

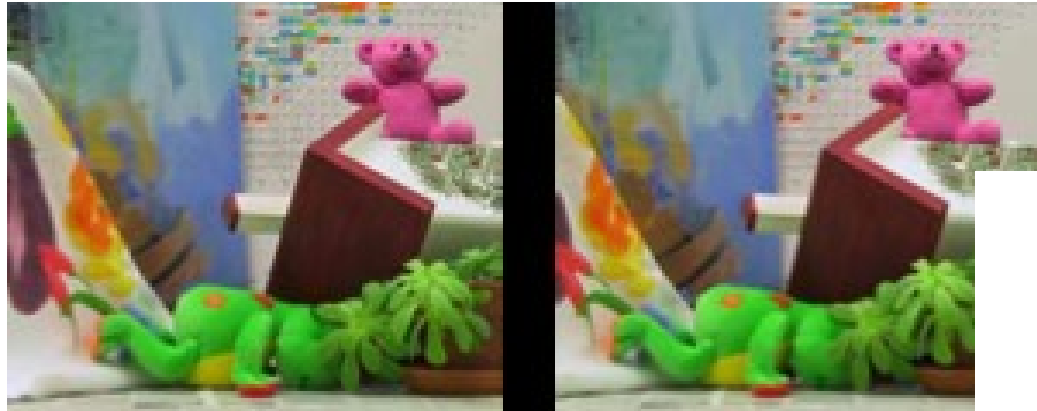
# Stereo

EECS 442 –David Fouhey

Fall 2019, University of Michigan

[http://web.eecs.umich.edu/~fouhey/teaching/EECS442\\_F19/](http://web.eecs.umich.edu/~fouhey/teaching/EECS442_F19/)

# Two-View Stereo



# Stereo

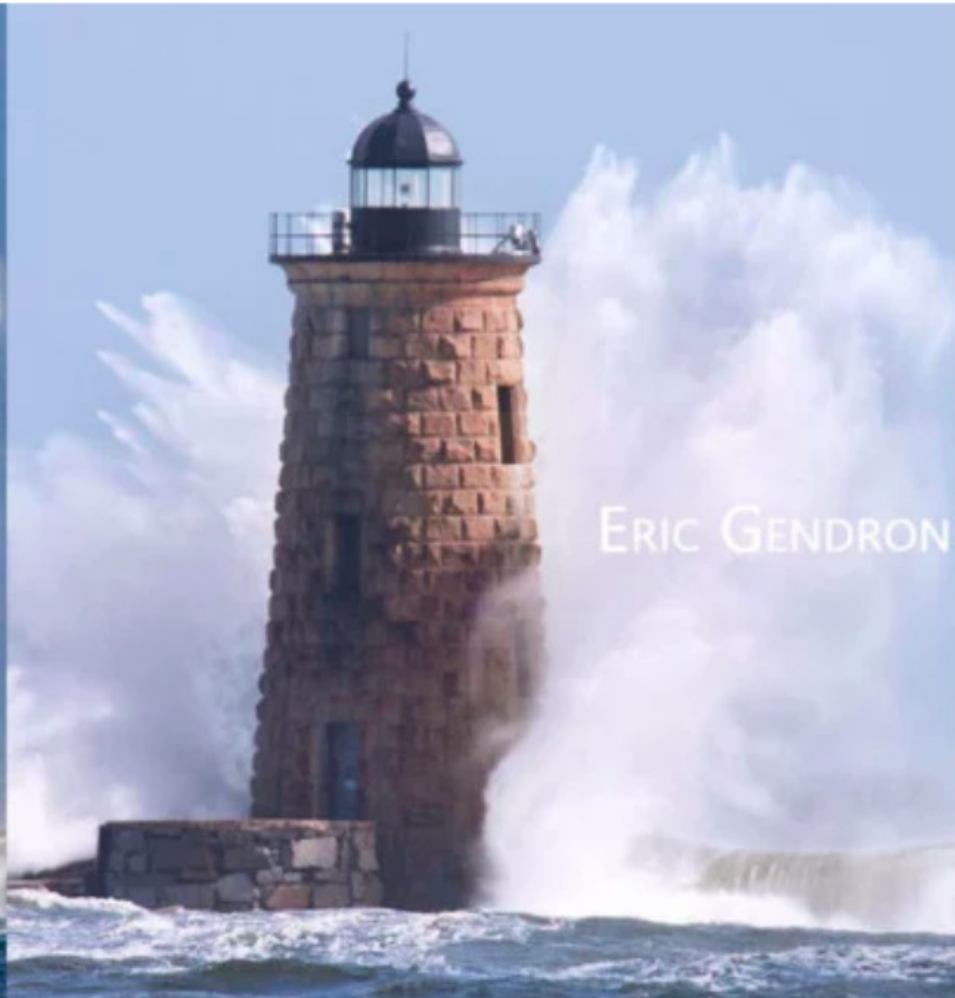


# How Two Photographers Unknowingly Shot the Same Millisecond in Time

PetaPixel

MAR 07, 2018

RON RISMAN



<https://petapixel.com/2018/03/07/two-photographers-unknowingly-shot-millisecond-time/>

# How Two Photographers Unknowingly Shot the Same Millisecond in Time

PetaPixel

MAR 07, 2018

RON RISMAN



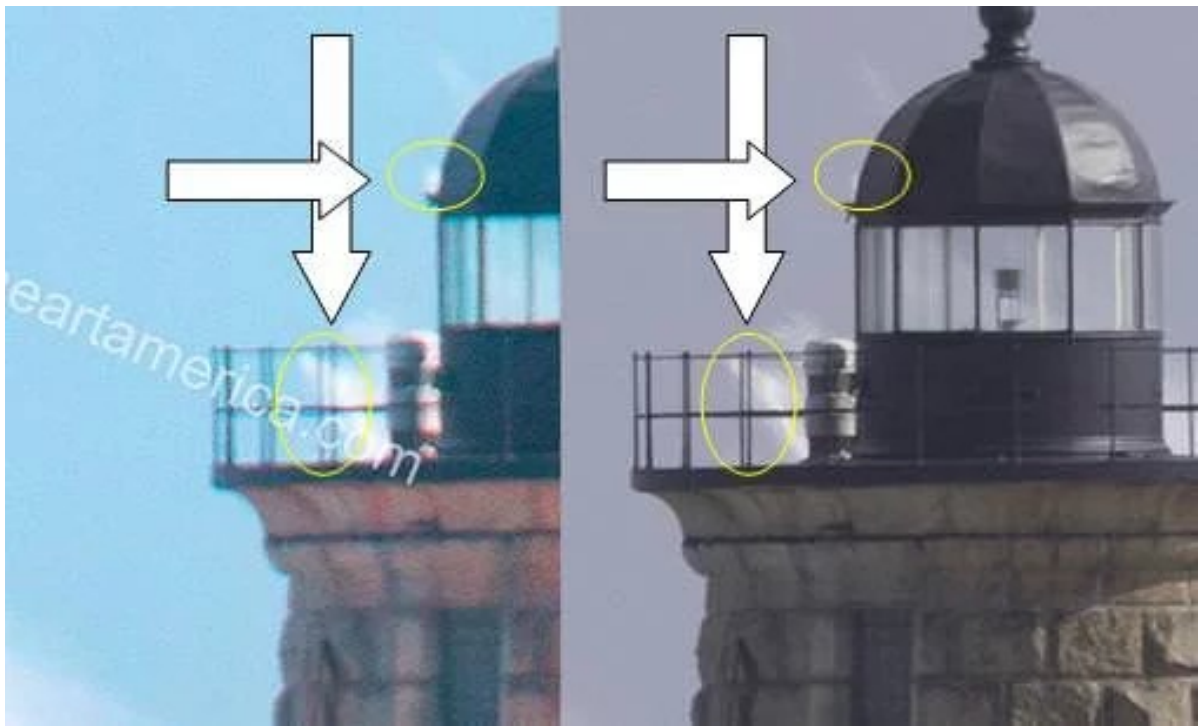
<https://petapixel.com/2018/03/07/two-photographers-unknowingly-shot-millisecond-time/>

# How Two Photographers Unknowingly Shot the Same Millisecond in Time

MAR 07, 2018

RON RISMAN

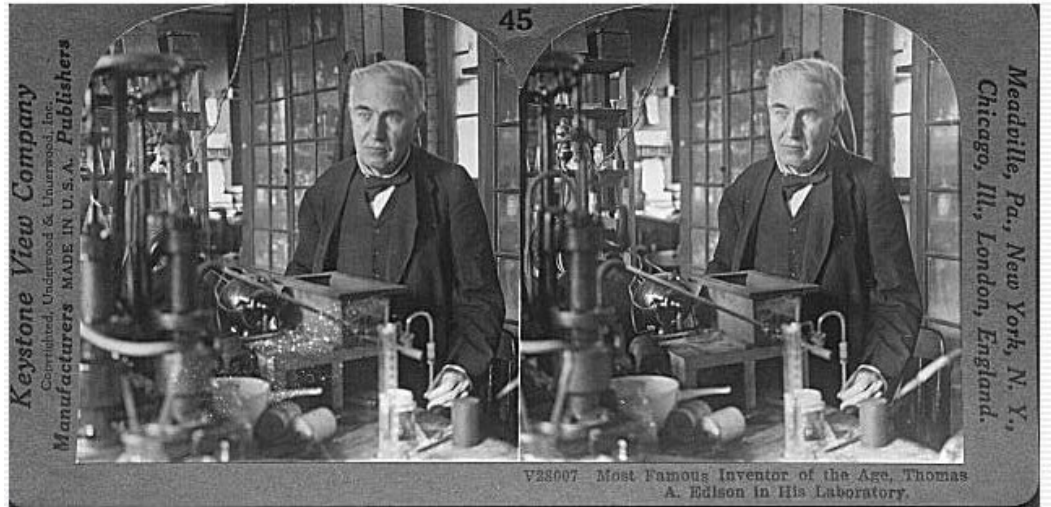
PetaPixel



<https://petapixel.com/2018/03/07/two-photographers-unknowingly-shot-millisecond-time/>

# Stereograms

Humans can fuse pairs of images to get a sensation of depth



Stereograms: Invented by Sir Charles Wheatstone, 1838

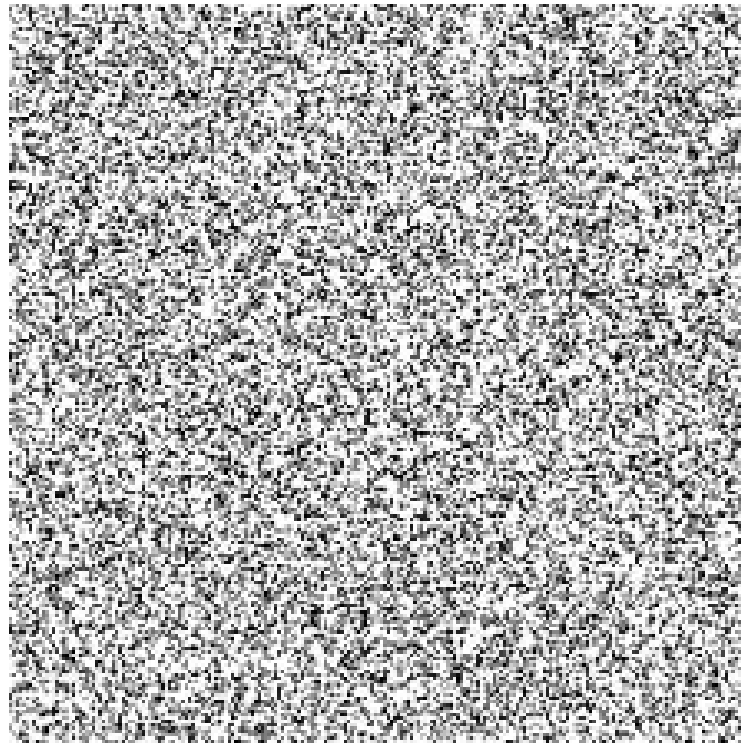
# Stereograms





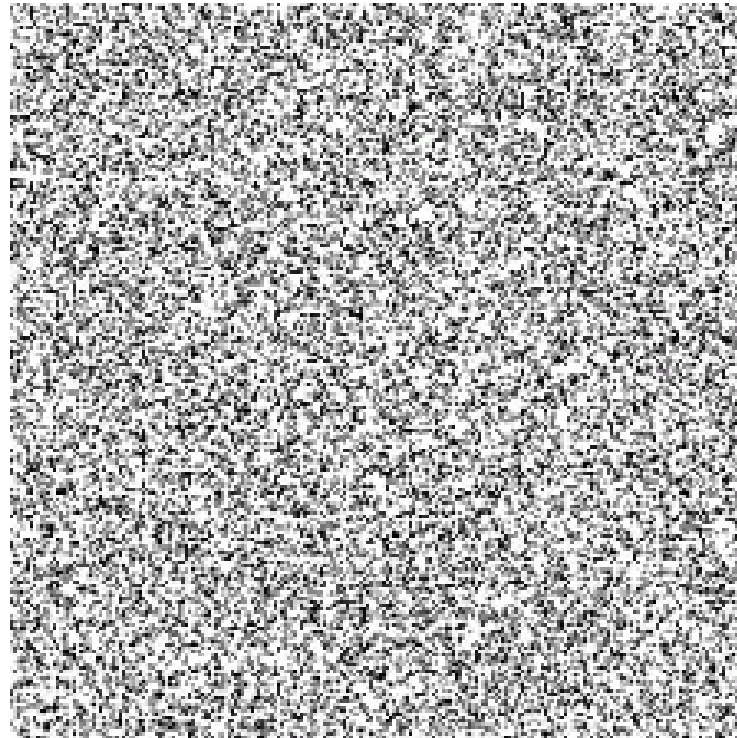
# Stereograms

What about this?



