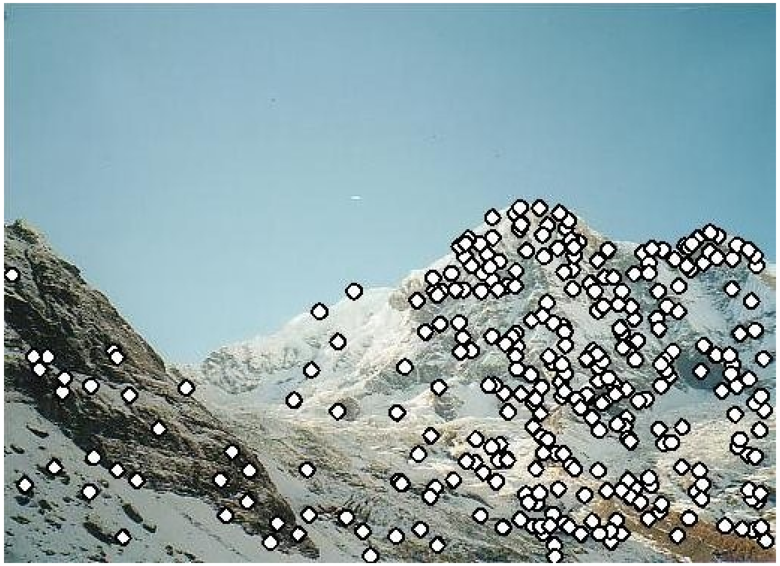
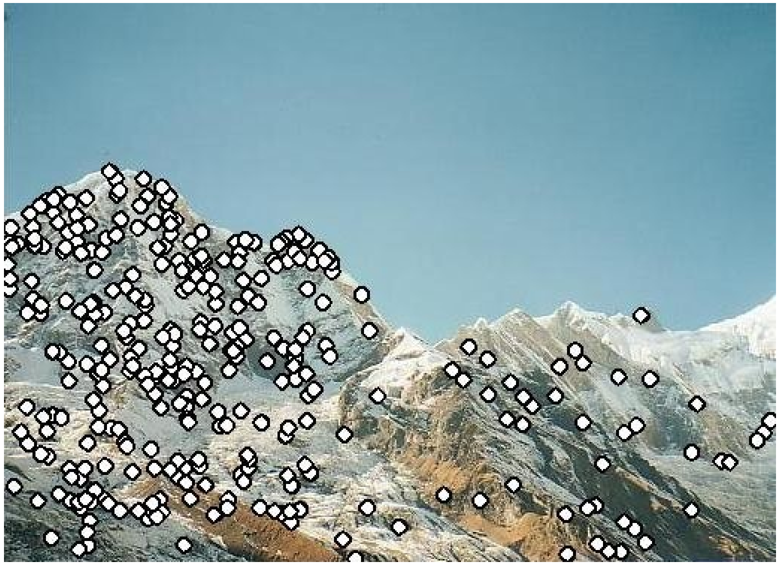
Scales and Descriptors

EECS 442 – David Fouhey
Fall 2019, University of Michigan

http://web.eecs.umich.edu/~fouhey/teaching/EECS442_F19/

Recap: Motivation



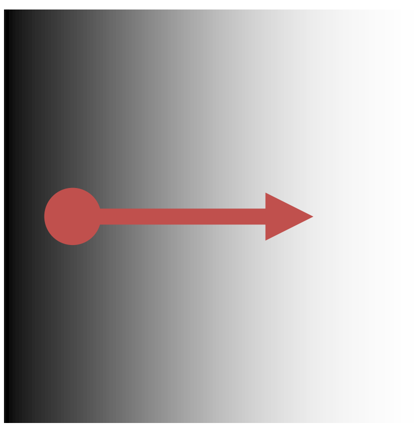


1: find corners+features

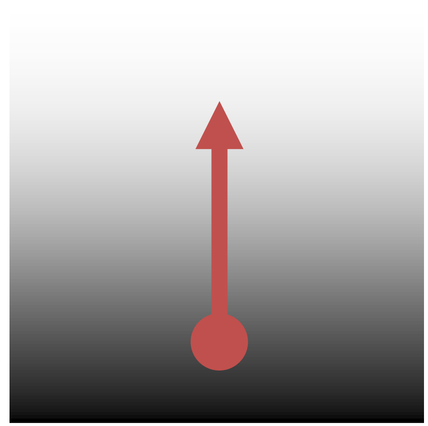
Image credit: M. Brown

Last Time

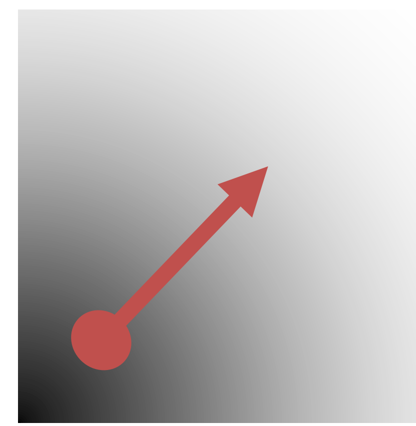
Image gradients – treat image like function of x,y – gives edges, corners, etc.



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$

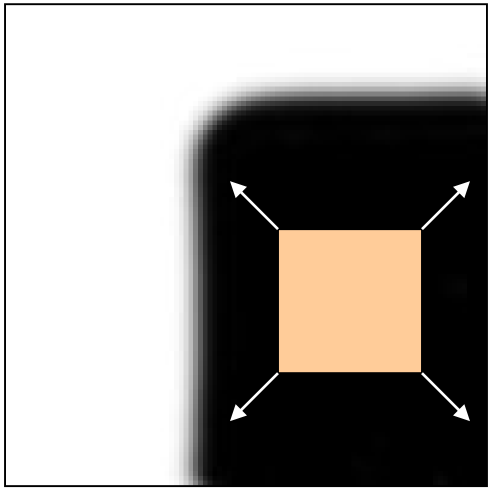


$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

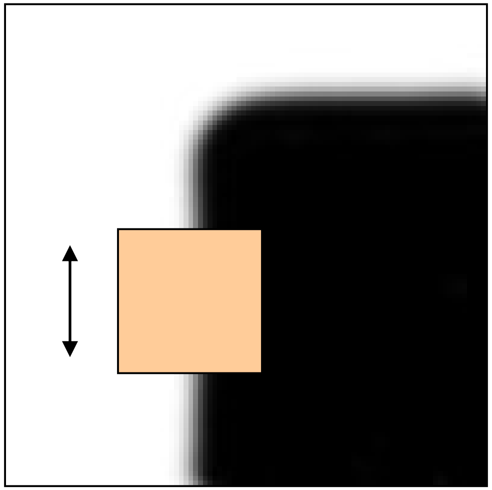
Figure credit: S. Seitz

Last Time – Corner Detection

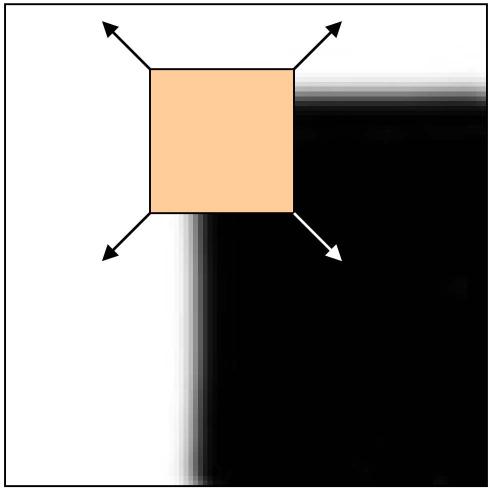
Can localize the location, or any shift → big intensity change.



"flat" region: no change in all directions



"edge": no change along the edge direction



"corner": significant change in all directions

Diagram credit: S. Lazebnik

Corner Detection

By doing a taylor expansion of the image, the second moment matrix tells us how quickly the image changes and in which directions.

Can compute at each pixel

Directions

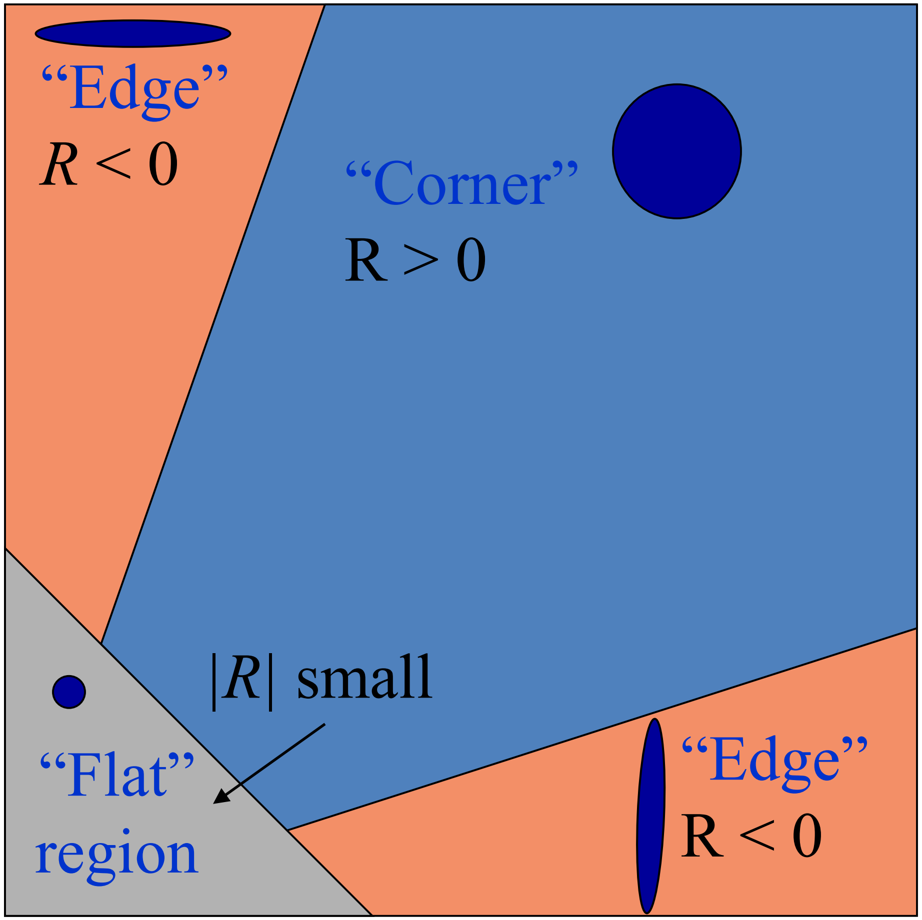
$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \boldsymbol{R}^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{R}$$

Amounts

Putting Together The Eigenvalues

$$R = \det(\boldsymbol{M}) - \alpha\, trace(\boldsymbol{M})^2$$
$$= \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)



Slide credit: S. Lazebnik; Note: this refers to visualization ellipses, not original M ellipse. Other slides on the internet may vary

In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} w(x,y) I_x^2 & \sum_{x,y \in W} w(x,y) I_x I_y \\ \sum_{x,y \in W} w(x,y) I_x I_y & \sum_{x,y \in W} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens. ["A Combined Corner and Edge Detector."] *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Slide credit: S. Lazebnik

In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R

$$R = \det(\boldsymbol{M}) - \alpha\, trace(\boldsymbol{M})^2$$
$$= \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

