Exercise 1 (Linear Term And Translation of Argument) Let \( f : \mathbb{R}^n \rightarrow \mathbb{R}, b, c \in \mathbb{R}^n \) and define \( g : \mathbb{R}^n \rightarrow \mathbb{R} \) as \( g(x) = c^\top x + f(x + b) \). Show that

\[
g^*(y) = -(y - c)^\top b + f^*(y - c), \quad \text{dom}(g^*) = c + \text{dom}(f^*)
\]

Exercise 2 (Conjugate of Log Moment Generating Function) Let \( X \) be a random \( n \)-dimensional vector. The moment generating function associated with \( X \) is defined as:

\[
M_X(\lambda) \doteq \mathbb{E}[e^{\lambda^\top X}]
\]

The logarithmic moment generating function associated with \( X \) is defined as:

\[
\Lambda_X(\lambda) \doteq \log M_X(\lambda)
\]

For each of the following random variables find the conjugate of \( \Lambda_X \).

1. \( X \) is supported on the standard basis \( \{e_1, \ldots, e_n\} \) of \( \mathbb{R}^n \) and \( \mathbb{P}(X = e_i) = p_i \).
2. \( X \sim N(\mu, \Sigma) \), where \( \mu \in \mathbb{R}^n \) is the mean vector, and \( \Sigma \in \mathbb{S}_+^n \) is the covariance matrix.