# Stereograms

Bela Julesz: Random Dot Stereogram  
Shows that stereo can operate *without* recognition



# Stereograms

Humans can fuse pairs of images to get a sensation of depth



Autostereograms: [www.magiceye.com](http://www.magiceye.com)

# Stereograms

Humans can fuse pairs of images to get a sensation of depth



Autostereograms: [www.magiceye.com](http://www.magiceye.com)

# Problem formulation

Given a calibrated binocular stereo pair, fuse it to produce a depth image

image 1



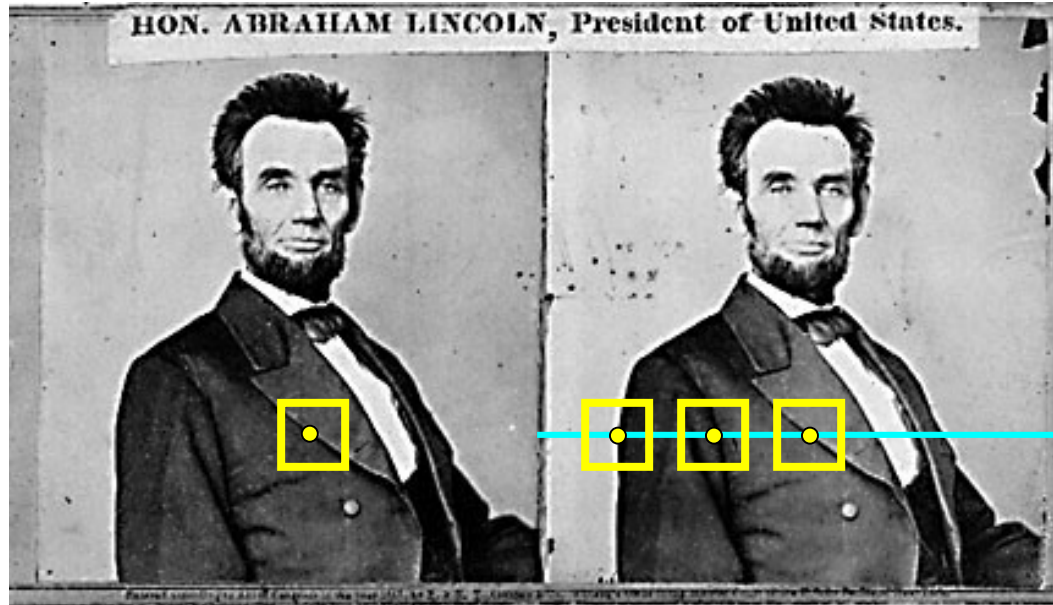
image 2



Dense depth map

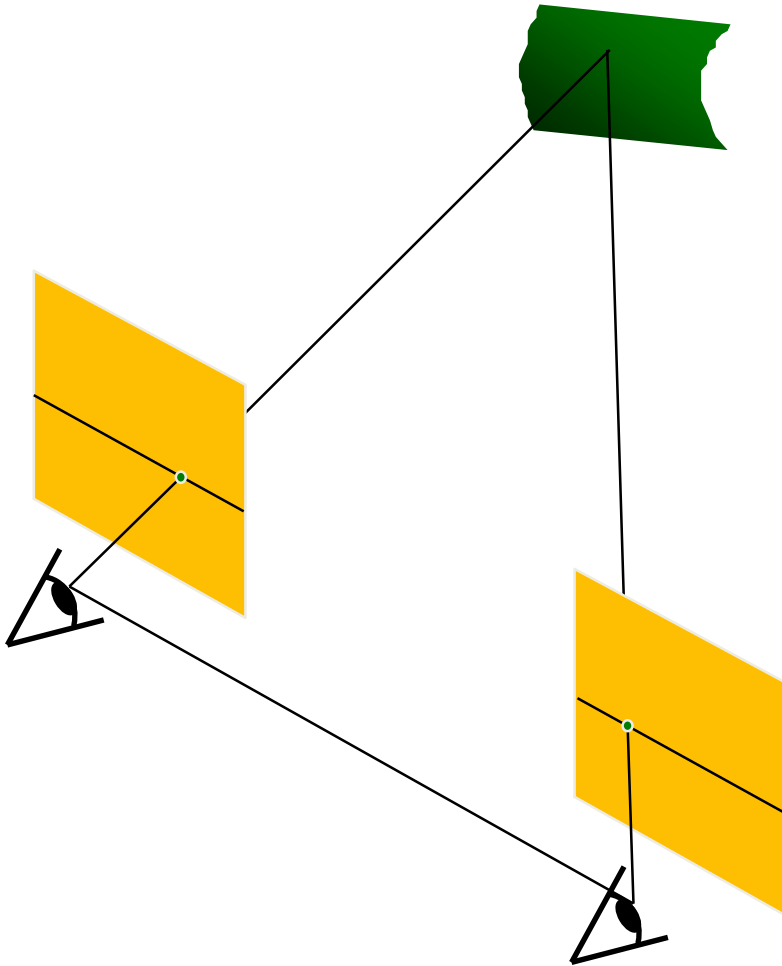


# Basic stereo matching algorithm



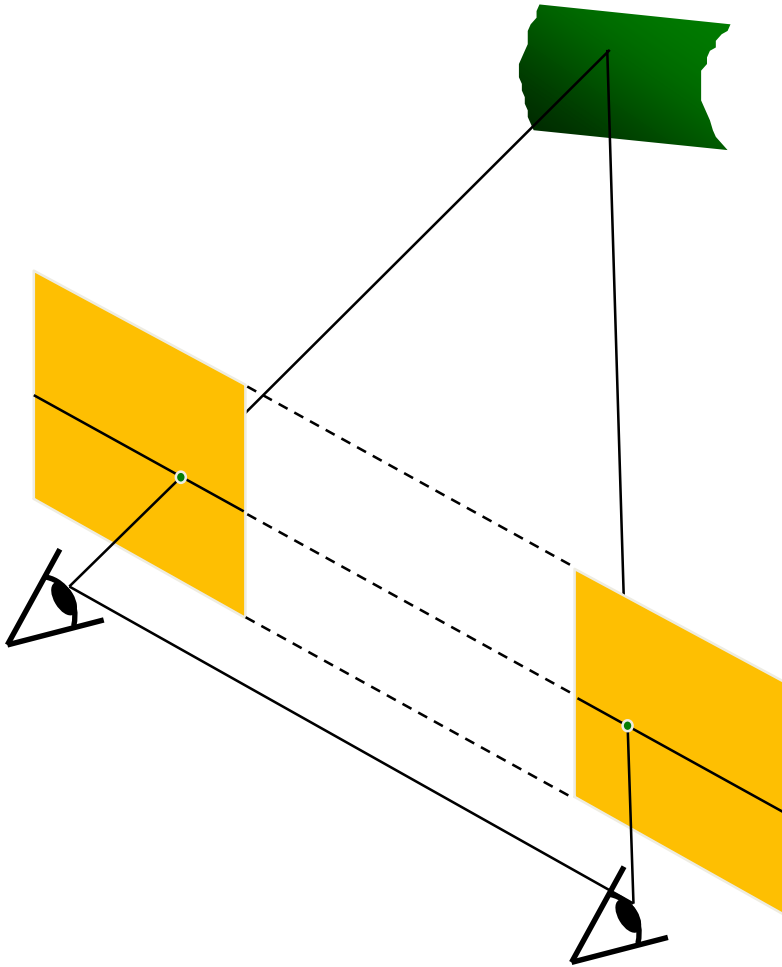
- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match
  - Triangulate the matches to get depth information
- Simplest case: epipolar lines = corresponding scanlines
  - When does this happen?

# Simplest Case: Parallel images



- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths the same

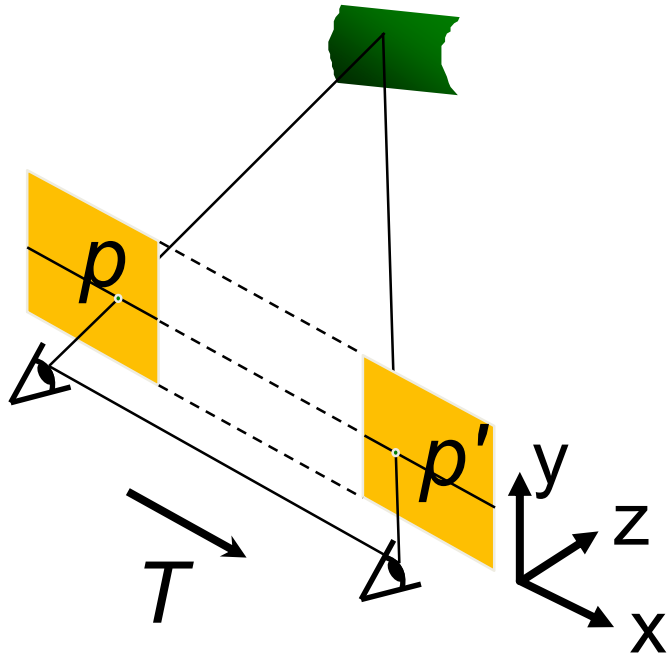
# Simplest Case: Parallel images



- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths the same
- Then epipolar lines fall along the horizontal scan lines of the images



# Essential matrix for parallel images



$$pEp' = 0 \quad E = [t_x]R$$

**What's R?      What's t?**

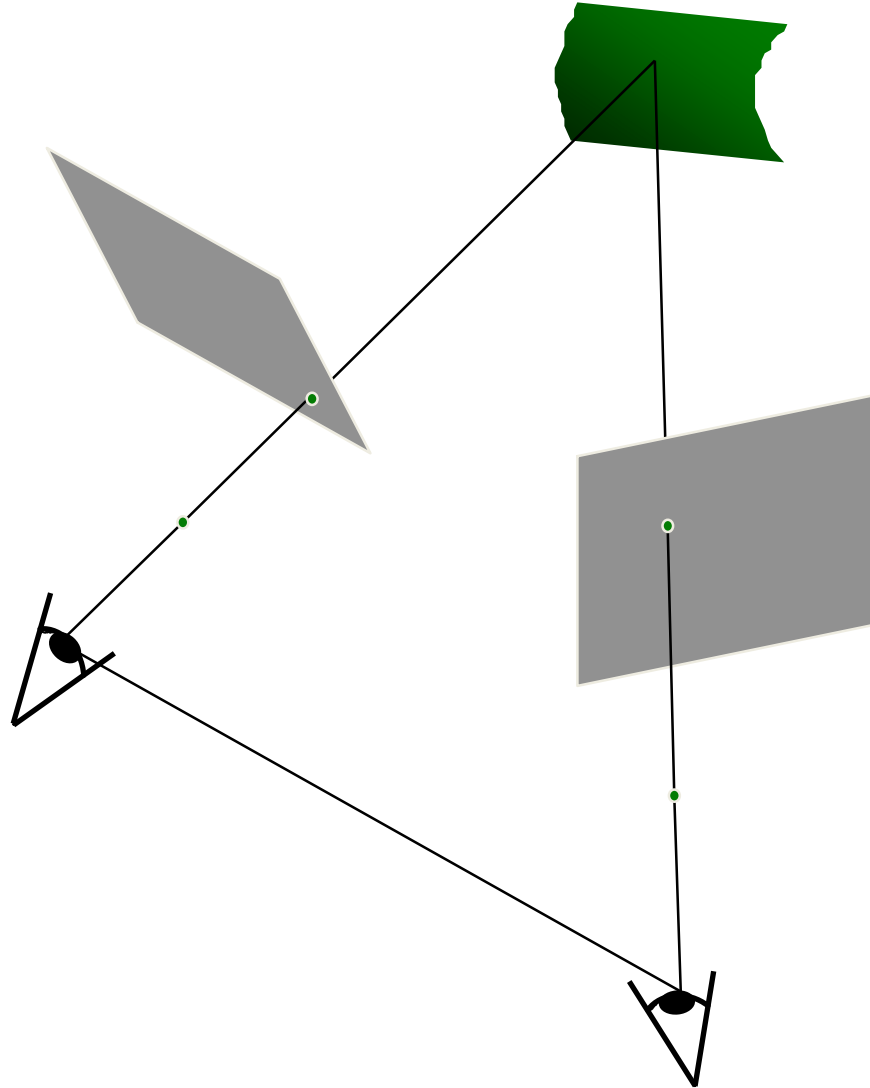
$$R = I \quad t = [T, 0, 0]$$

$$E = [t_x]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

$$[u \ v \ 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = 0 \quad \rightarrow \quad [u \ v \ 1] \begin{bmatrix} 0 \\ -T \\ Tv' \end{bmatrix} = 0 \quad \rightarrow \quad \begin{matrix} -Tv + Tv' = 0 \\ Tv' = Tv \end{matrix}$$

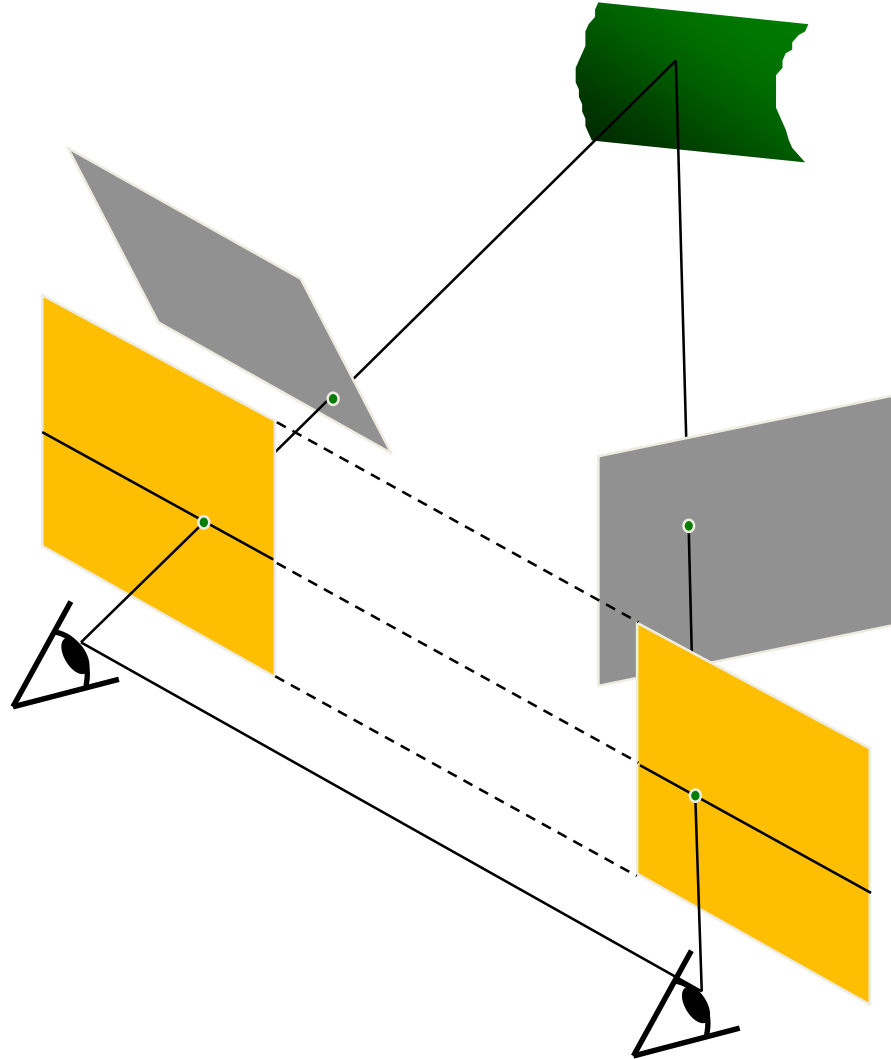
The y-coordinates of corresponding points are the same!