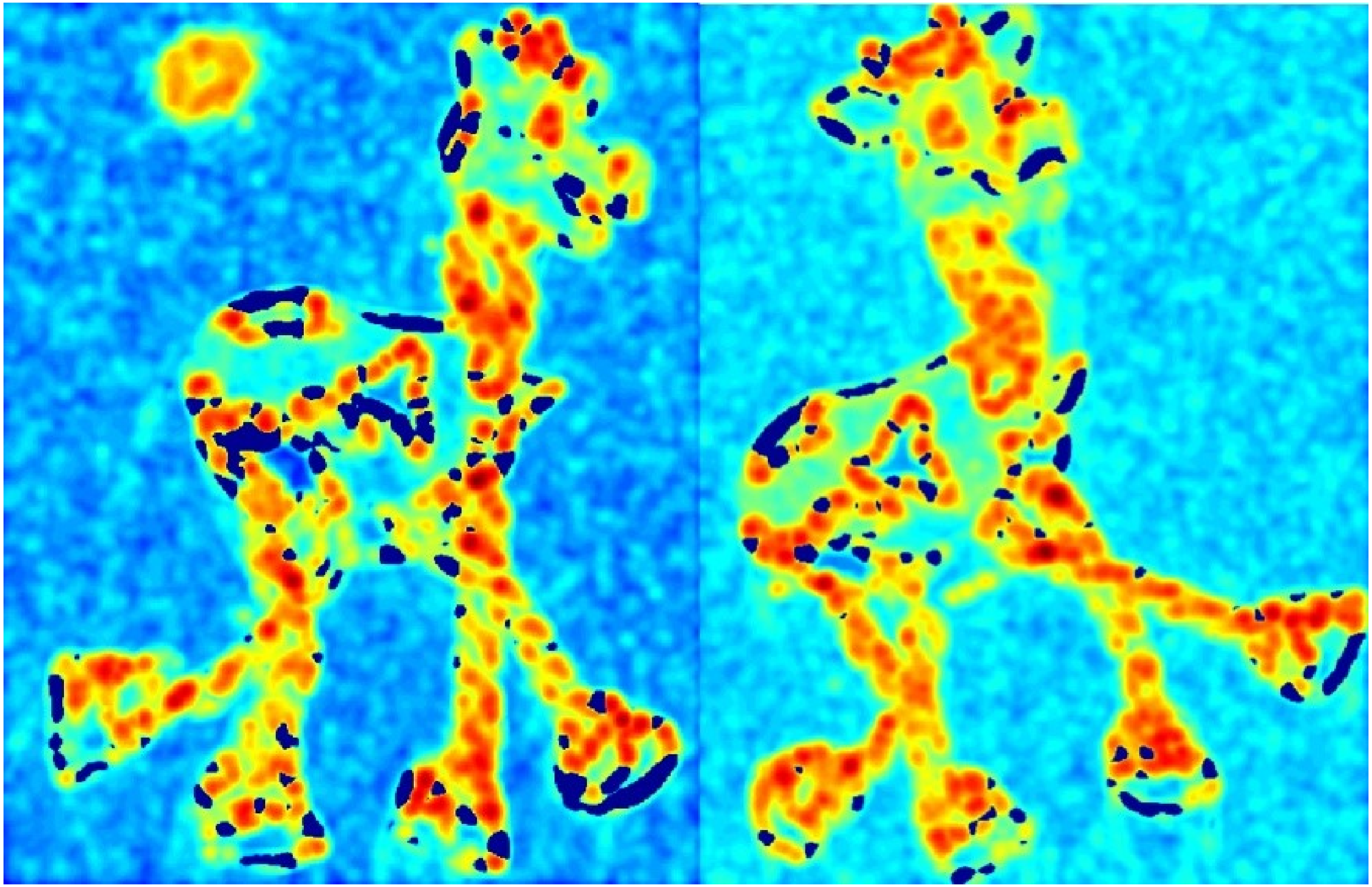
C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Slide credit: S. Lazebnik

Computing R

Slide credit: S. Lazebnik

Computing R



Slide credit: S. Lazebnik

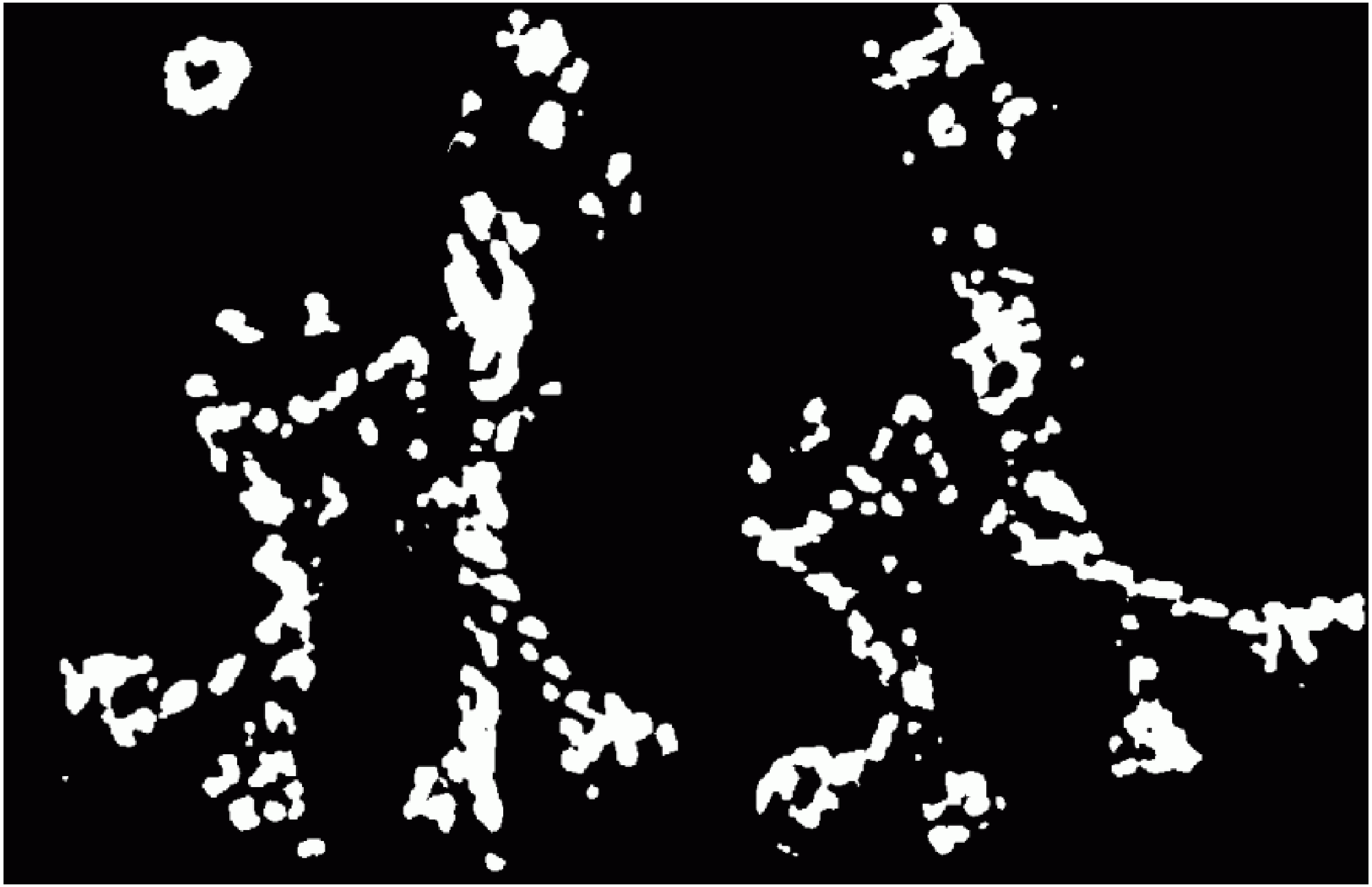
In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R
4. Threshold R

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Slide credit: S. Lazebnik

Thresholded R



Slide credit: S. Lazebnik

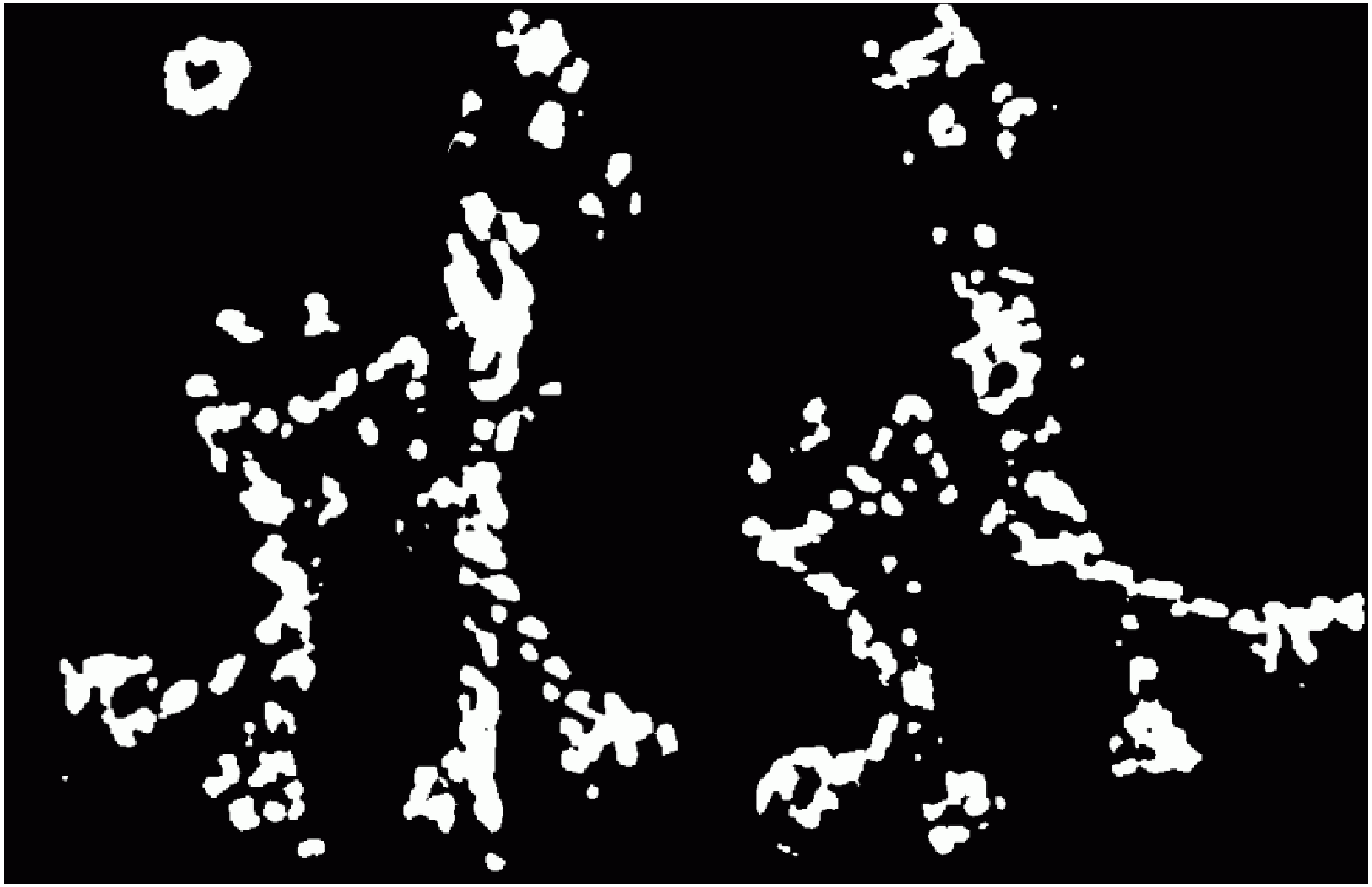
In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R
4. Threshold R
5. Take only local maxima (called non-maxima suppression)

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Slide credit: S. Lazebnik

Thresholded



Slide credit: S. Lazebnik

Final Results

Slide credit: S. Lazebnik

Desirable Properties

If our detectors are repeatable, they should be:

- **Invariant** to some things: image is transformed and corners remain the same
- **Covariant/equivariant** with some things: image is transformed and corners transform with it.

