, j \leq n \), and where \( a^{(0)}_{00} \) is taken to be 1. The division is always exact. The result, \( a^{(n)}_{nn} \) is the determinant of \( A^{(1)} \).

Devise an appropriate data structure for a matrix and write a Common Lisp program to compute determinants. How long does your program take as a function of \( n \)? Is it recomputing things?
4. How hard would it be to change your program if the entries were not integers, but symbols?

5. (optional) How hard would it be to change your program to do “pivoting” if you know what that means?