# Stereo image rectification



# Stereo image rectification

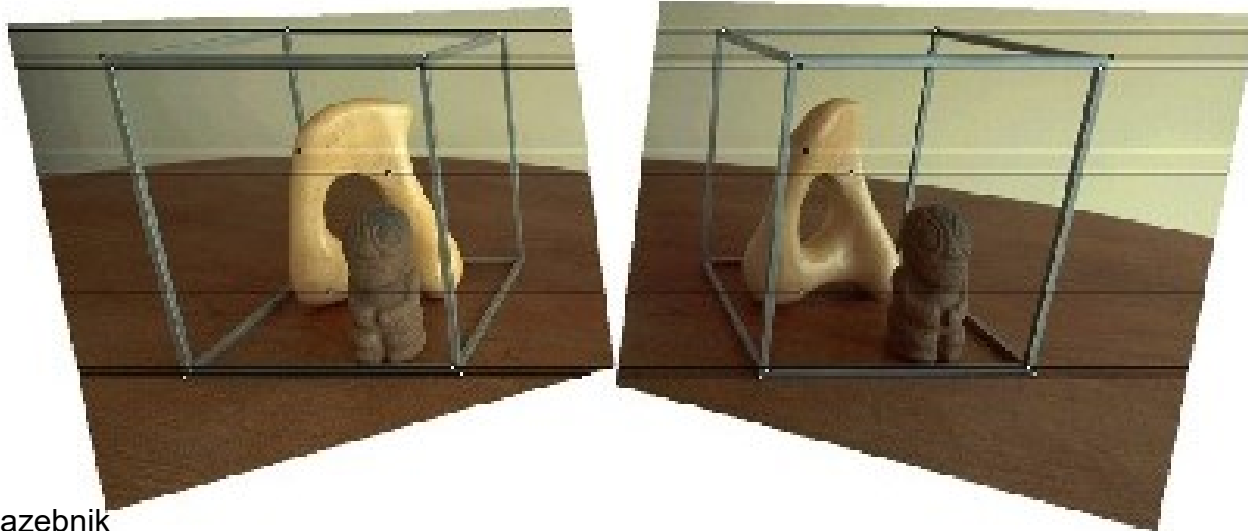
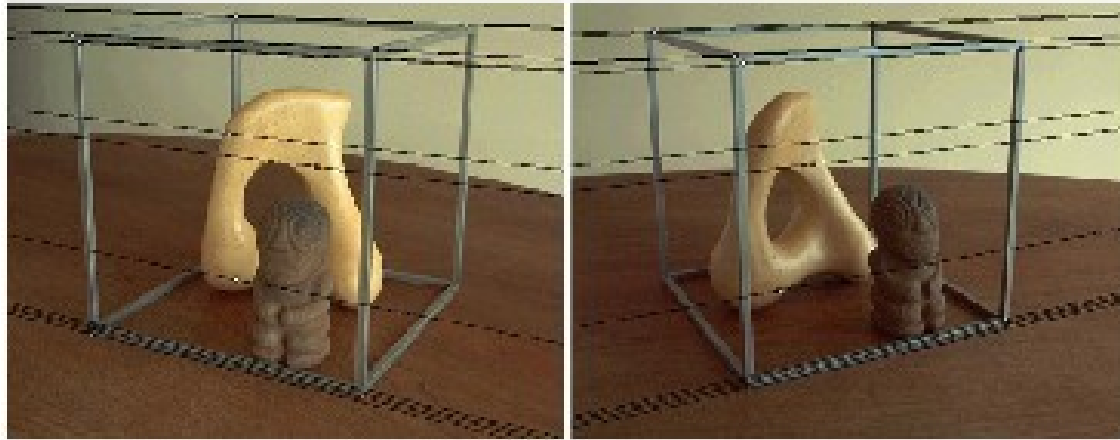
Reproject image planes onto a common plane parallel to the line between optical centers



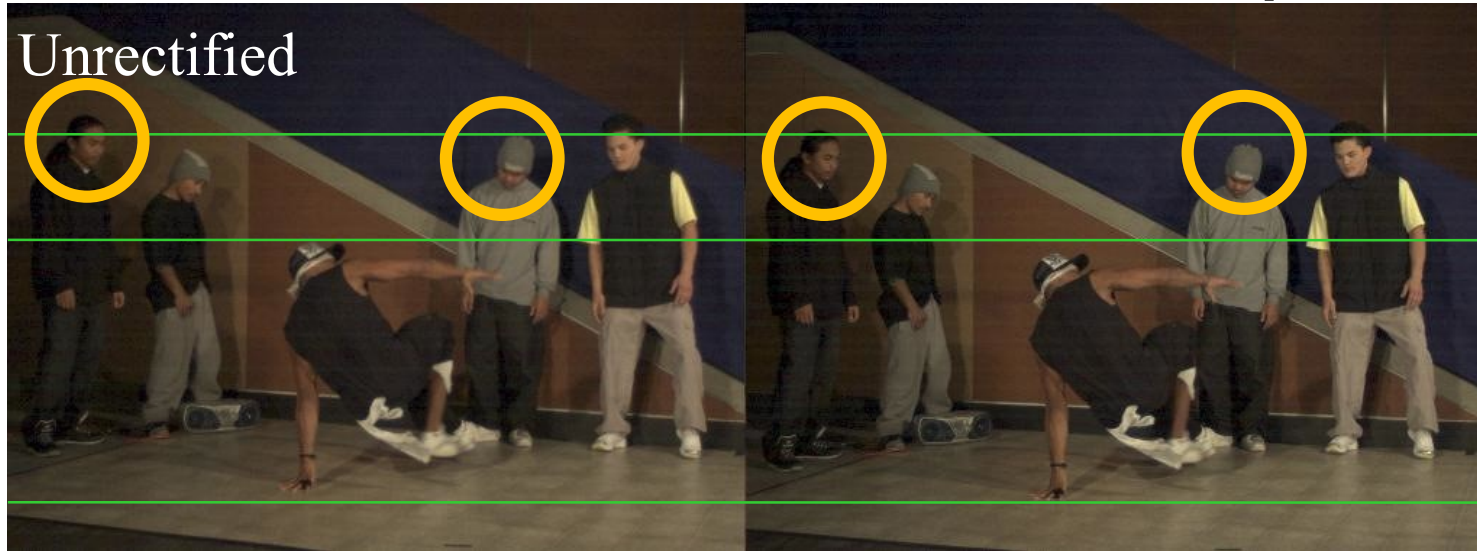
C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision](#). CVPR 1999

Slide credit: S. Lazebnik

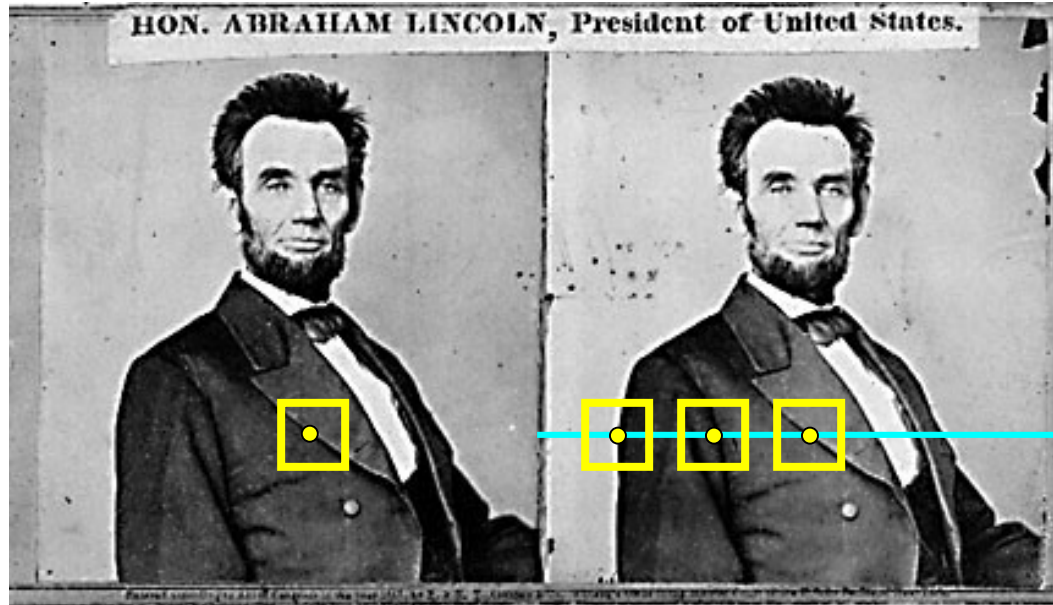
# Rectification example



# Another rectification example

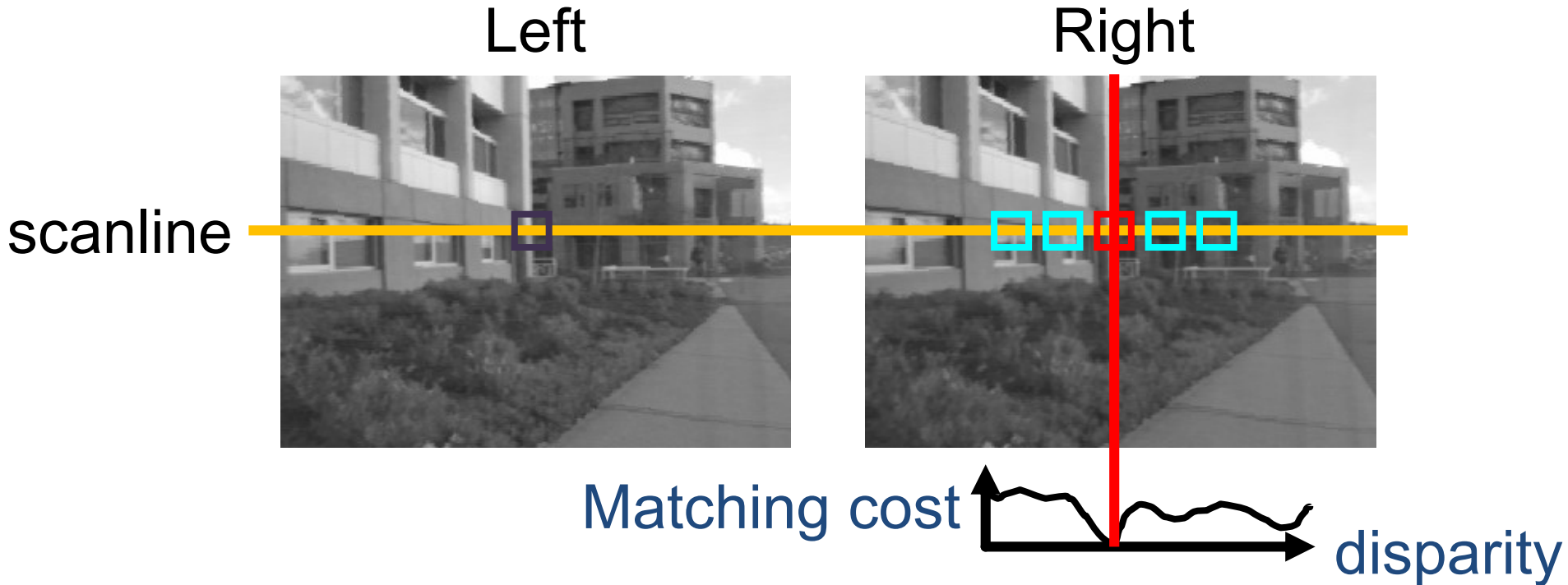


# Basic stereo matching algorithm



- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match

# Correspondence Search



Slide window along the right scanline, compare contents of that window with reference window on left

Matching cost: SSD or normalized correlation

# Correspondence Search

Left

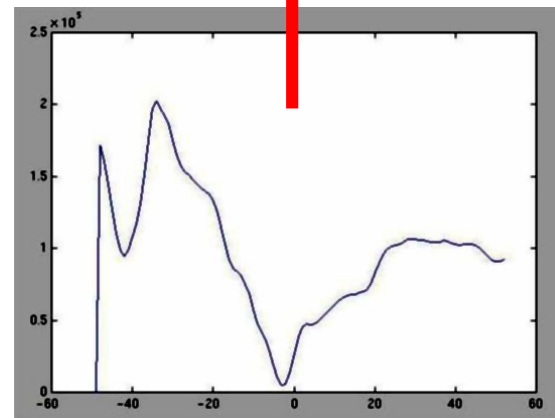
Right

scanline



Matching cost  
Sum of squared differences

$$\sum_i (l_i - r_i)^2$$



Disparity



# Correspondence Search

Left

Right

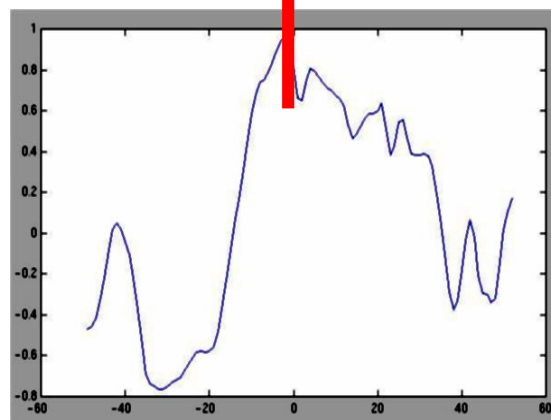
scanline



Matching cost  
Normalized correlation

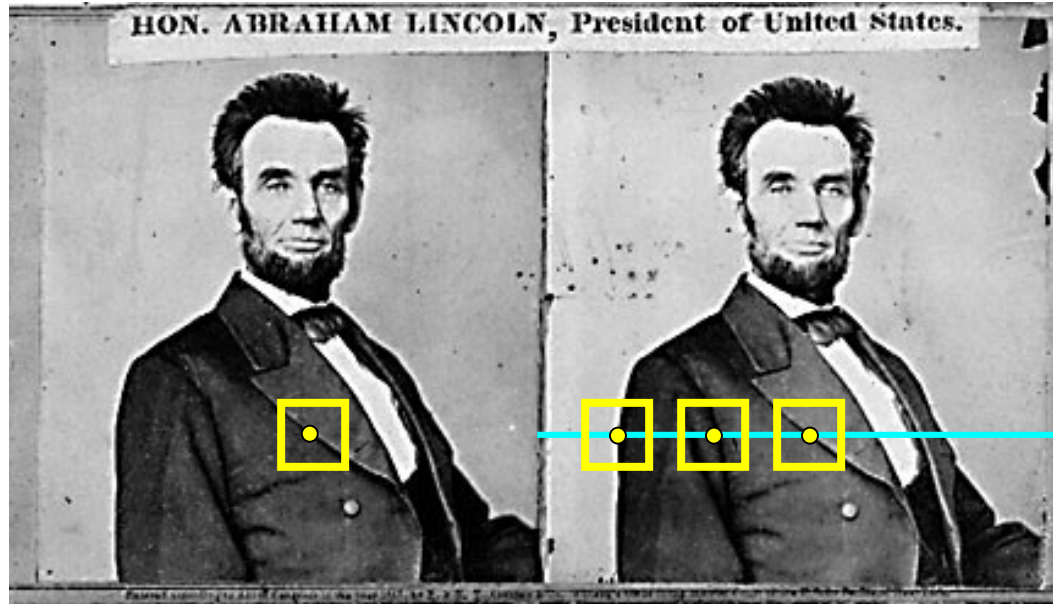
$$\hat{x}_i = \frac{x_i - \text{mean}(x)}{\text{std}(x)}$$

$$\hat{l} \cdot \hat{r}$$



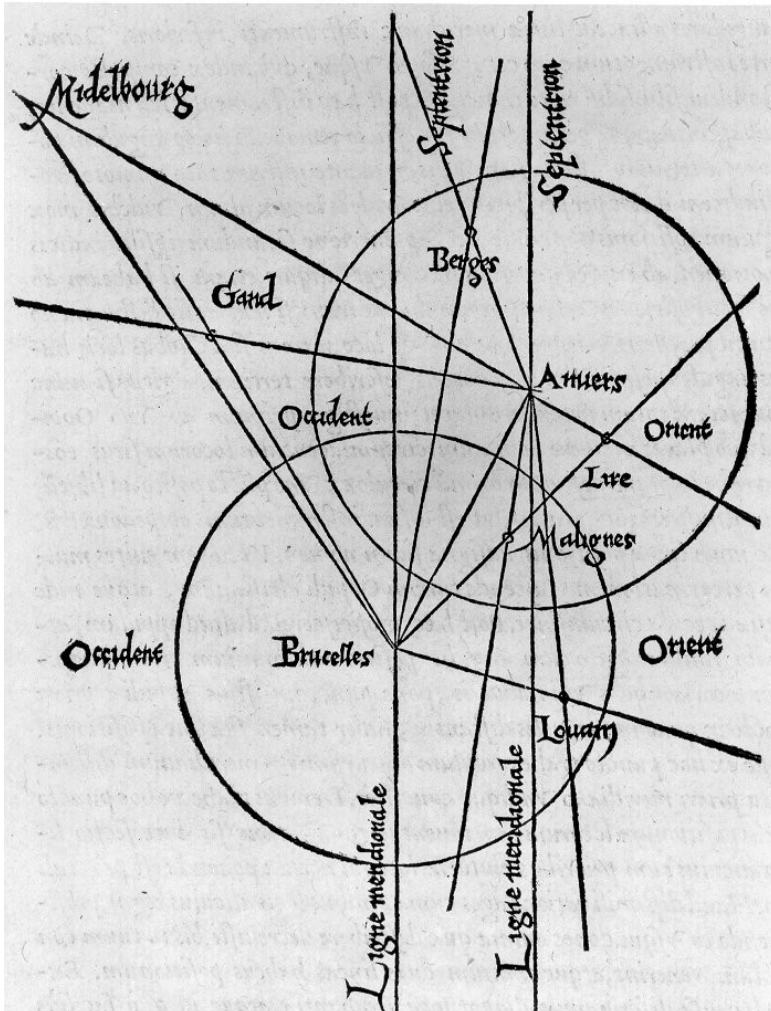
Disparity

# Basic stereo matching algorithm



- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel  $x$  in the first image
  - Find corresponding epipolar scanline in the right image
  - Examine all pixels on the scanline and pick the best match  $x'$
  - Triangulate the matches to get depth information