Slide credit: S. Lazebnik

Recall Motivating Problem

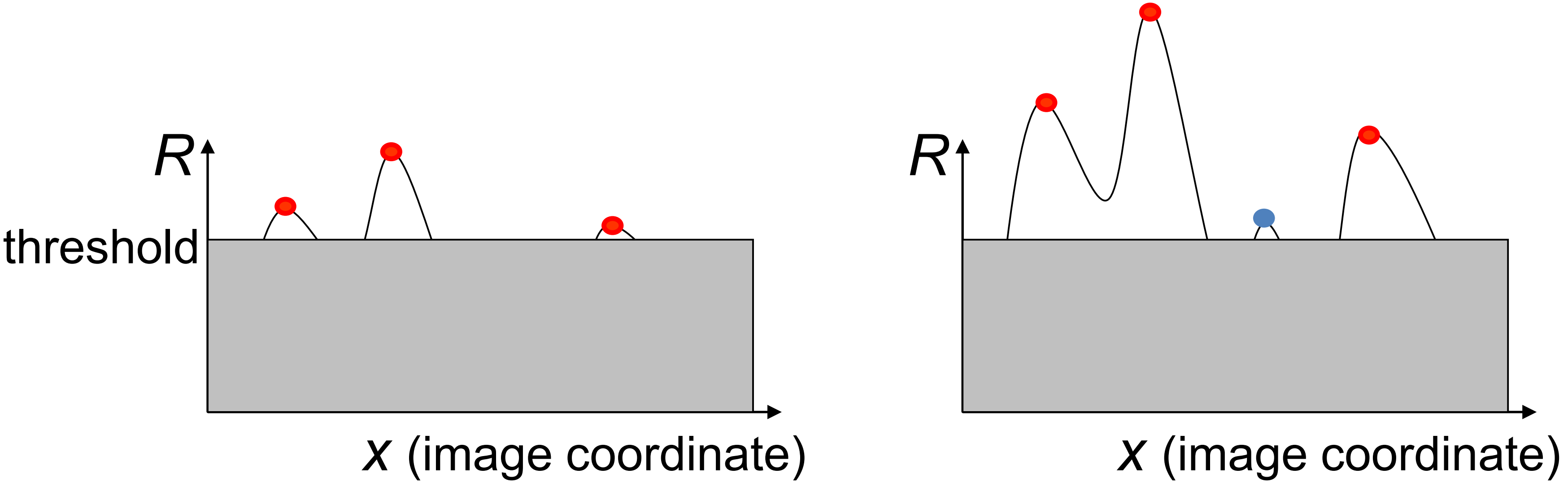
Images may be different in lighting and geometry

Affine Intensity Change

$$I_{new} = aI_{old} + b$$

M only depends on derivatives, so *b* is irrelevant

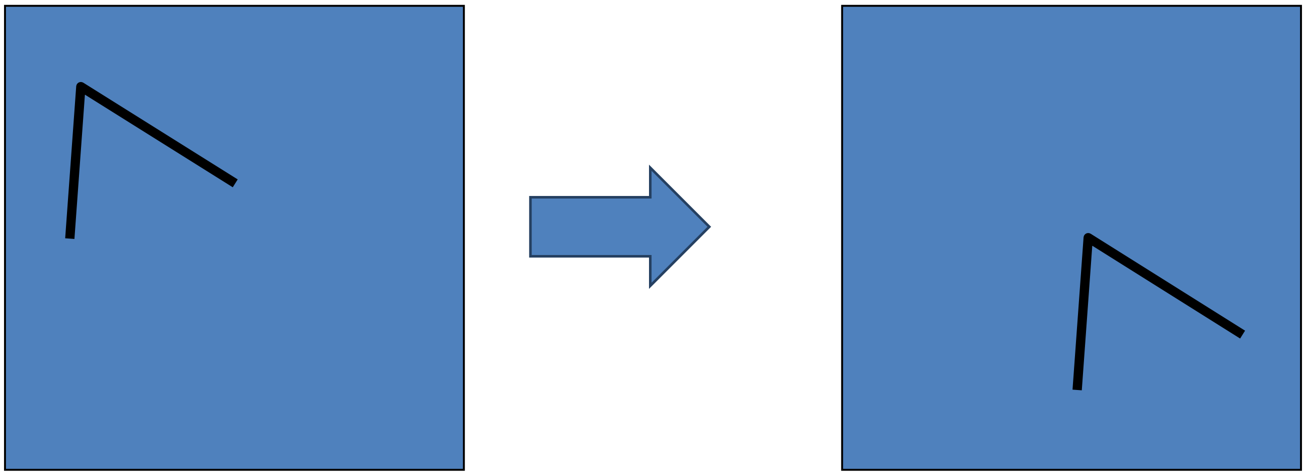
But *a* scales derivatives and there's a threshold



Partially invariant to affine intensity changes

Slide credit: S. Lazebnik

Image Translation

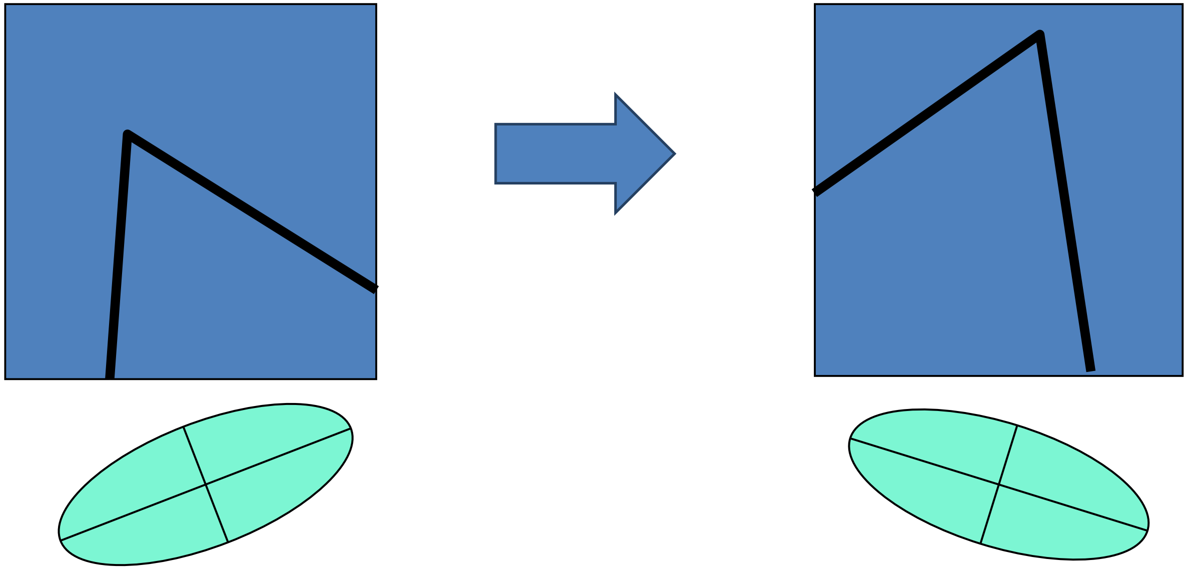


All done with convolution. Convolution is translation equivariant.

Equivariant with translation

Slide credit: S. Lazebnik

Image Rotation

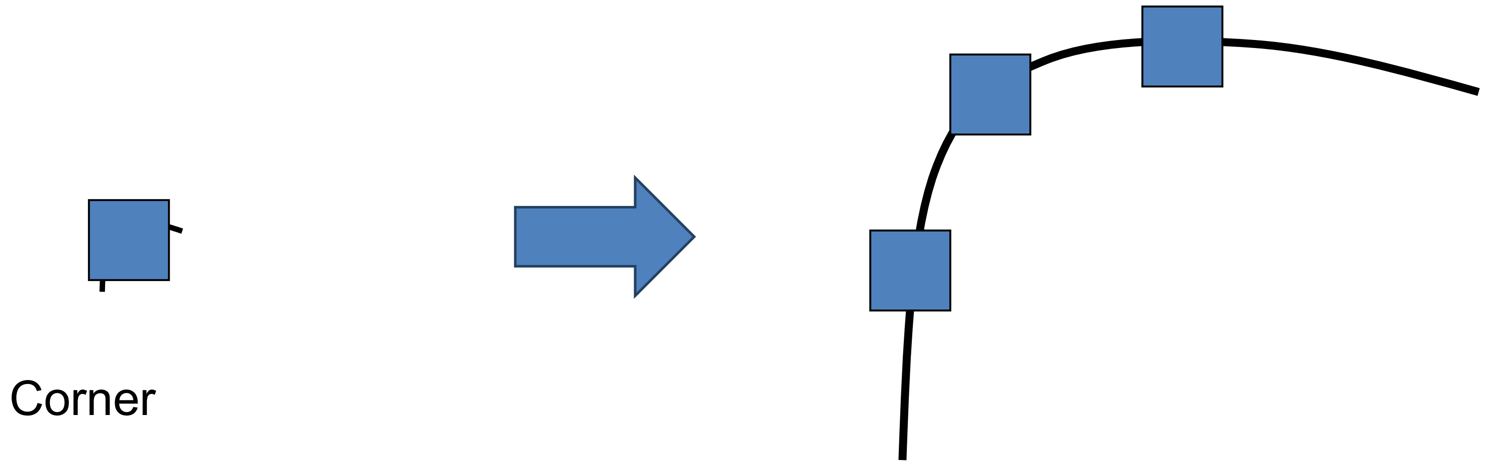


Rotations just cause the corner rotation matrix to change. Eigenvalues remain the same.

Equivariant with rotation

Slide credit: S. Lazebnik

Image Scaling



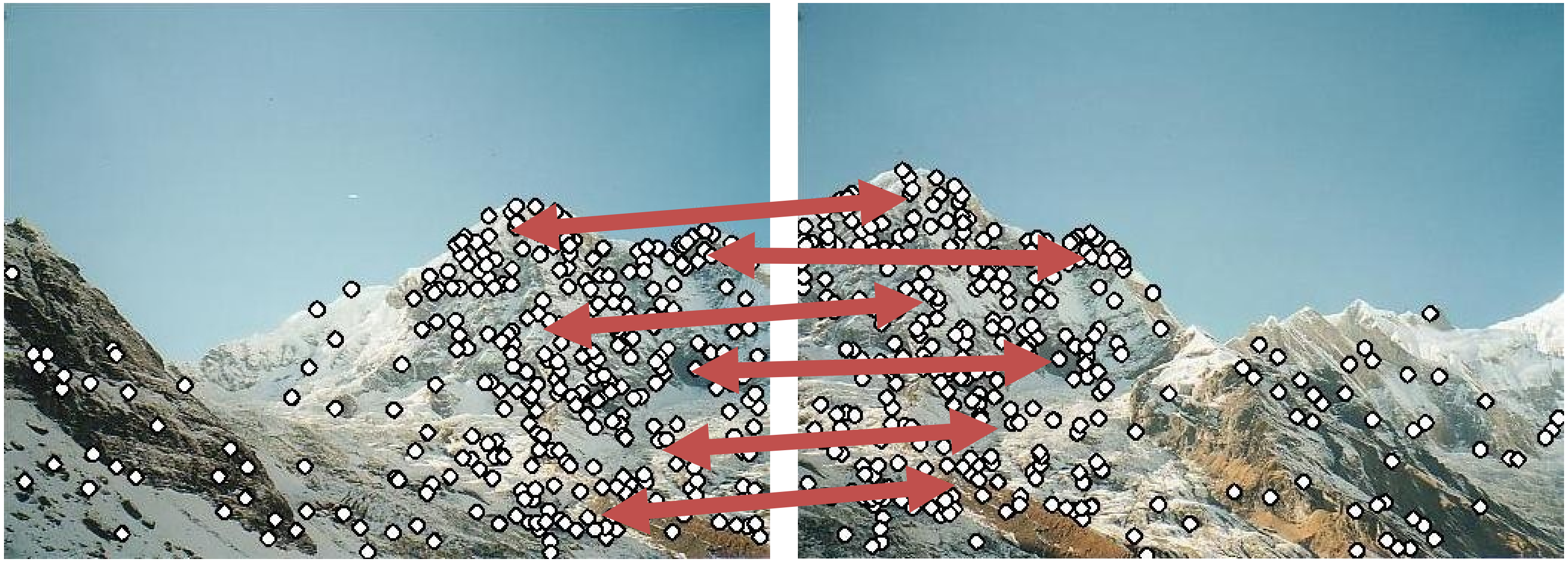
One pixel can become many pixels and vice-versa.