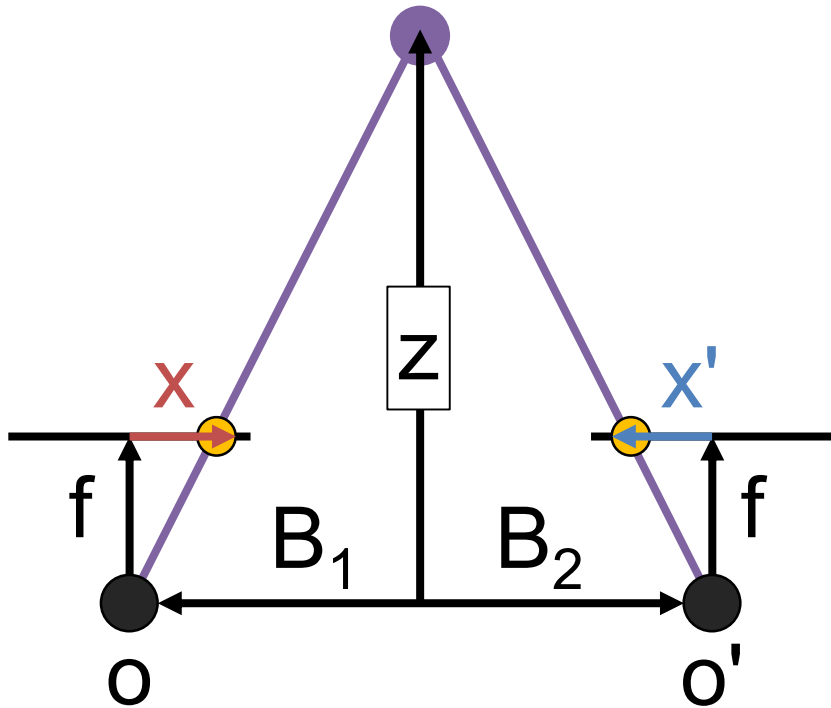
# Triangulation: Older History



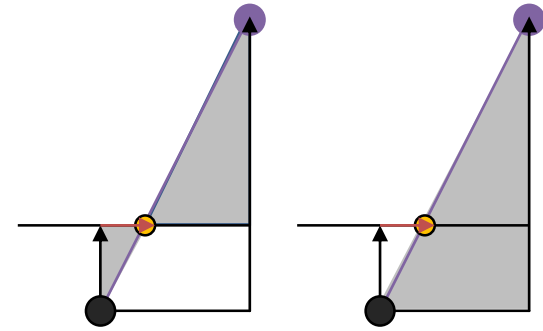
- From [Wikipedia](#): Gemma Frisius's 1533 diagram introducing the idea of triangulation into the science of surveying. Having established a baseline, e.g. the cities of Brussels and Antwerp, the location of other cities, e.g. Middelburg, Ghent etc., can be found by taking a compass direction from each end of the baseline, and plotting where the two directions cross. This was only a theoretical presentation of the concept — due to topographical restrictions, it is impossible to see Middelburg from either Brussels or Antwerp. Nevertheless, the figure soon became well known all across Europe.

# Triangulation: Modern History

# Depth from disparity



$$\frac{x}{f} = \frac{B_1}{z}$$

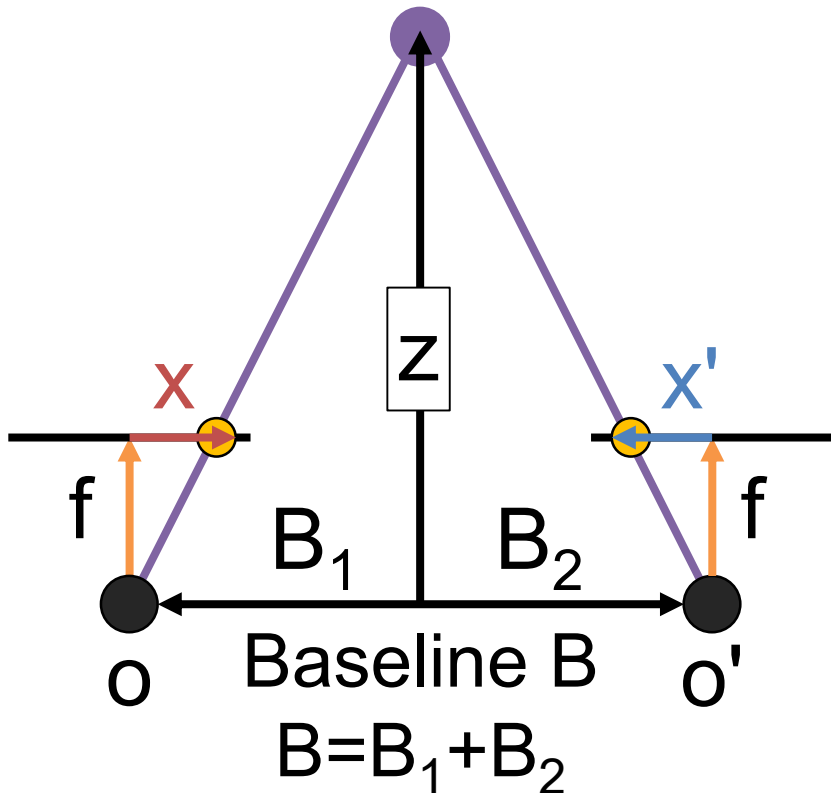


By similar triangles

$$\frac{-x'}{f} = \frac{B_2}{z}$$

Similarly by  
similar triangles

# Depth from disparity



$$\frac{x}{f} = \frac{B_1}{z} \quad \frac{-x'}{f} = \frac{B_2}{z}$$

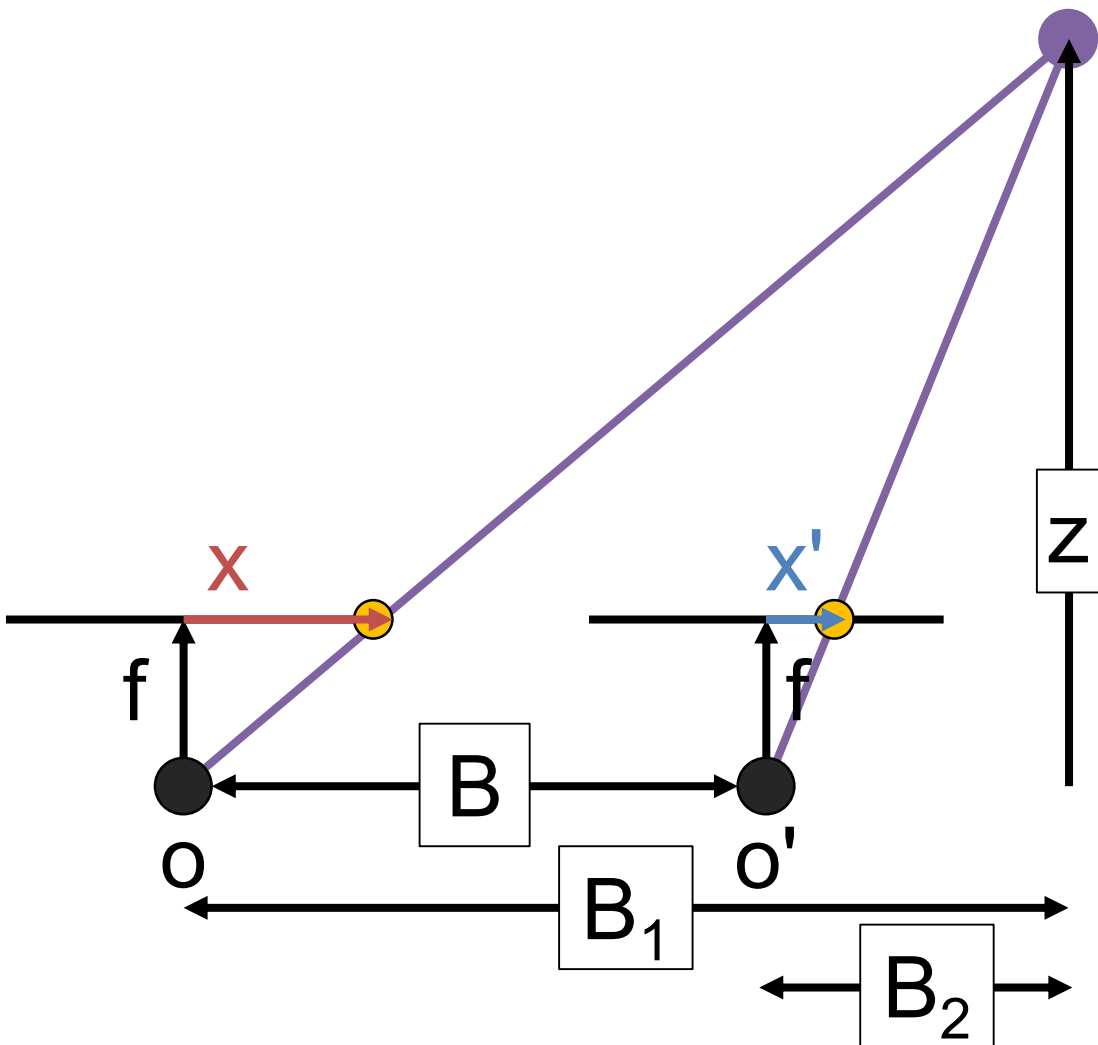
Add them

$$\frac{x - x'}{f} = \frac{B_1 + B_2}{z}$$

$$\underbrace{x - x'}_{\text{Disparity}} = \frac{fB}{z}$$

Disparity

# Depth from disparity



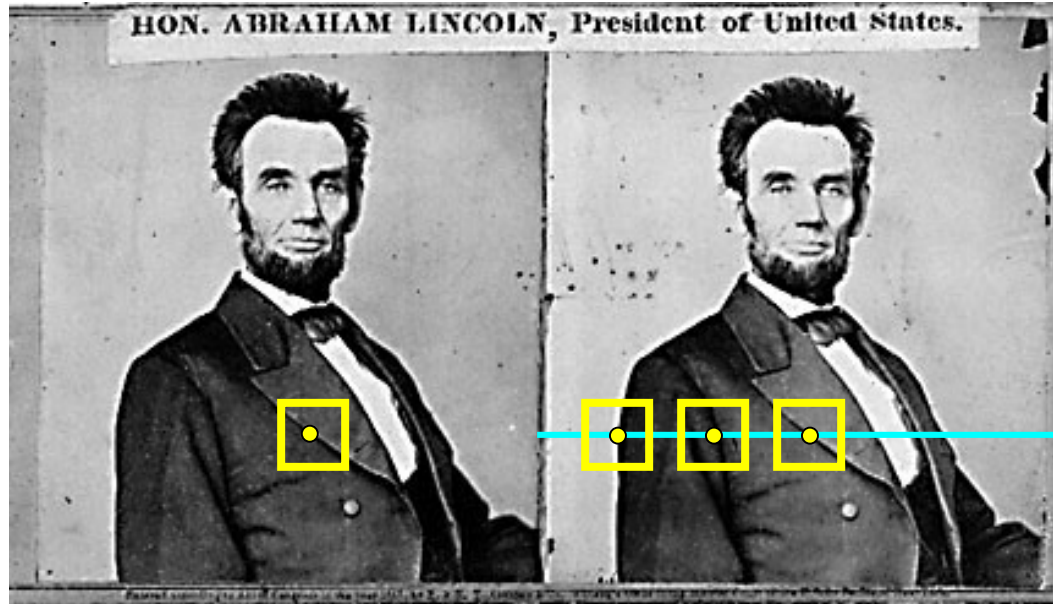
$$\frac{x}{f} = \frac{B_1}{z} \quad \frac{x'}{f} = \frac{B_2}{z}$$

Subtract them

$$\frac{x - x'}{f} = \frac{B_1 - B_2}{z}$$

$$x - x' = \frac{fB}{z}$$

# Basic stereo matching algorithm

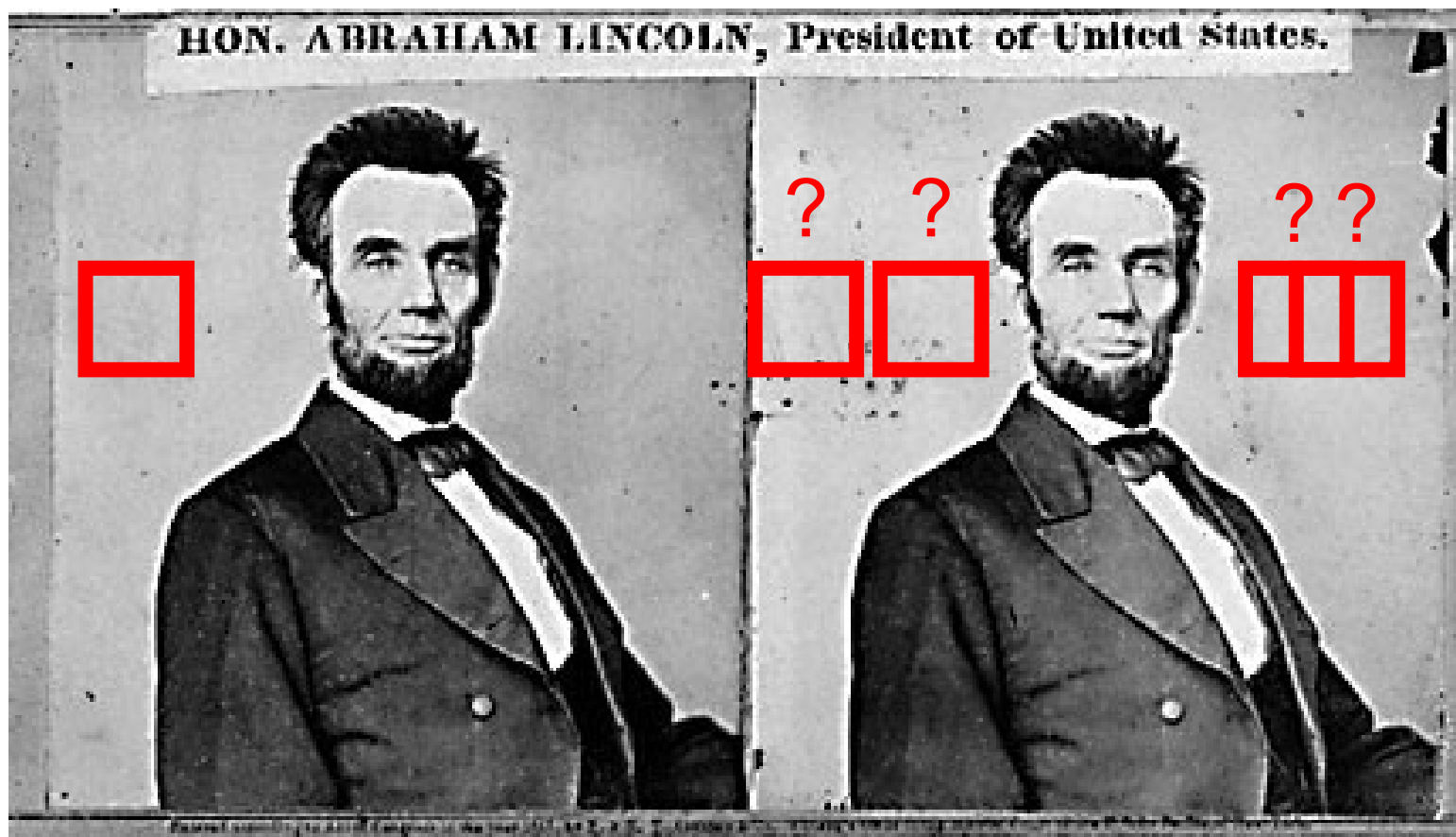


- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel  $x$  in the first image
  - Find corresponding epipolar scanline in the right image
  - Examine all pixels on the scanline and pick the best match  $x'$
  - Compute disparity  $x-x'$  and set  $\text{depth}(x) = B \cdot f / (x-x')$