Not equivariant with scaling

How do we fix this?

Slide credit: S. Lazebnik

Recap: Motivation



1: find corners+features

2: match based on local image data

How?

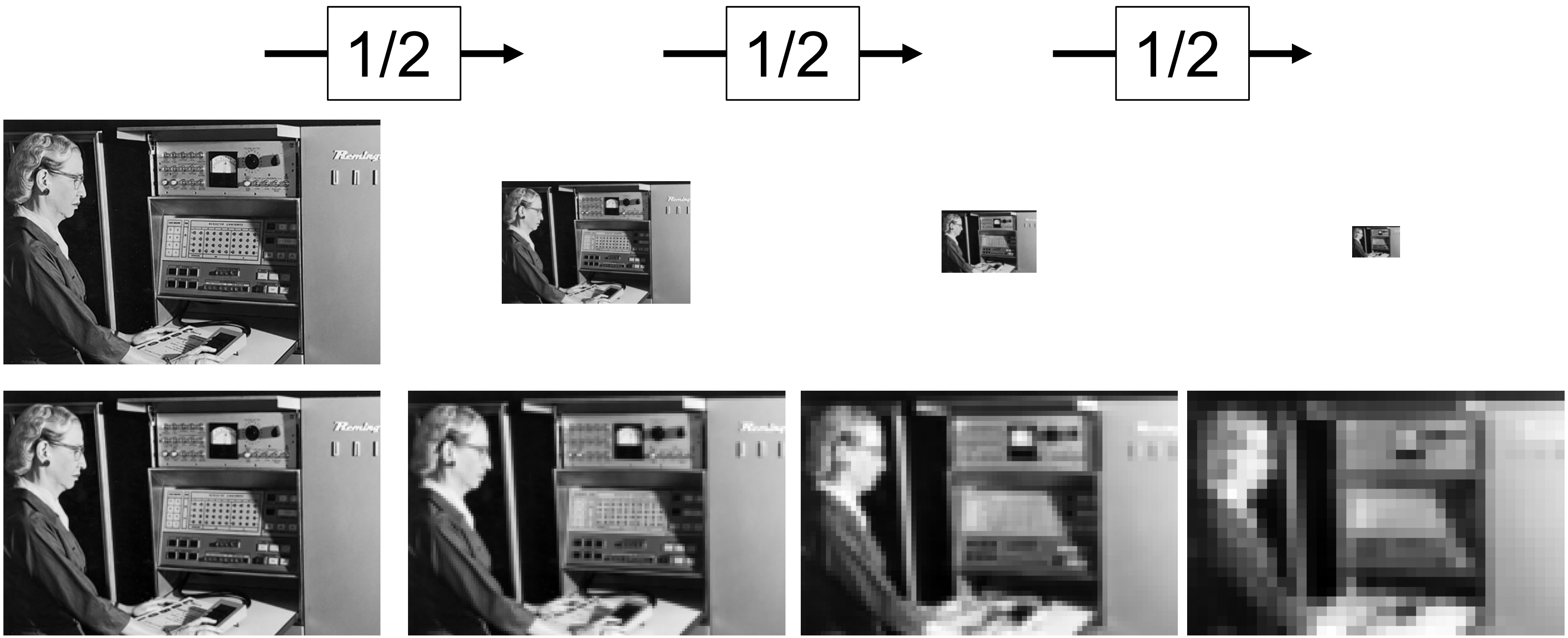
Image credit: M. Brown

Today

- Fixing scaling by making detectors in both location **and scale**
- Enabling matching between features by **describing regions**

Key Idea: Scale

Left to right: each image is half-sized
Upsampled with big pixels below

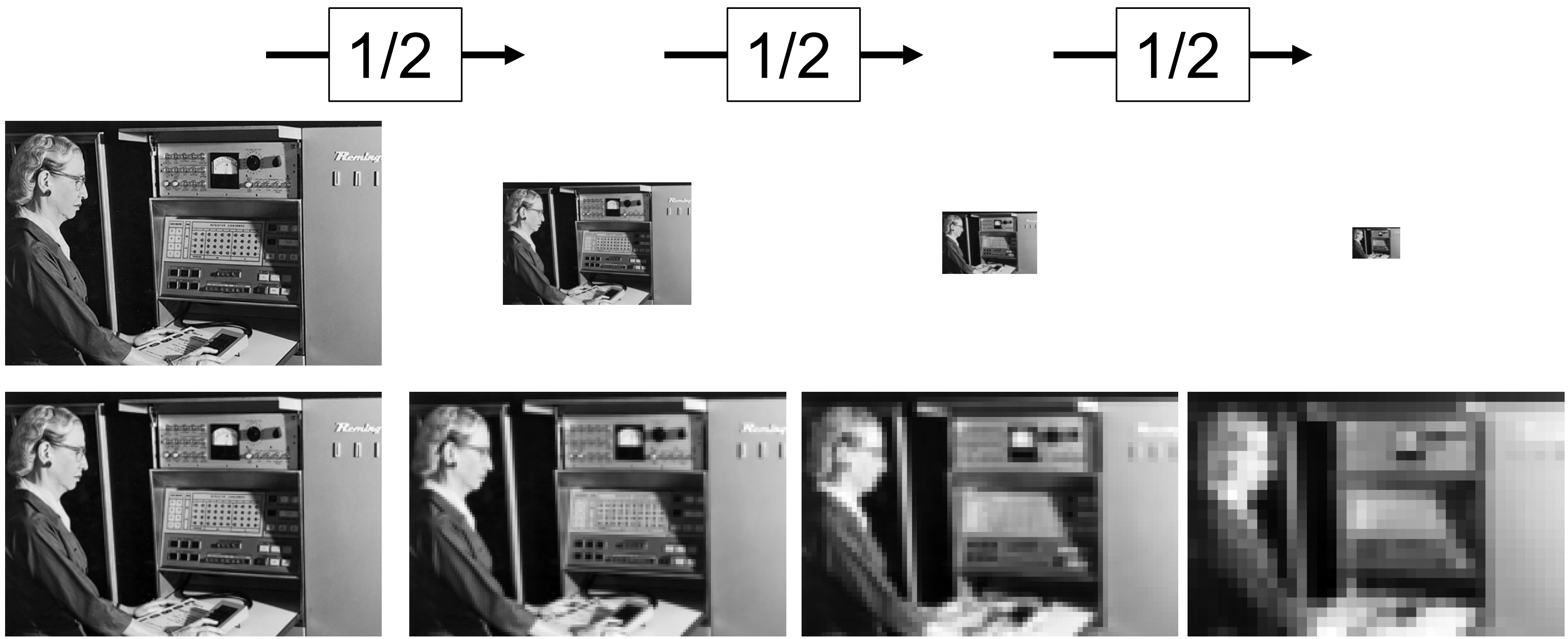


Note: I'm also slightly blurring to prevent aliasing (https://en.wikipedia.org/wiki/Aliasing)

Key Idea: Scale

Left to right: each image is half-sized

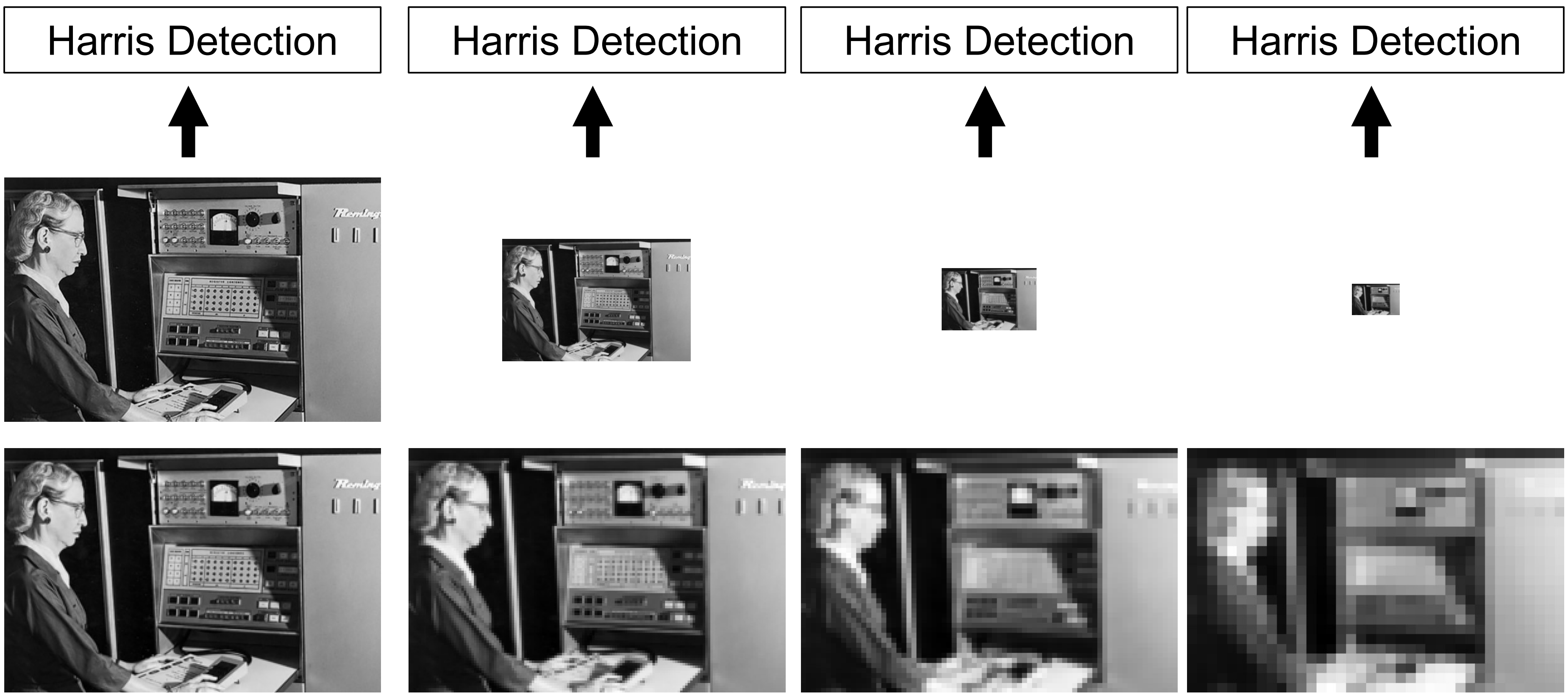
If I apply a KxK filter, how much of the original image does it see in each image?



Note: I’m also slightly blurring to prevent aliasing (https://en.wikipedia.org/wiki/Aliasing)

Solution to Scales

Try them all!



See: Multi-Image Matching using Multi-Scale Oriented Patches, Brown et al. CVPR 2005

Aside: This Trick is Common

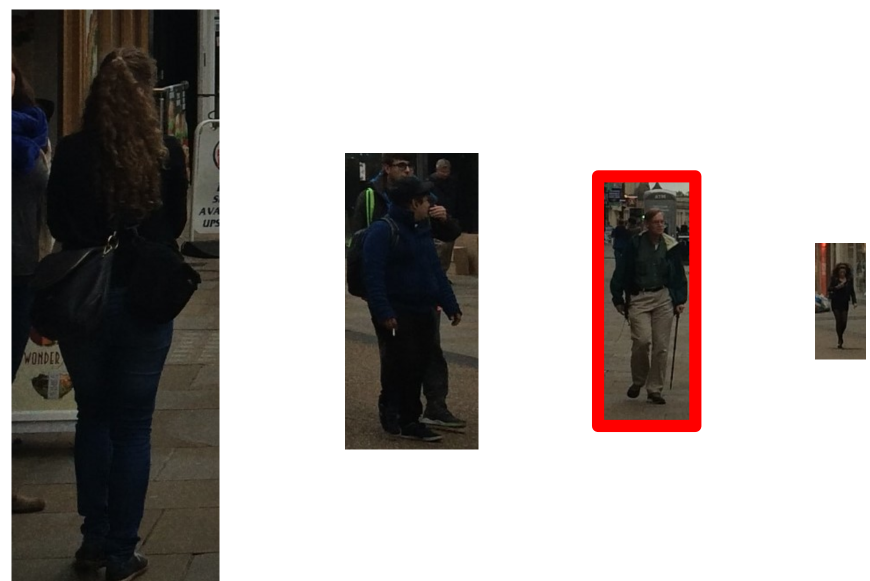
Given a 50x16 person detector, how do I detect:
(a) 250x80 (b) 150x48 (c) 100x32 (d) 25x8 people?

Sample people from image

Aside: This Trick is Common

Detecting all the people
The red box is a fixed size

Sample people from image



Aside: This Trick is Common

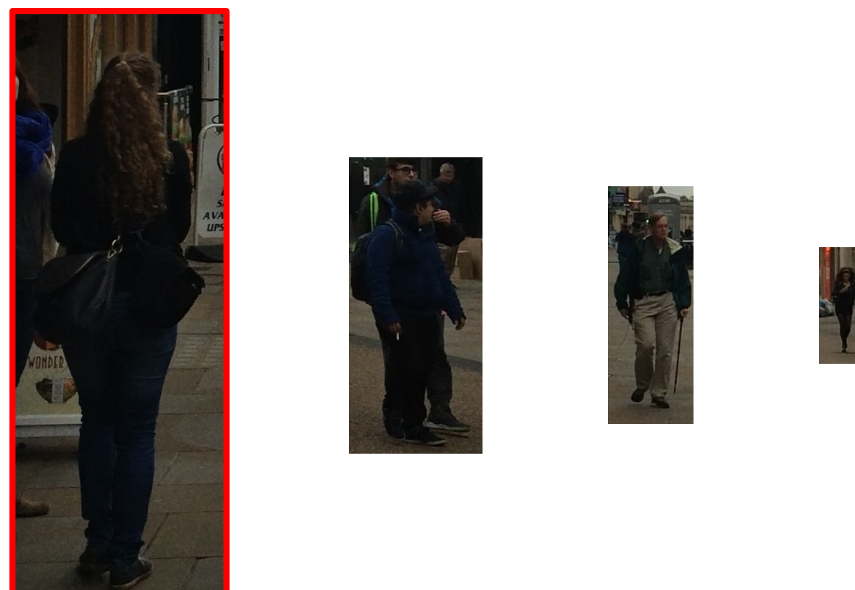
Detecting all the people
The red box is a fixed size

Sample people from image

Aside: This Trick is Common

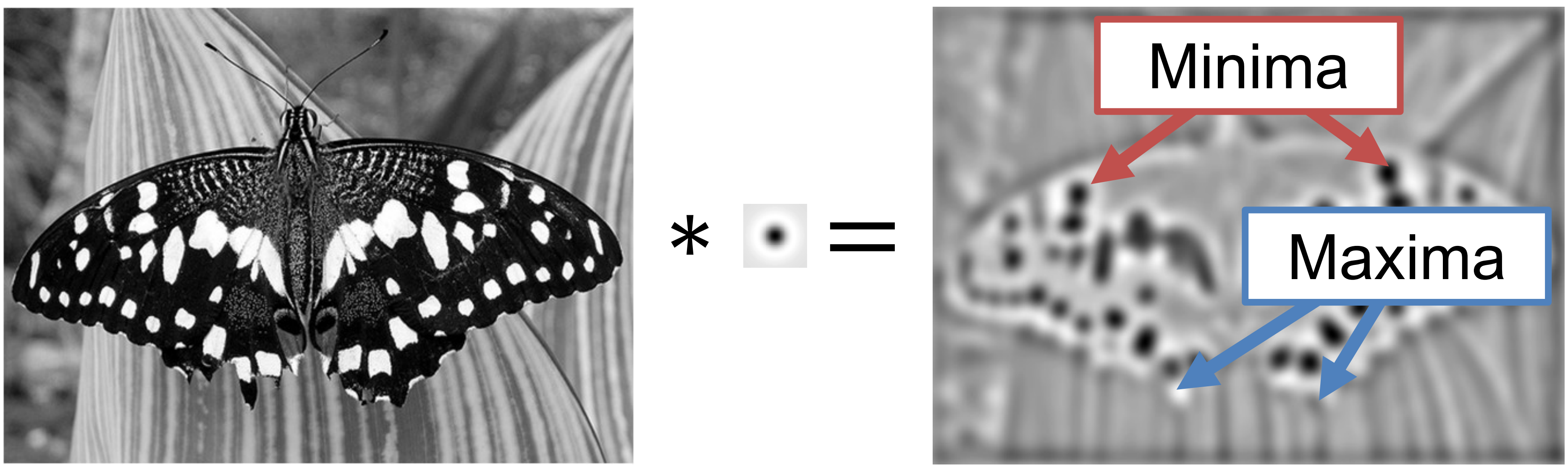
Detecting all the people
The red box is a fixed size

Sample people from image



Blob Detection

Another detector (has some nice properties)



Find maxima ***and minima*** of blob filter response in scale ***and space***

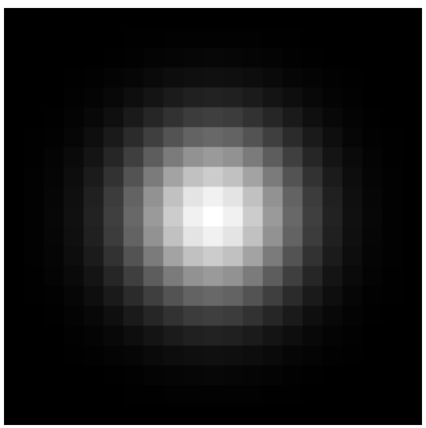
Slide credit: N. Snavely

Gaussian Derivatives

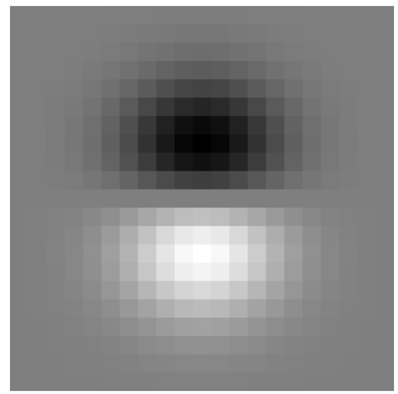
1st Deriv

2nd Deriv

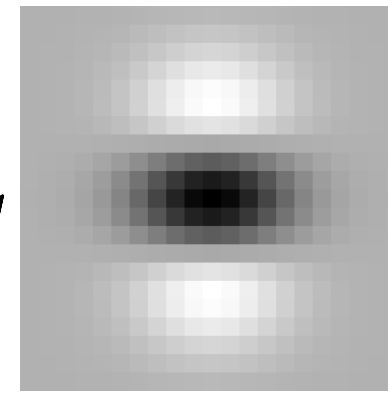
Gaussian



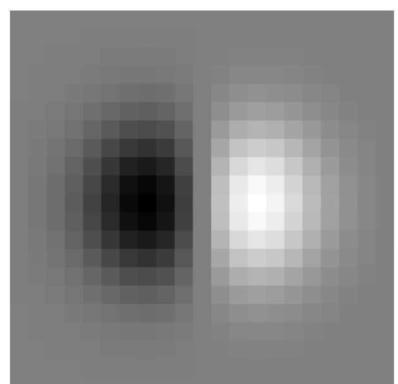
$$\frac{\partial}{\partial y} g$$



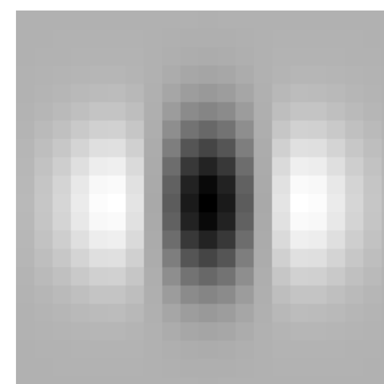
$$\frac{\partial^2}{\partial^2 y} g$$



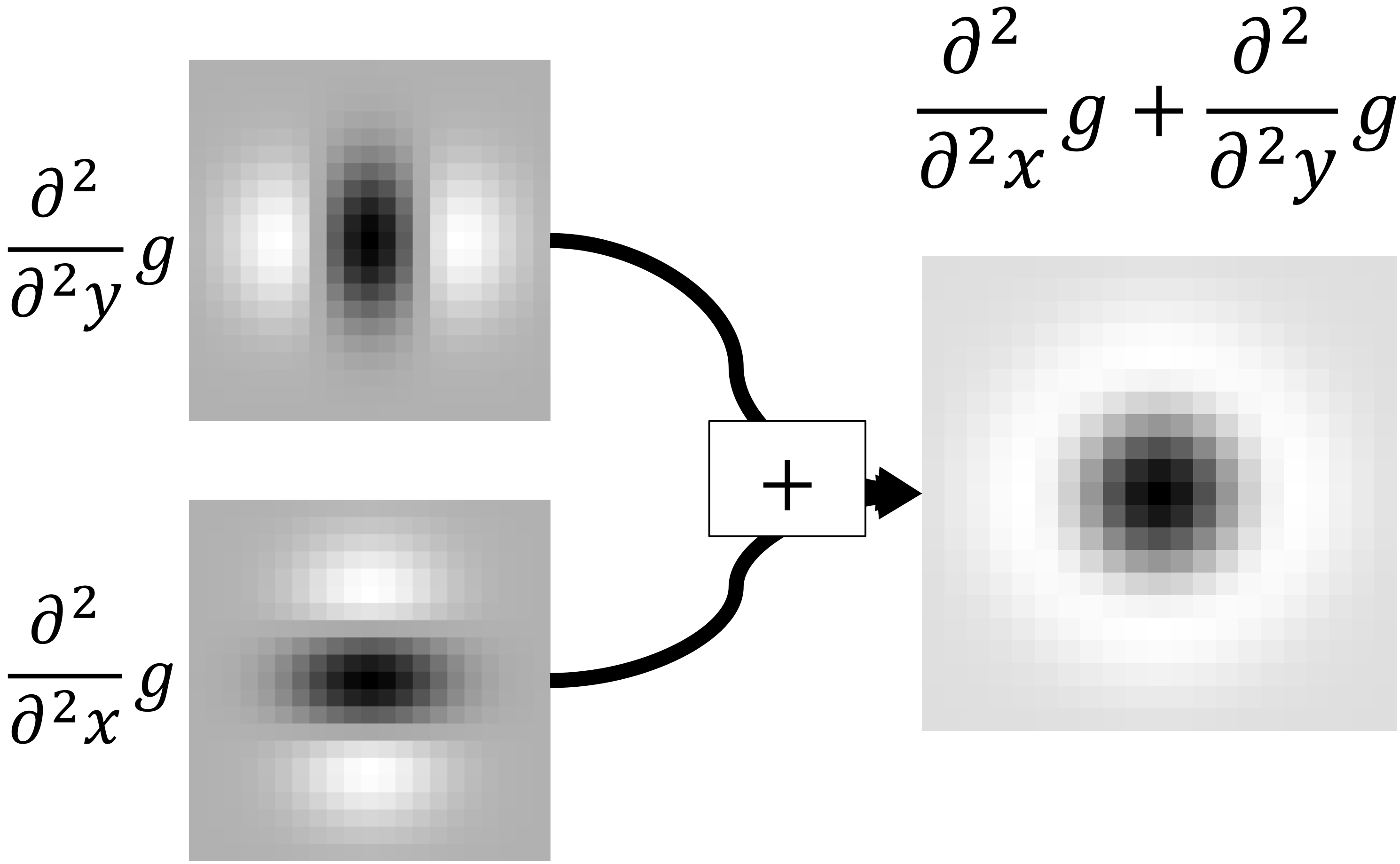
$$\frac{\partial}{\partial x} g$$



$$\frac{\partial^2}{\partial^2 x} g$$



Laplacian of Gaussian



Slight detail: for technical reasons, you need to scale the Laplacian.