# Failures of Correspondence Search

Textureless regions. **Why?**



# Failures of Correspondence Search

Repeated Patterns. **Why?**

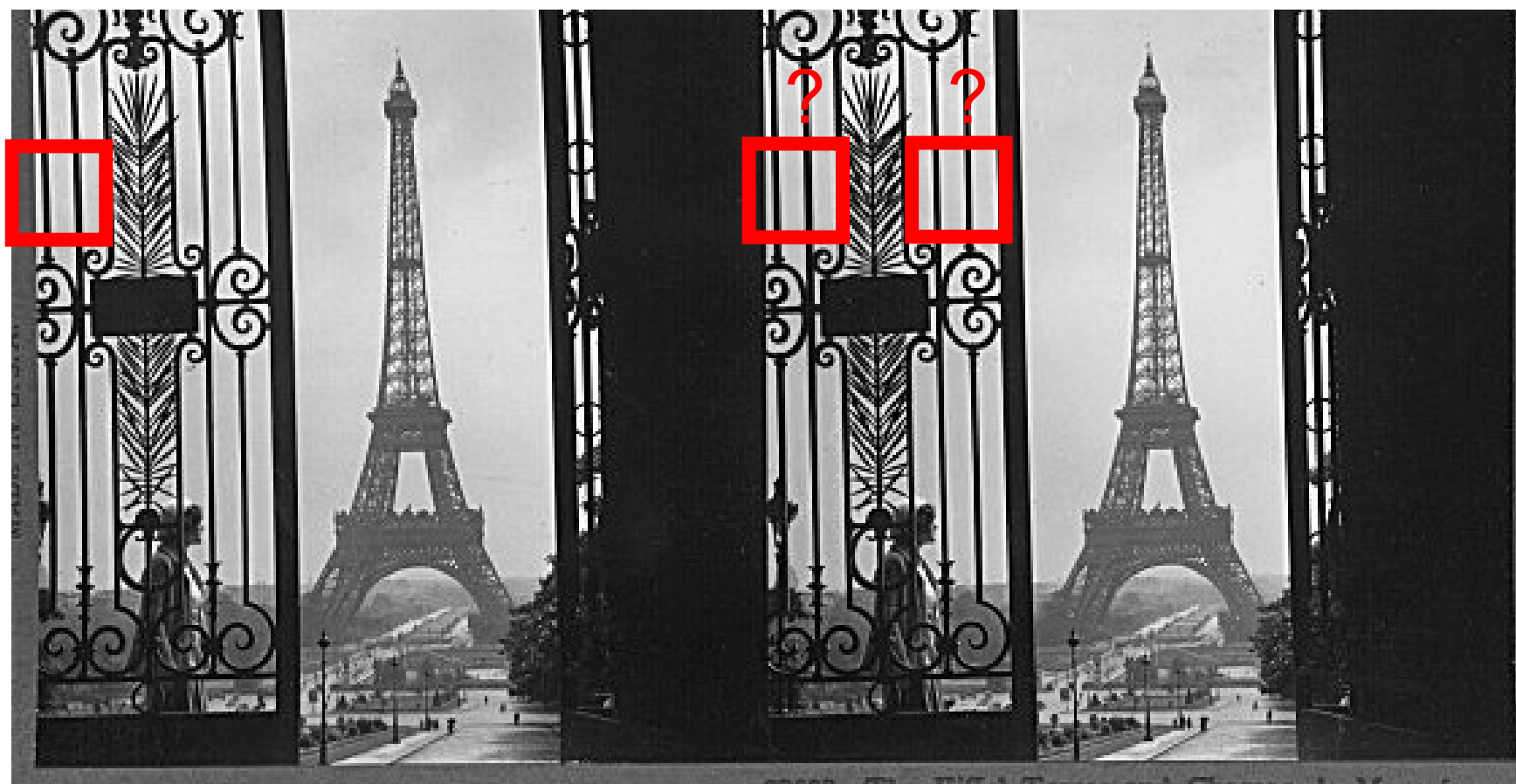


Image credit: S. Lazebnik

# Failures of Correspondence Search

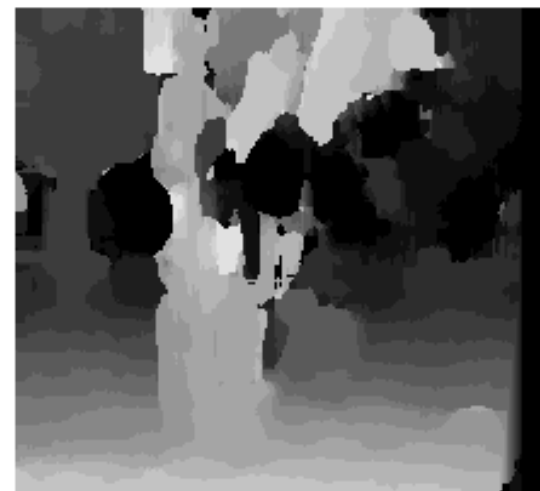
## Specular Surfaces. **Why?**



# Effect of window size



$W = 3$



$W = 20$

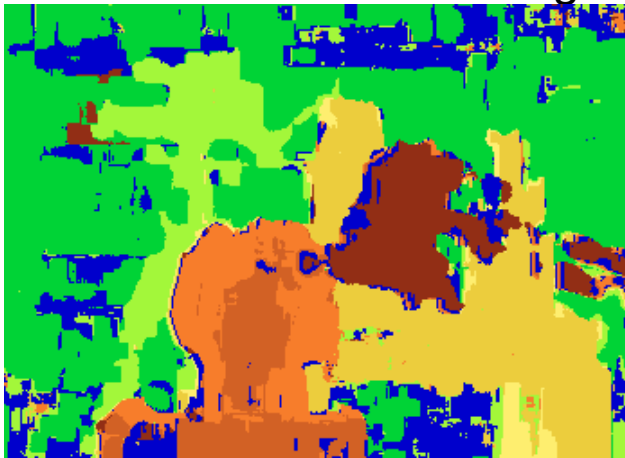
- Smaller window
  - + More detail
  - More noise
- Larger window
  - + Smoother disparity maps
  - Less detail

# Results with window search

Data



Window-based matching



Ground truth



# Better methods exist...



Graph cuts



Ground truth

Y. Boykov, O. Veksler, and R. Zabih, [Fast Approximate Energy Minimization via Graph Cuts](#), PAMI 2001

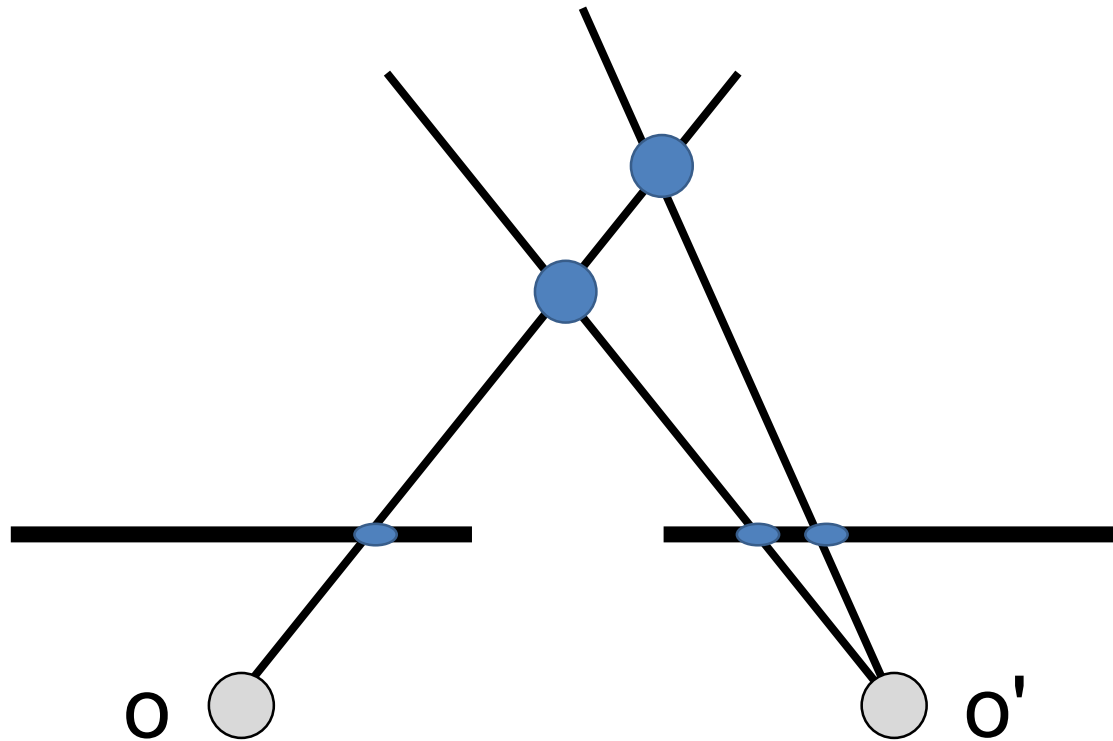
For the latest and greatest: <http://www.middlebury.edu/stereo/>

# Improving Window-based Matching

- Similarity is **local** (each window independent)
- Need non-local correspondence constraints / cues.

# Uniqueness

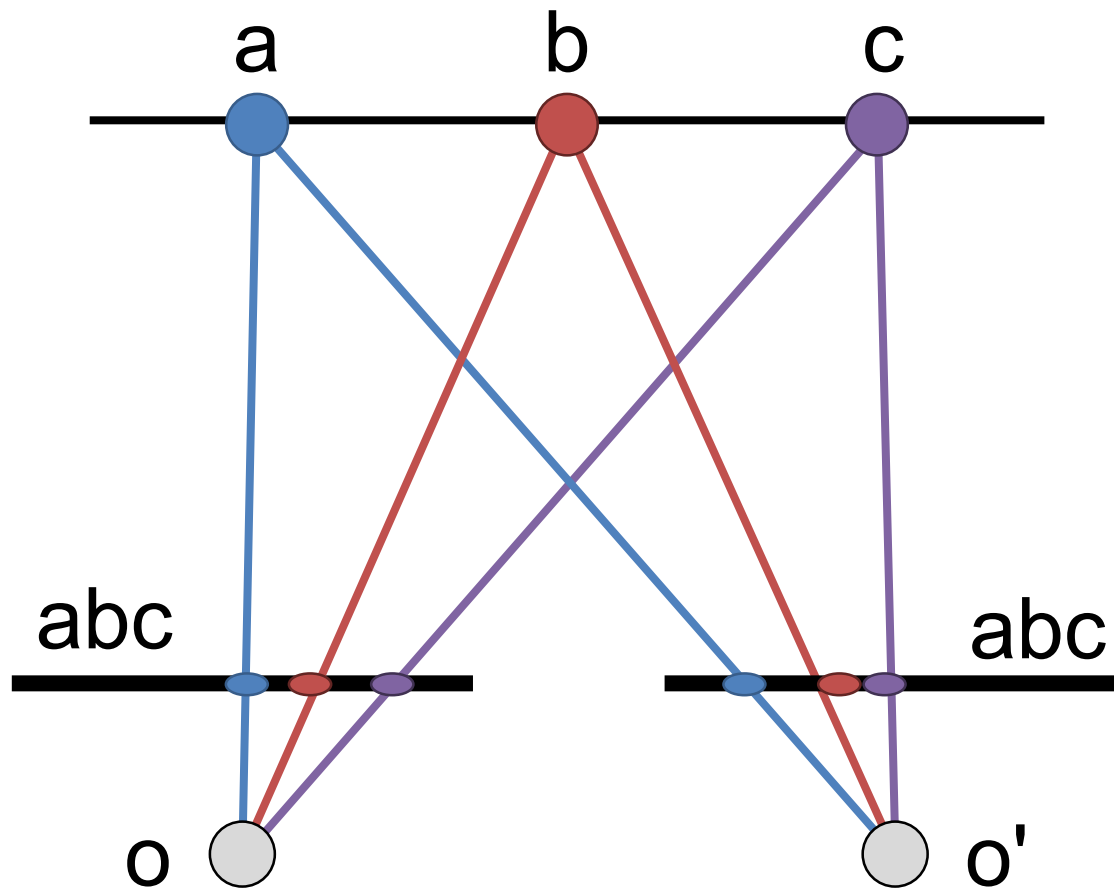
- Each point in one image should match at most one point in other image.
- **When might this not be true?**





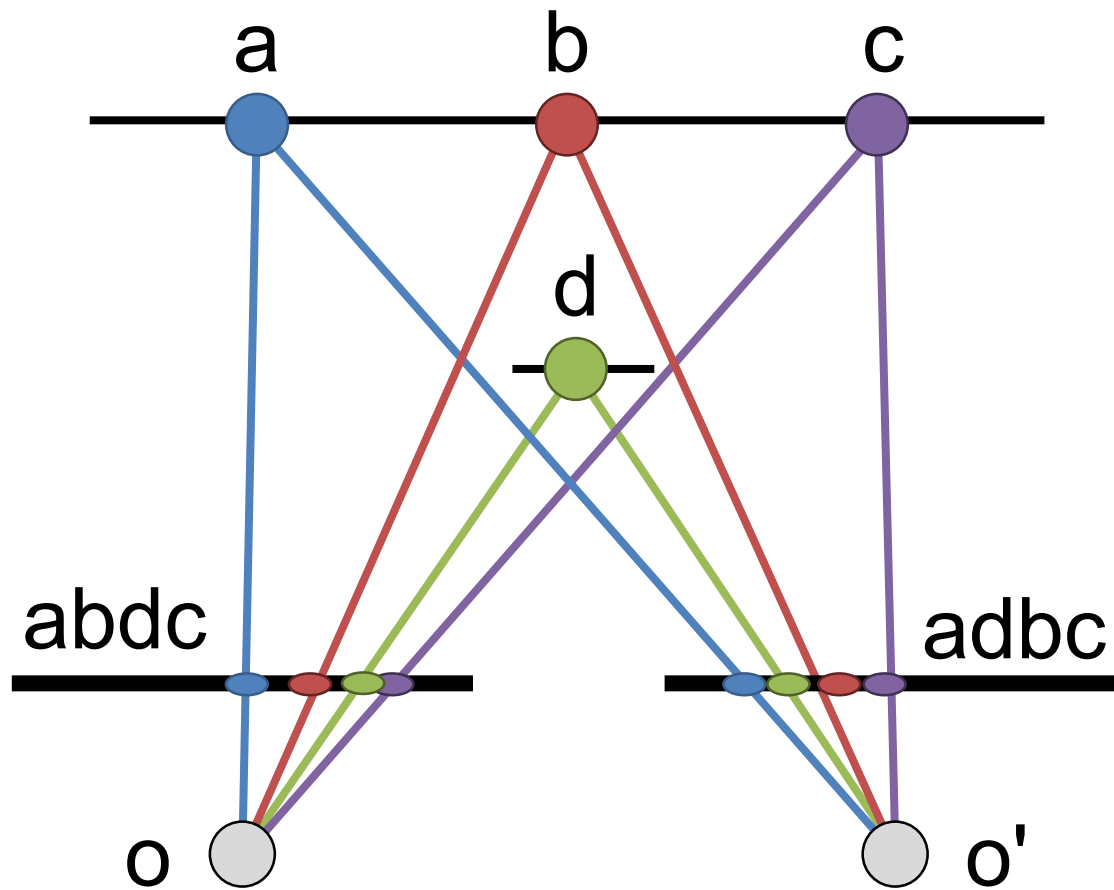
# Ordering

- Corresponding points should be in same order



# Ordering

- Not always true!