$$\nabla^2_{norm} = \sigma^2 \left(\frac{\partial^2}{\partial x^2} g + \frac{\partial^2}{\partial^2 y} g \right)$$

Edge Detection with Laplacian

f — Edge

$\frac{\partial^2}{\partial^2 x} g$ — Laplacian Of Gaussian

$f * \frac{\partial^2}{\partial^2 x} g$ — Edge = Zero-crossing

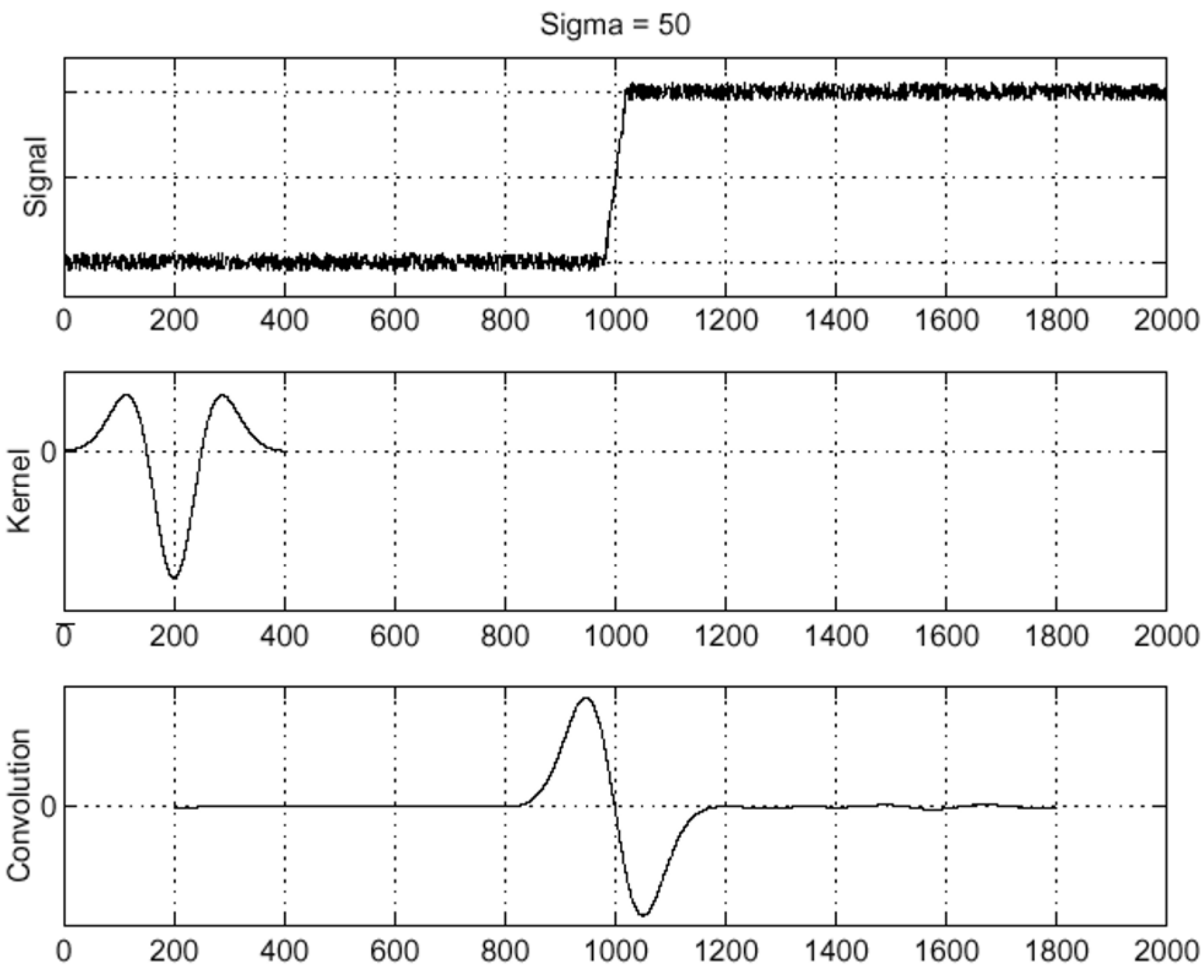


Figure credit: S. Seitz

Blob Detection with Laplacian

Edge: zero-crossing

Blob: superposition of zero-crossing

Remember: can scale signal or filter

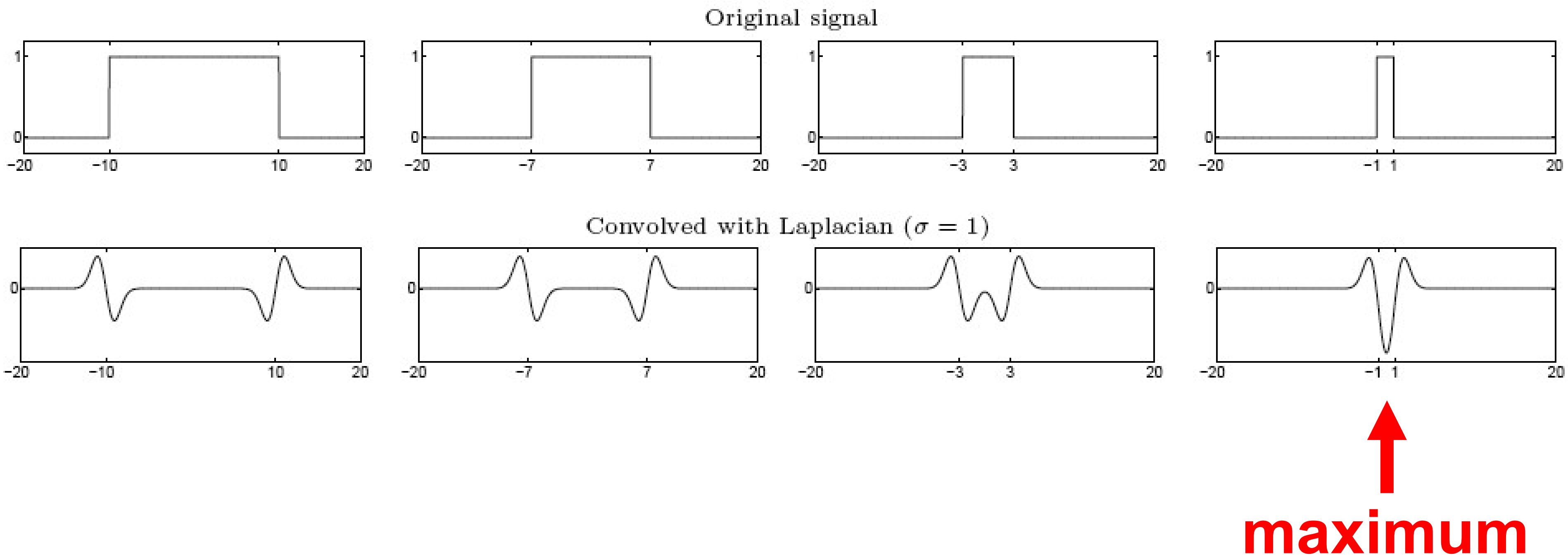
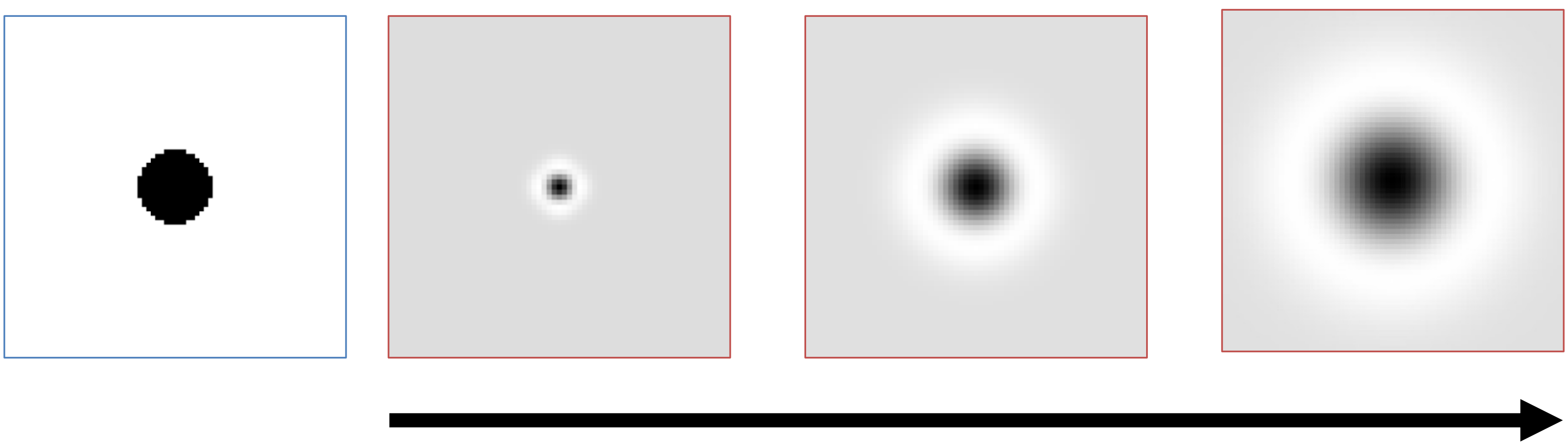


Figure credit: S. Lazebnik

Scale Selection

Given binary circle and Laplacian filter of scale σ, we can compute the response as a function of the scale.

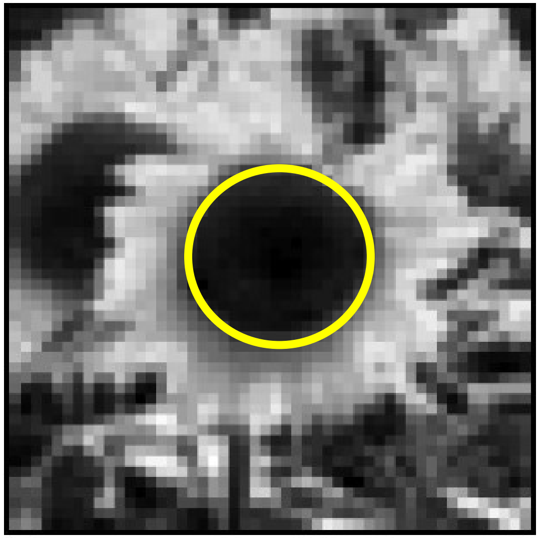
Image	$\sigma = 2$	$\sigma = 6$	$\sigma = 10$
Radius: 8	R: 0.02	R: 2.9	R: 1.8



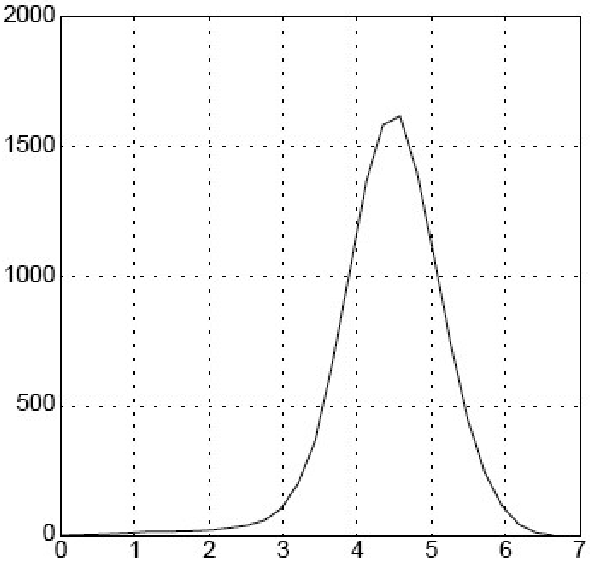
Characteristic Scale

Characteristic scale of a blob is the scale that produces the maximum response

Image



Abs. Response



Slide credit: S. Lazebnik. For more, see: T. Lindeberg (1998). "Feature detection with automatic scale selection." *International Journal of Computer Vision* **30** (2): pp 77--116.

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales

Slide credit: S. Lazebnik

Scale-space blob detector: Example

Slide credit: S. Lazebnik

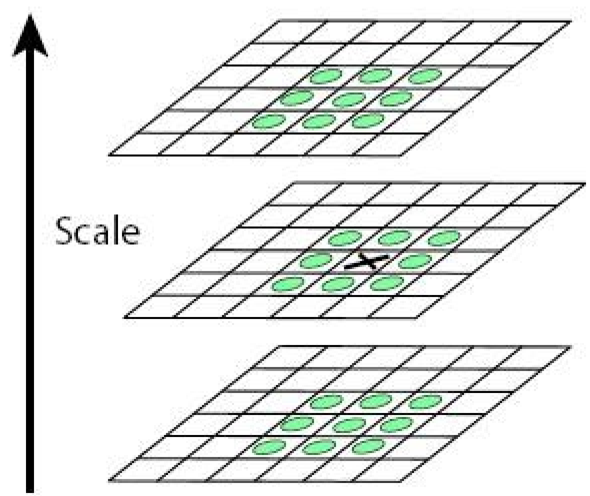
Scale-space blob detector: Example

sigma = 11.9912

Slide credit: S. Lazebnik

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales
2. Find maxima of squared Laplacian response in scale-space



Slide credit: S. Lazebnik

Finding Maxima

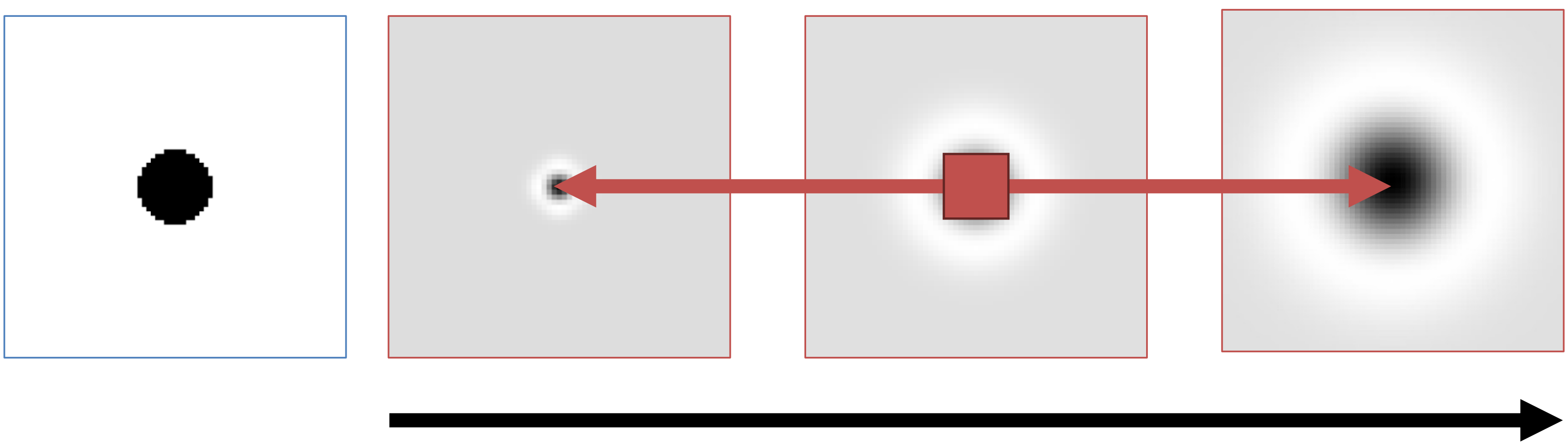
Point i,j is maxima (minima if you flip sign) in image I if:

```
for y=range(i-1,i+1+1):
    for x in range(j-1,j+1+1):
        if y == i and x== j: continue
        #below has to be true
        I[y,x] < I[i,j]
```

Scale Space

Red lines are the scale-space neighbors

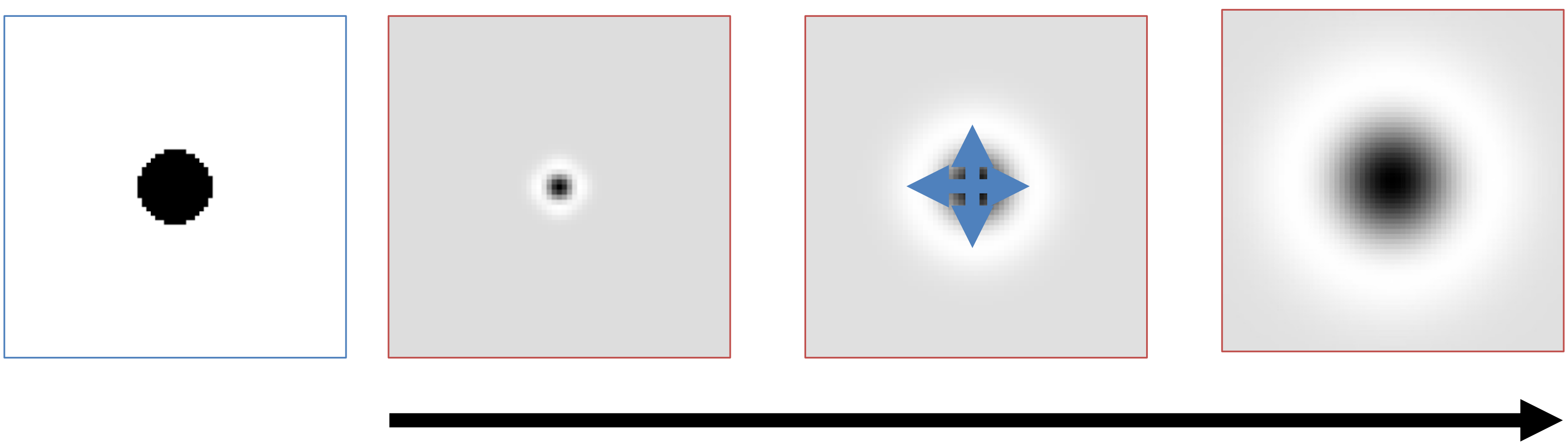
Image	$\sigma = 2$	$\sigma = 6$	$\sigma = 10$
Radius: 8	R: 0.02	R: 2.9	R: 1.8



Scale Space

Blue lines are image-space neighbors (should be just one pixel over but you should get the point)

Image	$\sigma = 2$	$\sigma = 6$	$\sigma = 10$
Radius: 8	R: 0.02	R: 2.9	R: 1.8

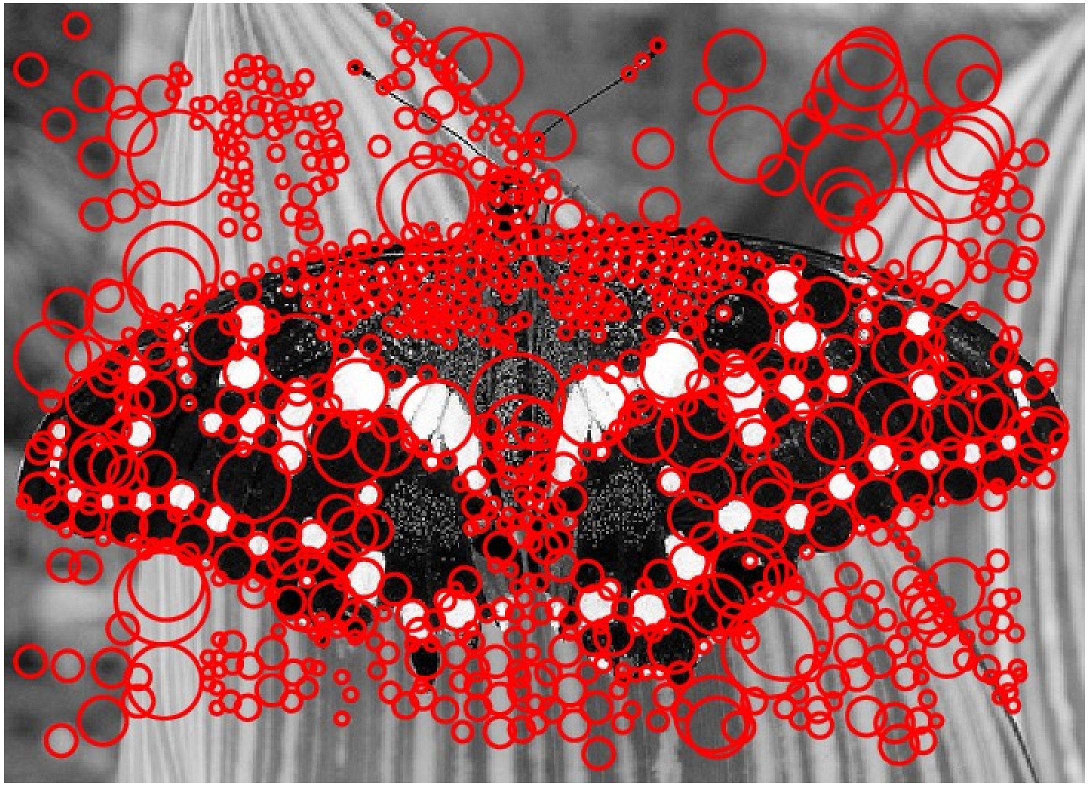


Finding Maxima

Suppose I[:,:,k] is image at scale k. Point i,j,k is maxima (minima if you flip sign) in image I if:

```
for y=range(i-1,i+1+1):
    for x in range(j-1,j+1+1):
        for c in range(k-1,k+1+1):
            if y == i and x== j and c==k:
                continue
        #below has to be true
        I[y,x,c] < I[i,j,k]
```

Scale-space blob detector: Example



Slide credit: S. Lazebnik

Efficient implementation

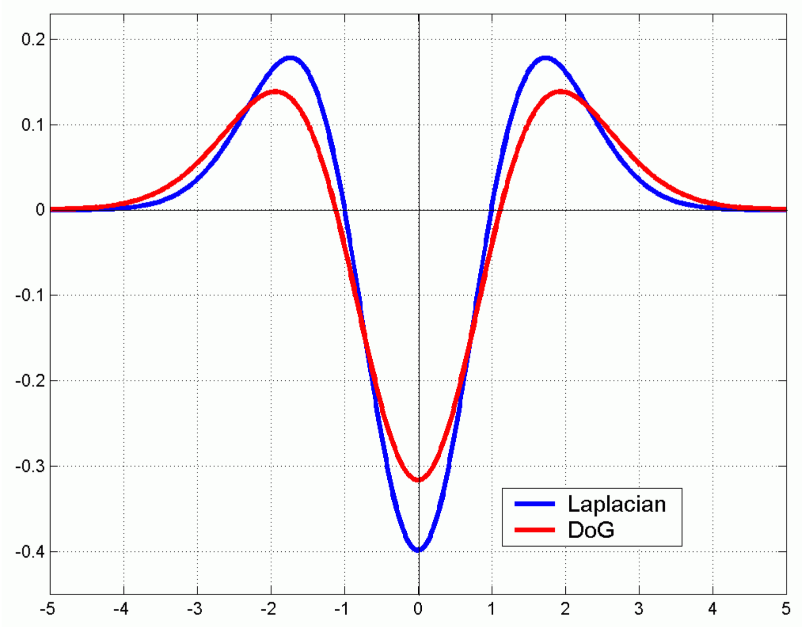
- Approximating the Laplacian with a difference of Gaussians:

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

(Laplacian)