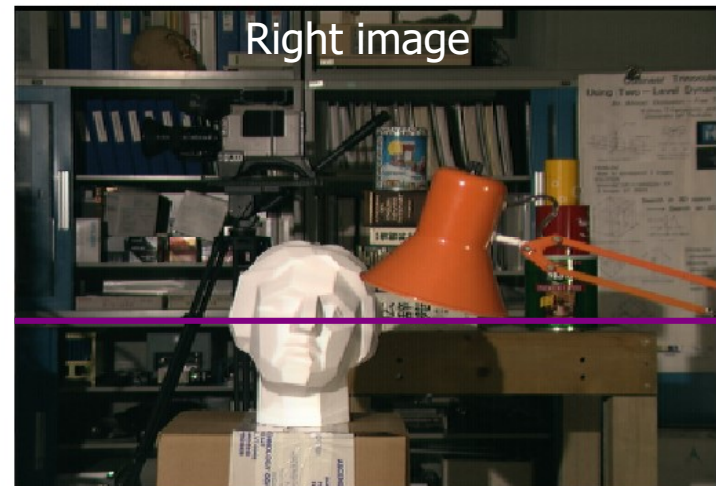
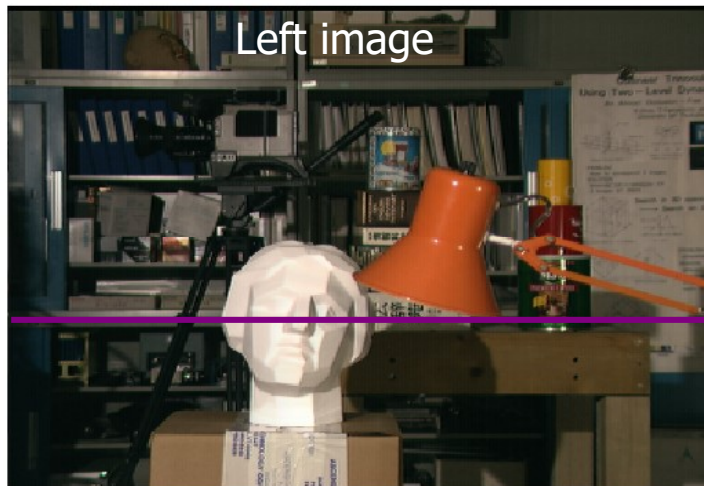


# Smoothness

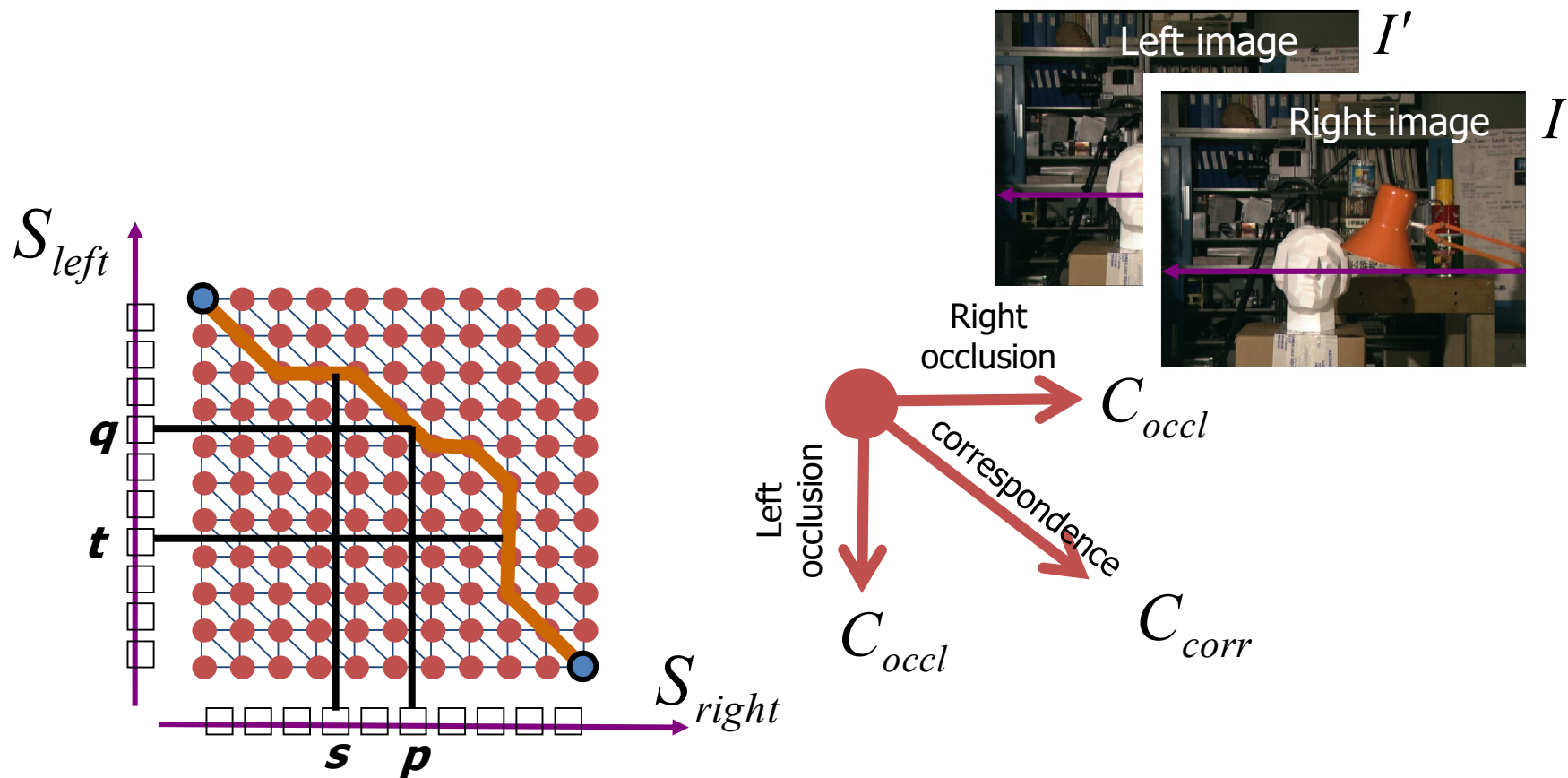
- We expect disparity values to change slowly (for the most part)
- **When is this not true?**

# Scanline Stereo

- Try to coherently match pixels on the entire scanline
- Different scanlines are optimized (by dynamic programming) independently



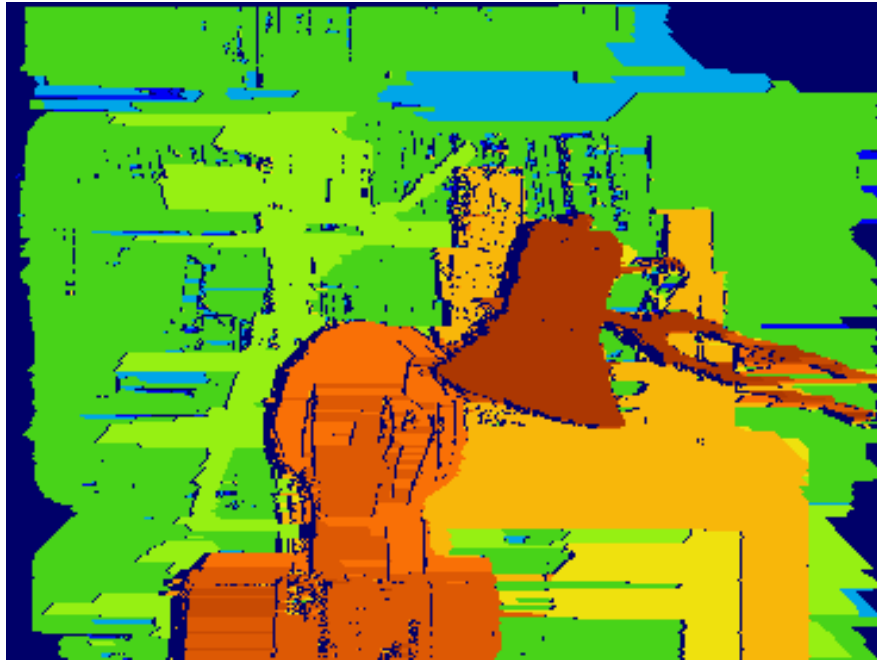
# “Shortest paths” for scan-line stereo



Can be implemented with dynamic programming  
Ohta & Kanade '85, Cox et al. '96

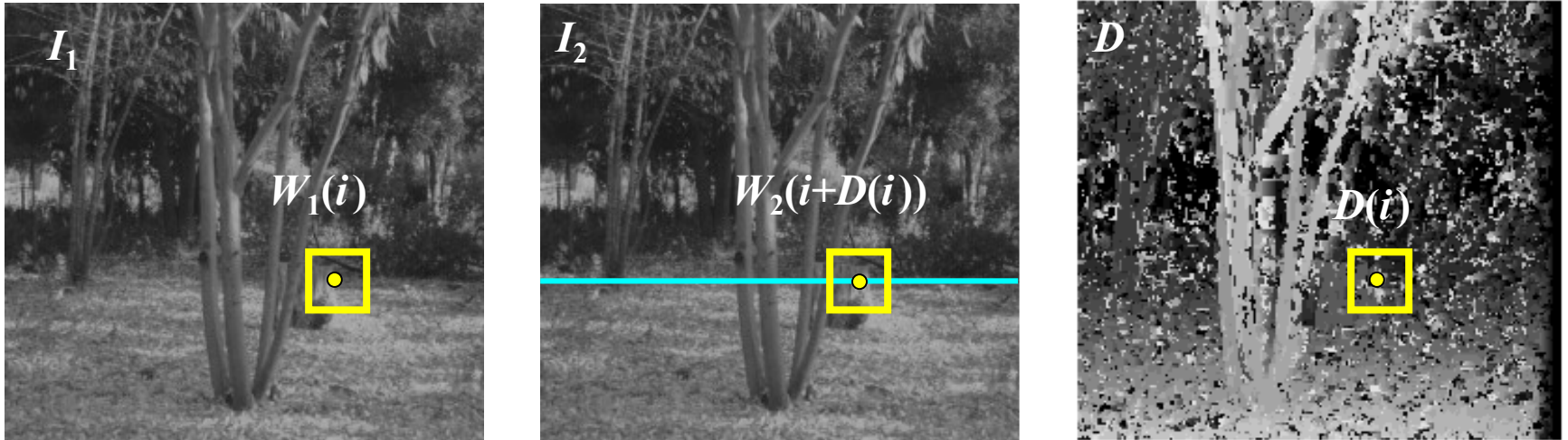
# Coherent Stereo on 2D Grid

- Scanline stereo generates streaking artifacts



- Can't use dynamic programming to find spatially coherent disparities on a 2D grid

# Stereo Matching as Optimization

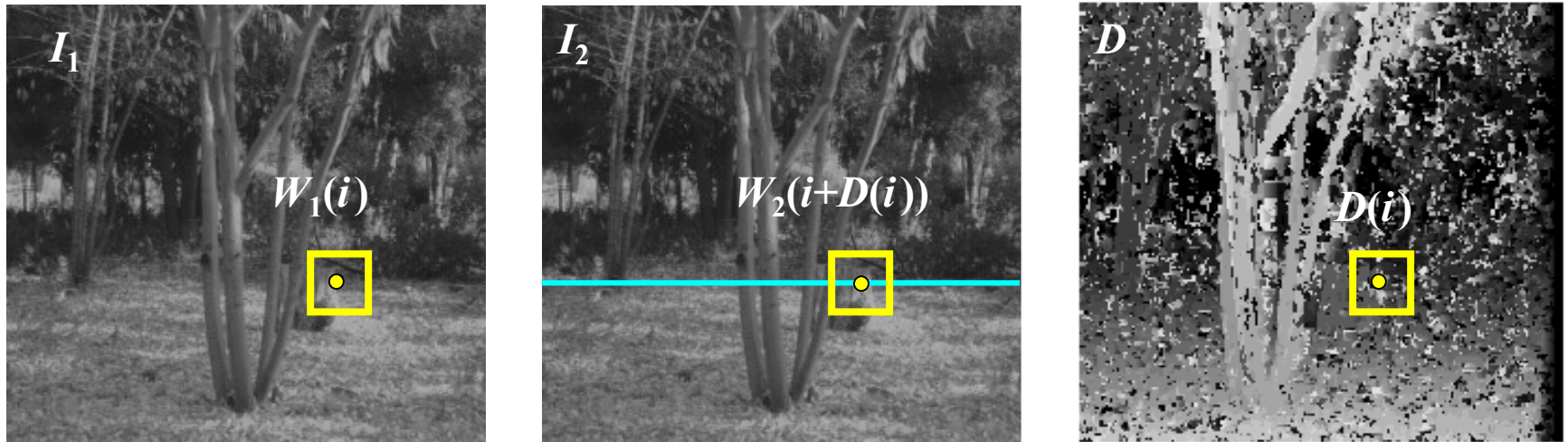


$$E(D) = \underbrace{\sum_i \left( W_1(i) - W_2(i + D(i)) \right)^2}_{\text{Data term}} + \lambda \underbrace{\sum_{\text{neighbors } i,j} \rho(D(i) - D(j))}_{\text{Smoothness term}}$$

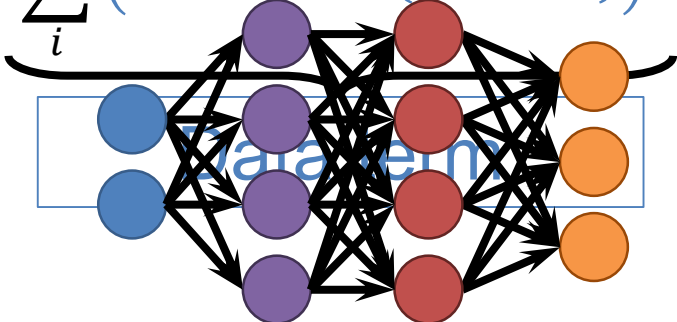
Solvable by graph cuts for certain smoothnesses  $\rho$

Y. Boykov, O. Veksler, and R. Zabih, [Fast Approximate Energy Minimization via Graph Cuts](#), PAMI 2001

# Is This Doable by Deep Network?



$$E(D) = \sum_i \left( W_1(i) - W_2(i + D(i)) \right)^2 + \lambda \sum_{\text{neighbors } i,j} \rho(D(i) - D(j))$$



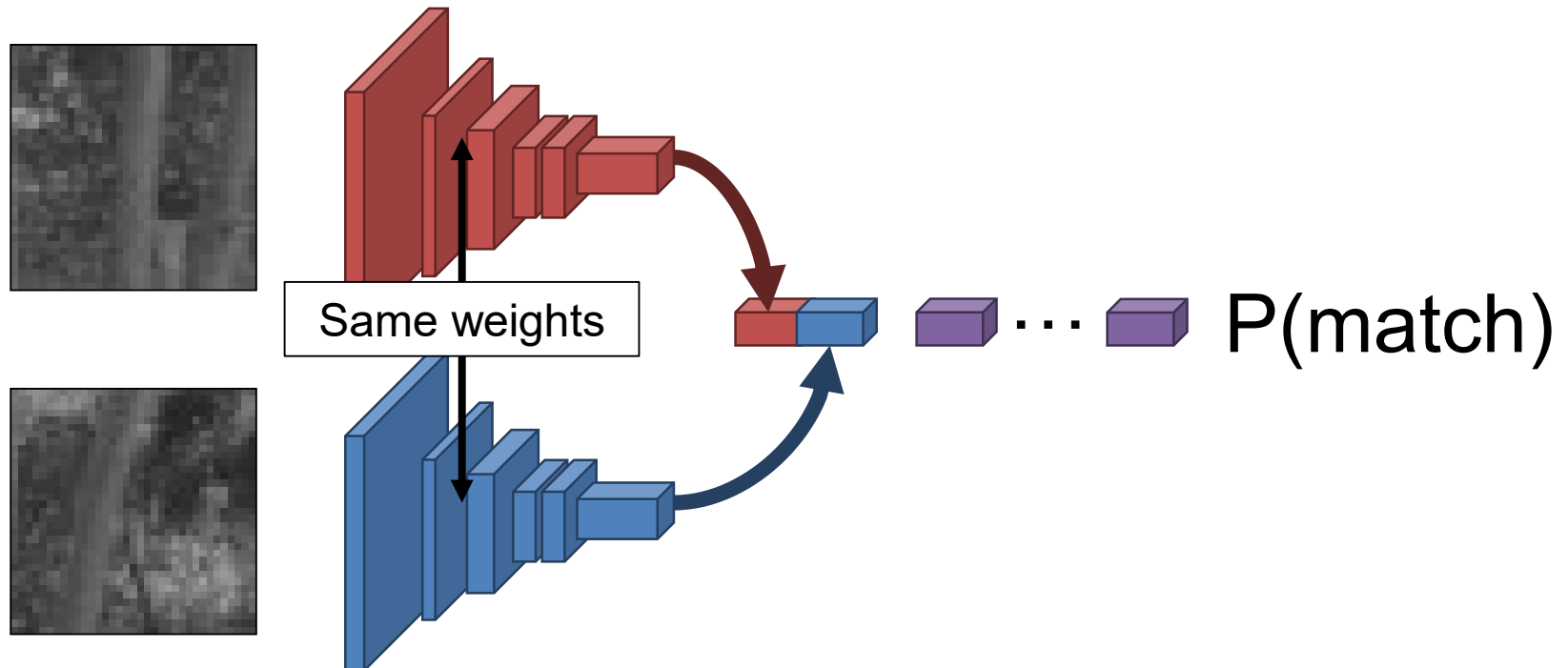
**Smoothness term**

Easy solution: replace the data term with a network



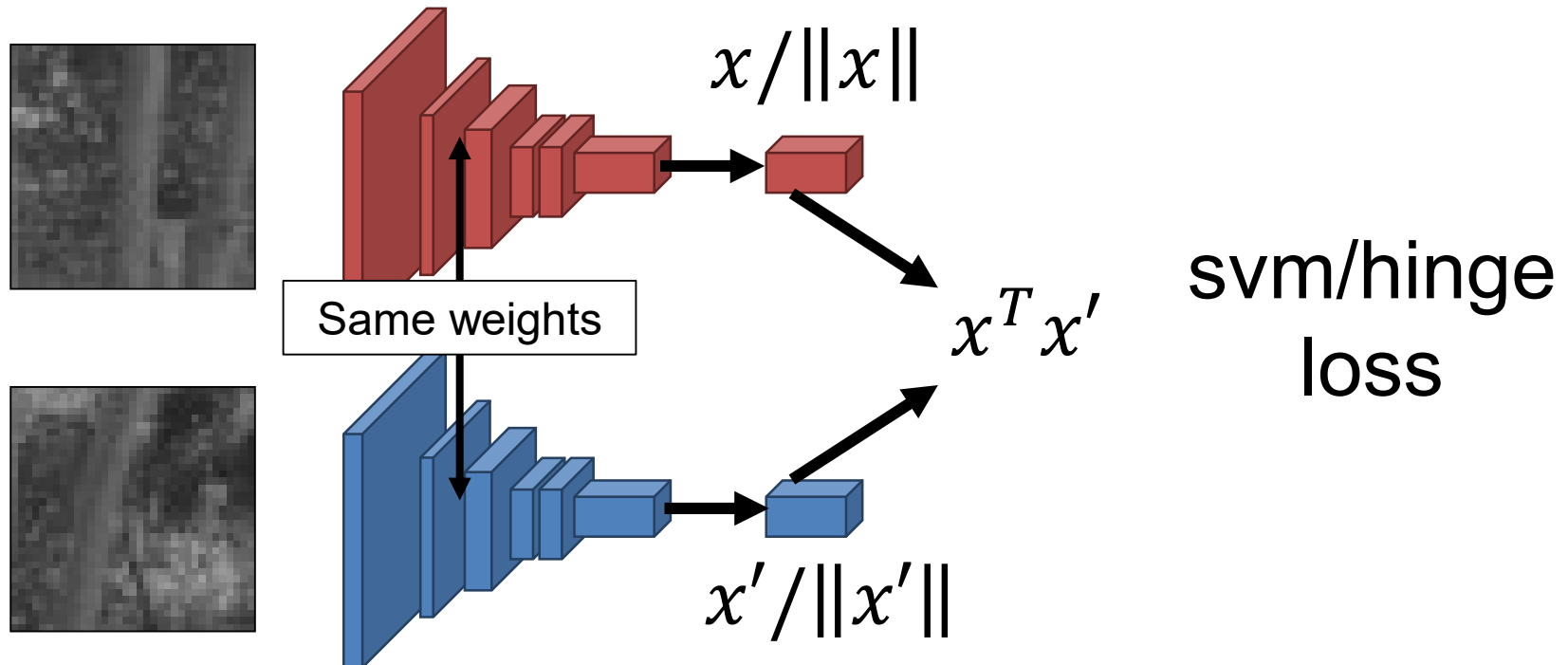
# Deep Learning For Stereo

- Feed in two images to identical networks, concatenate outputs, learn multilayer perception
- Slow: **why?**



# Deep Learning For Stereo

- Normalize outputs; treat dot product as prediction of match/no match
- Fast: **why?**

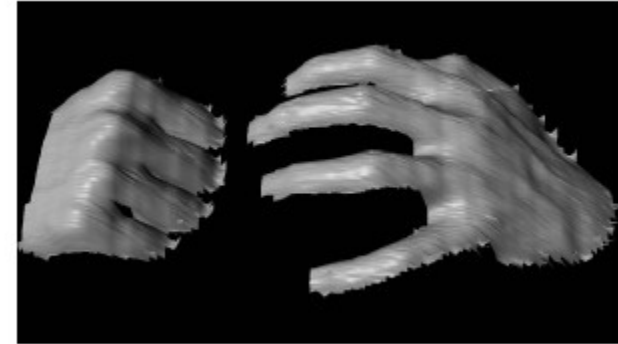
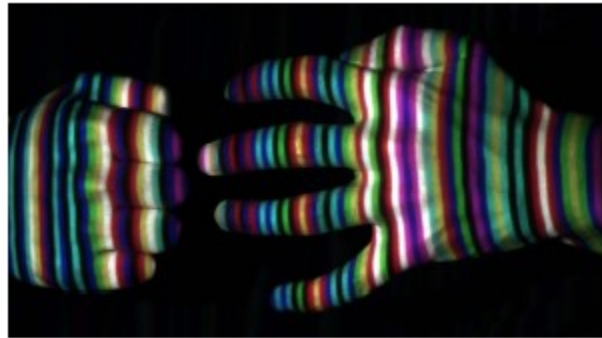


# Stereo datasets

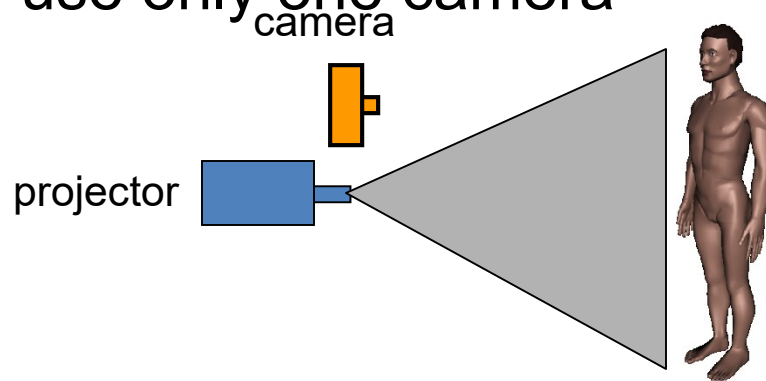
- [Middlebury stereo datasets](#)
- [KITTI](#)
- [Synthetic data?](#)



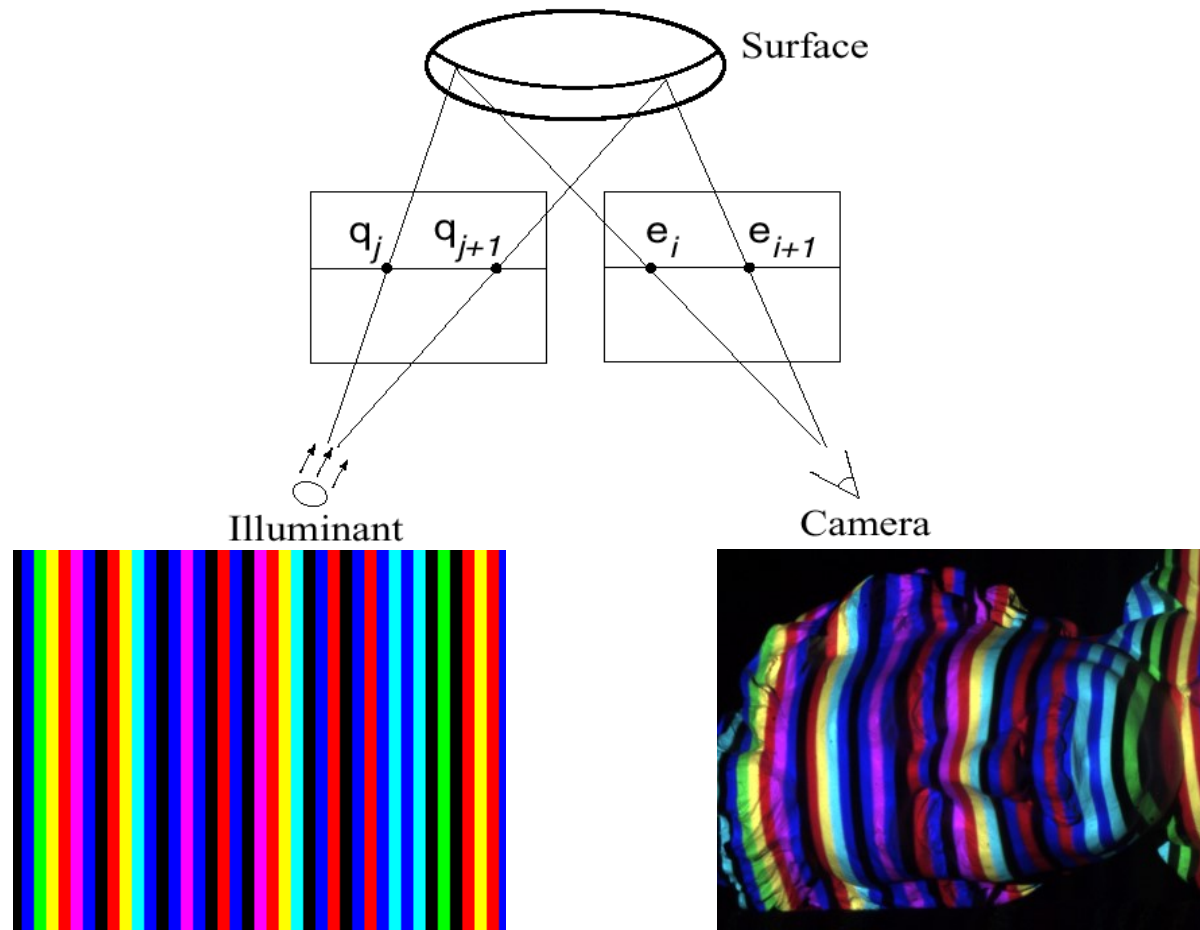
# Active stereo with structured light



- Project “structured” light patterns onto the object
  - Simplifies the correspondence problem
  - Allows us to use only one camera



# Active stereo with structured light



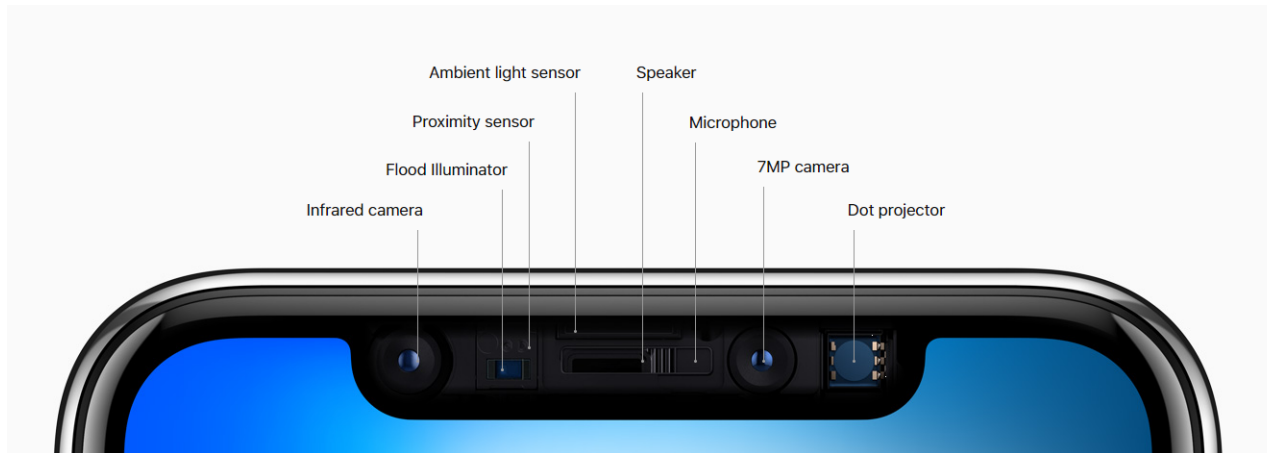
L. Zhang, B. Curless, and S. M. Seitz. [Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming.](#) 3DPVT 2002

Slide credit:  
S. Lazebnik

# Kinect: Structured infrared light



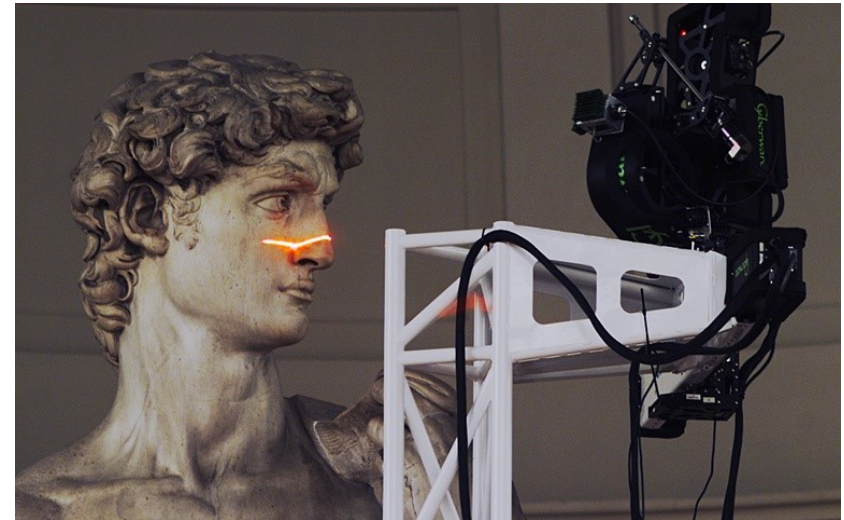
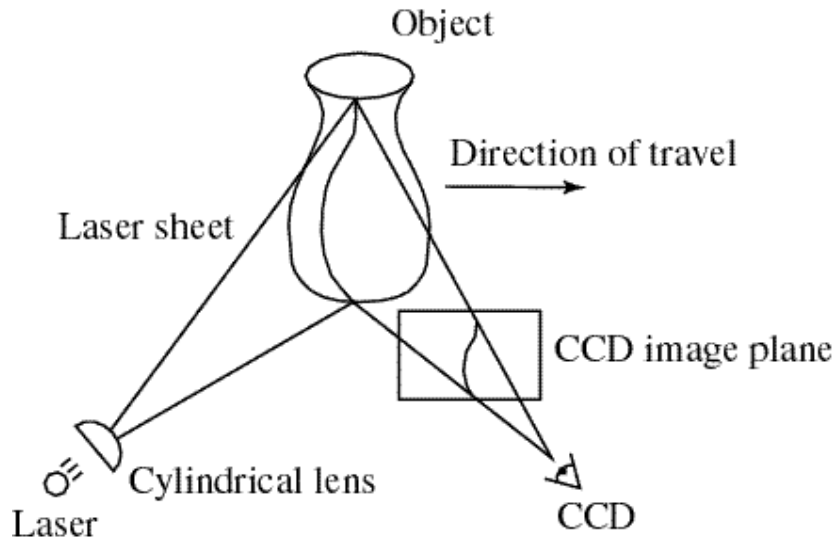
# Apple TrueDepth



<https://www.cnet.com/news/apple-face-id-truedepth-how-it-works/>



# Laser scanning



Digital Michelangelo Project  
Levoy et al.

<http://graphics.stanford.edu/projects/mich/>

- Optical triangulation
  - Project a single stripe of laser light
  - Scan it across the surface of the object
  - This is a very precise version of structured light scanning

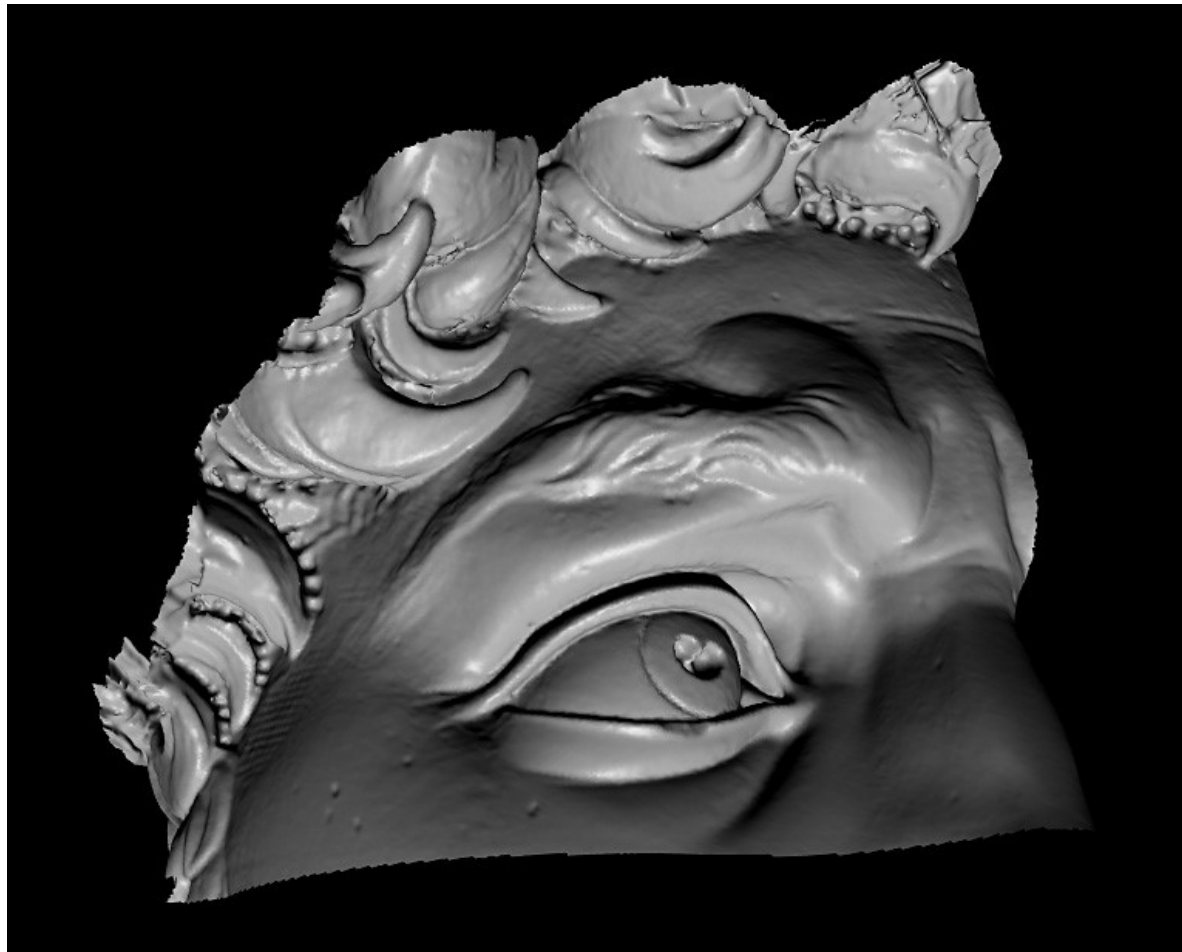


# Laser scanned models



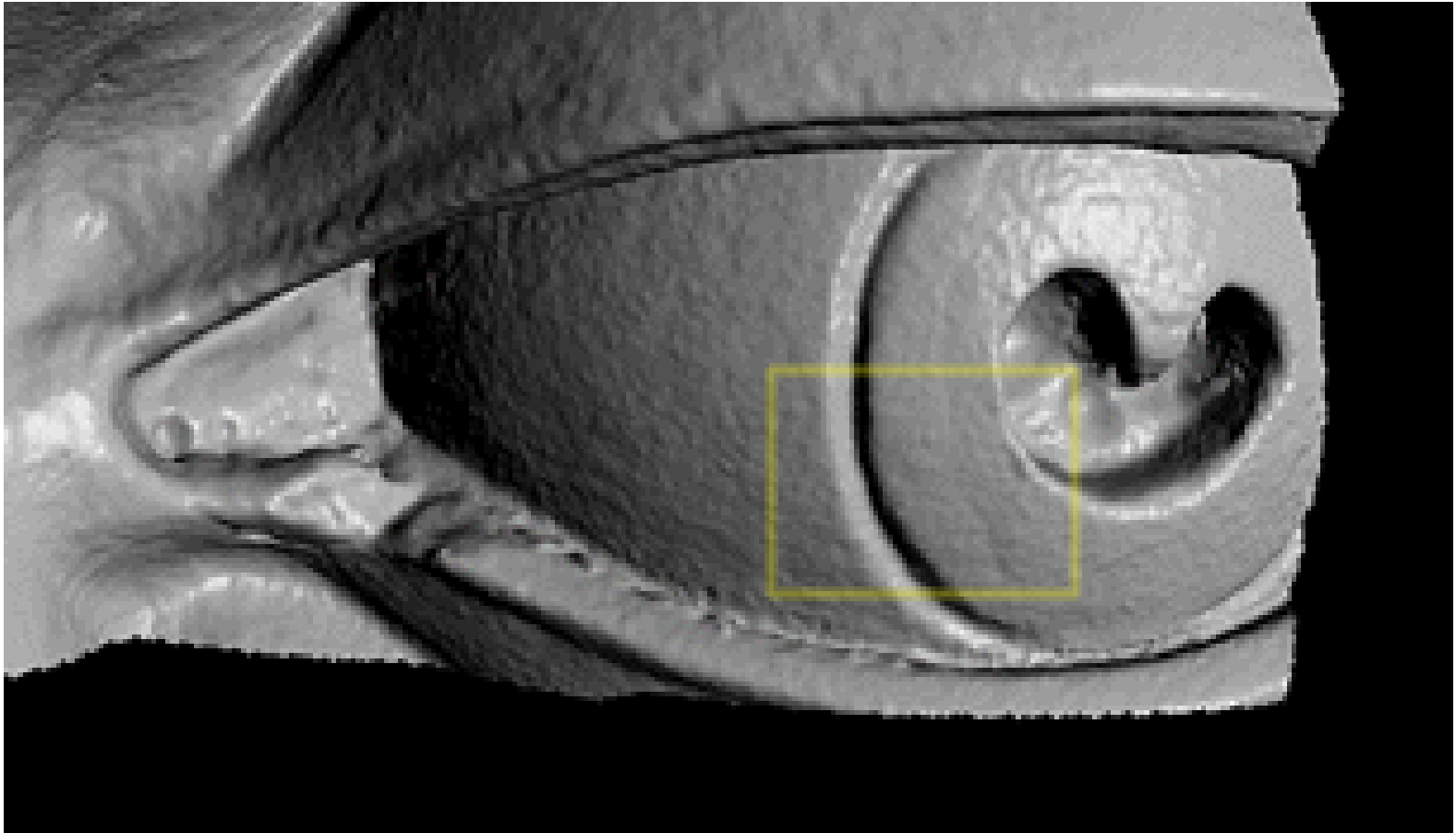
*The Digital Michelangelo Project, Levoy et al.*

# Laser scanned models



*The Digital Michelangelo Project, Levoy et al.*

# Laser scanned models

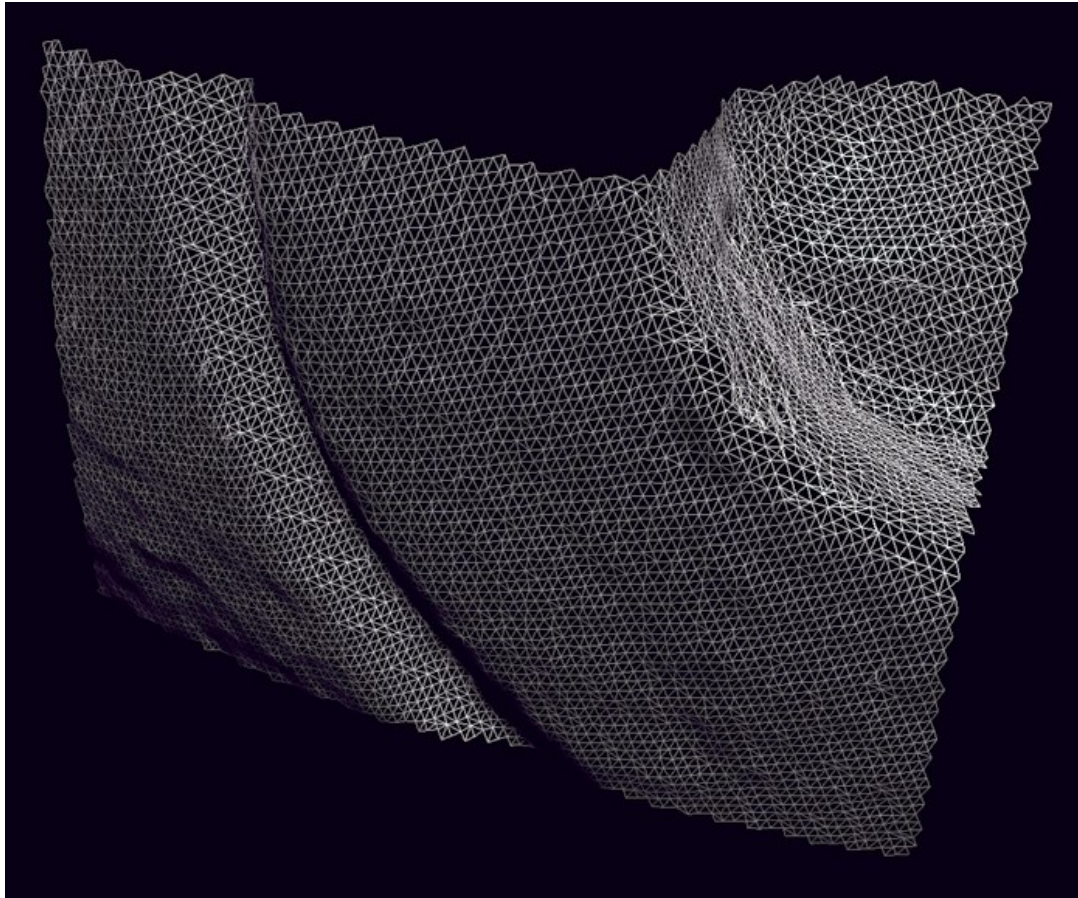


*The Digital Michelangelo Project, Levoy et al.*

Source: S. Seitz

# Laser scanned models

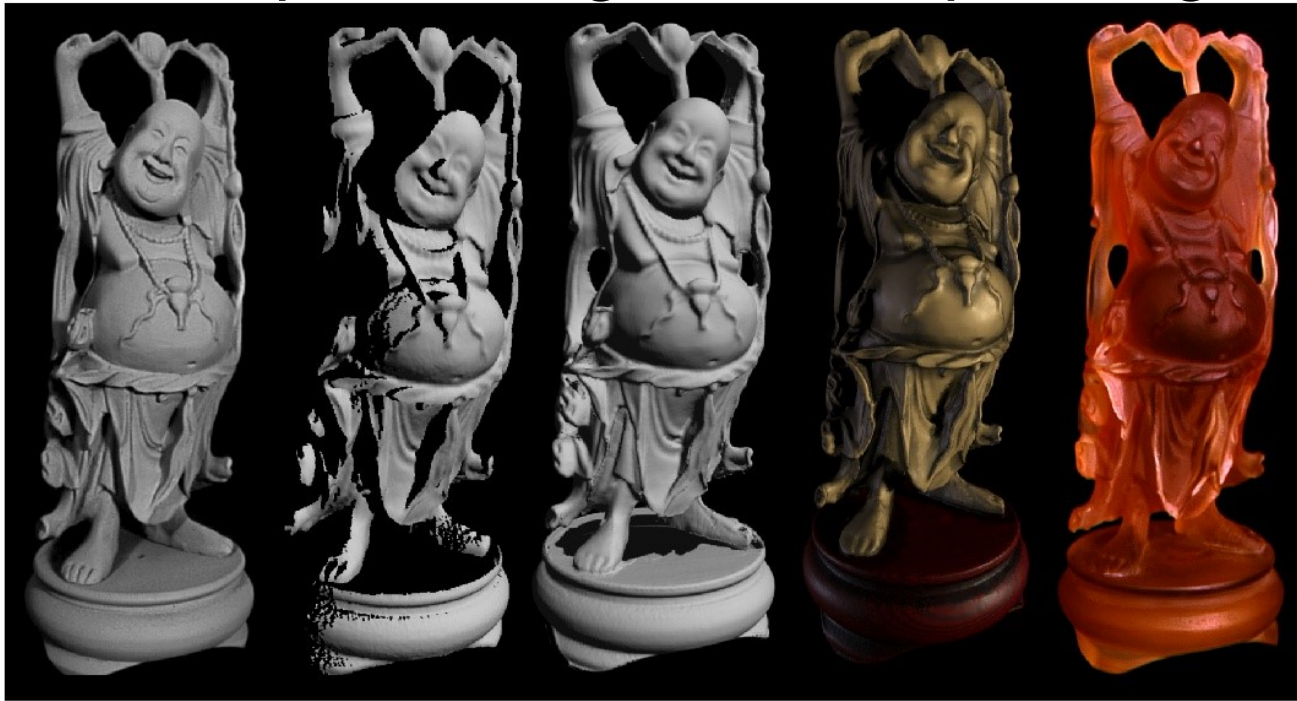
1.0 mm resolution (56 million triangles)



*The Digital Michelangelo Project, Levoy et al.*

# Aligning range images

- One range scan not enough for complex surfaces
- Need techniques to register multiple range images



B. Curless and M. Levoy, [A Volumetric Method for Building Complex Models from Range Images](#), SIGGRAPH 1996