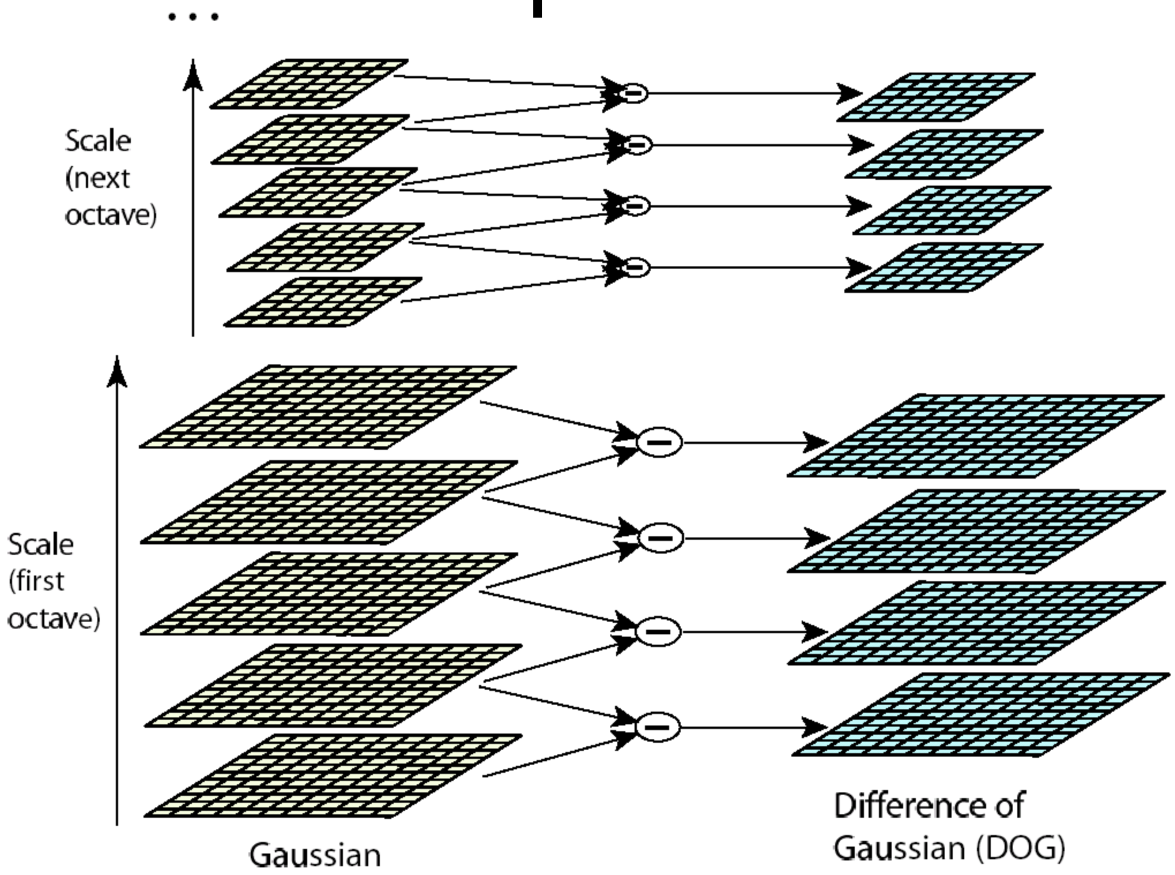
$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)



Slide credit: S. Lazebnik

Efficient implementation



David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

Slide credit: S. Lazebnik

Problem 1 Solved

- How do we deal with scales: try them all
- **Why is this efficient?**

Vast majority of effort is in the first and second scales

$$1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{4^i} \ldots = \frac{4}{3}$$

Problem 2 – Describing Features

Image – 40

1/2 size, rot. 45°
Lightened+40

Image

100x100 crop
at Glasses

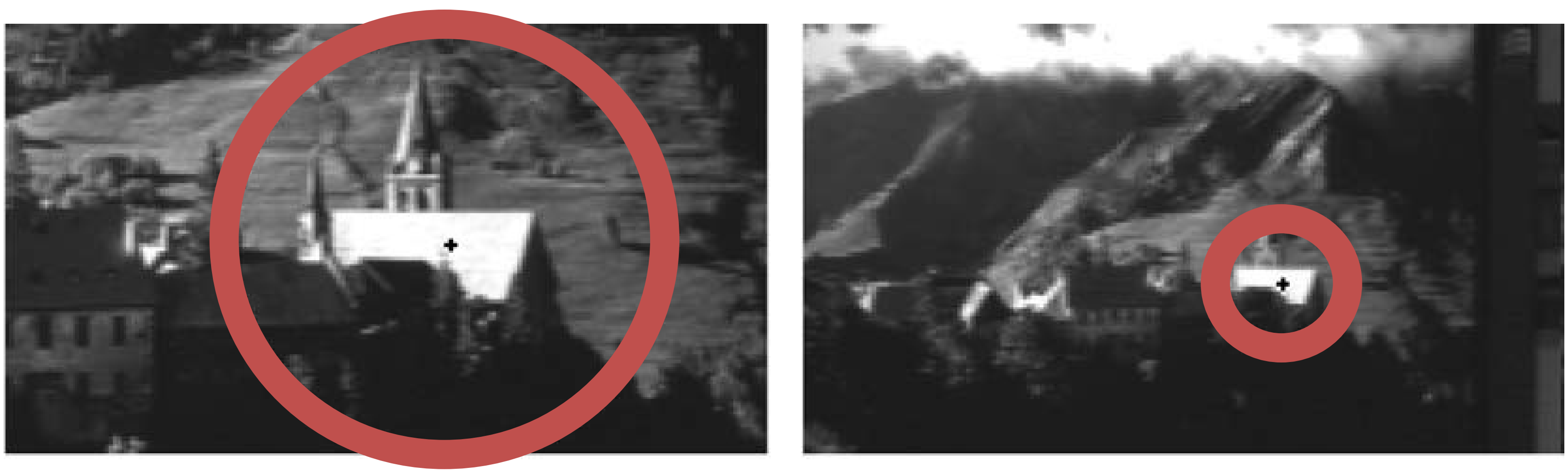
Problem 2 – Describing Features

Once we’ve found a corner/blobs, we can’t just use the image nearby. What about:

1. Scale?
2. Rotation?
3. Additive light?

Handling Scale

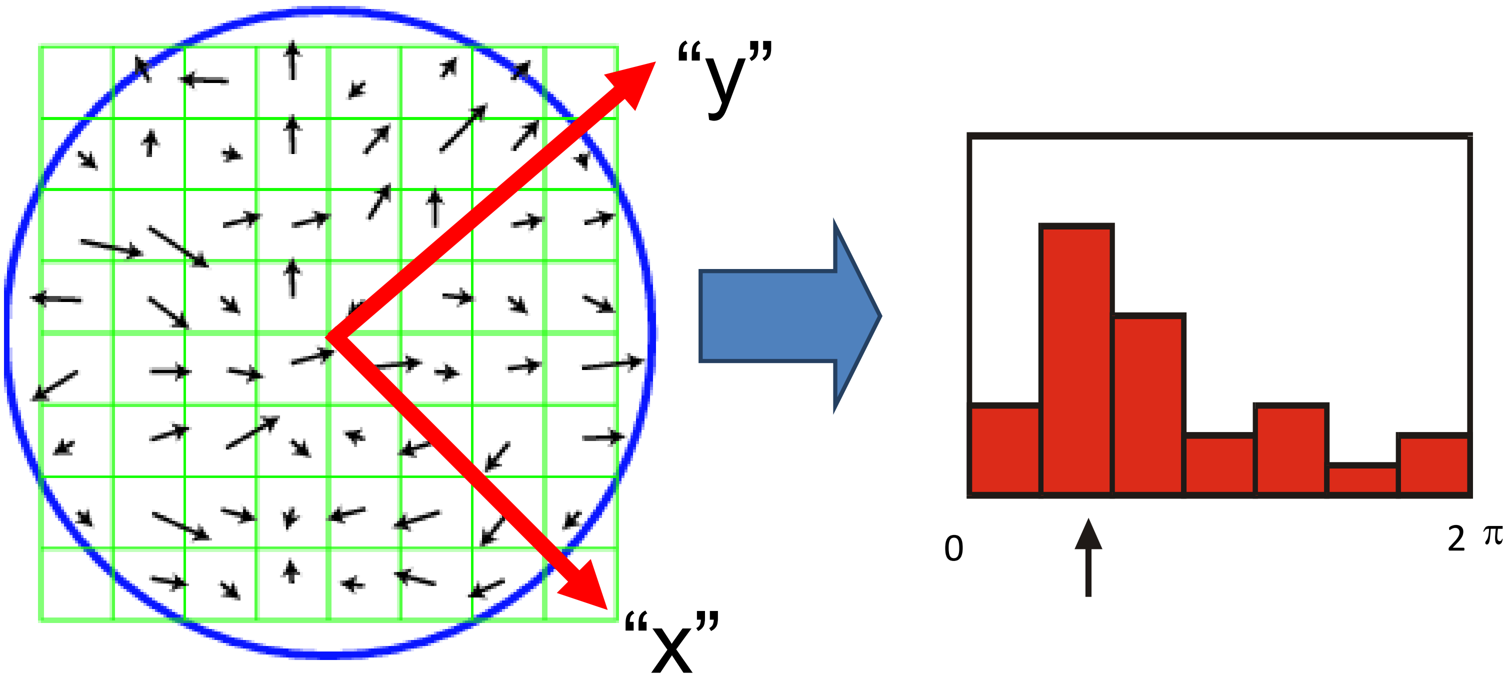
Given characteristic scale (maximum Laplacian response), we can just rescale image



Slide credit: S. Lazebnik

Handling Rotation

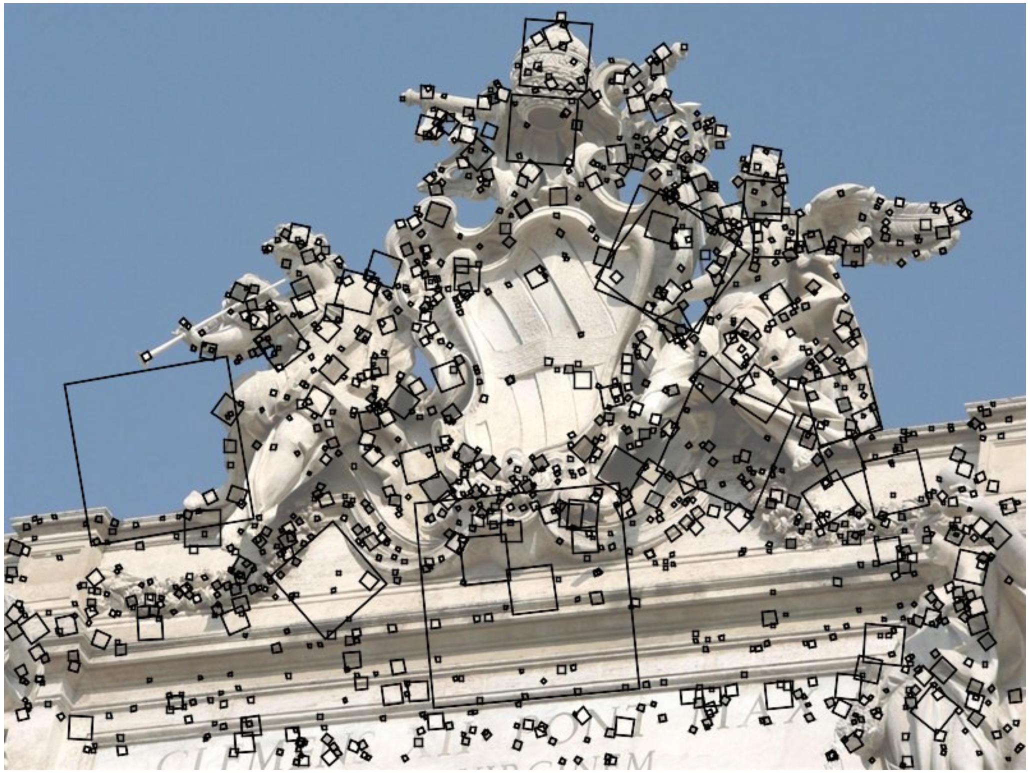
Given window, can compute dominant orientation and then rotate image



Slide credit: S. Lazebnik

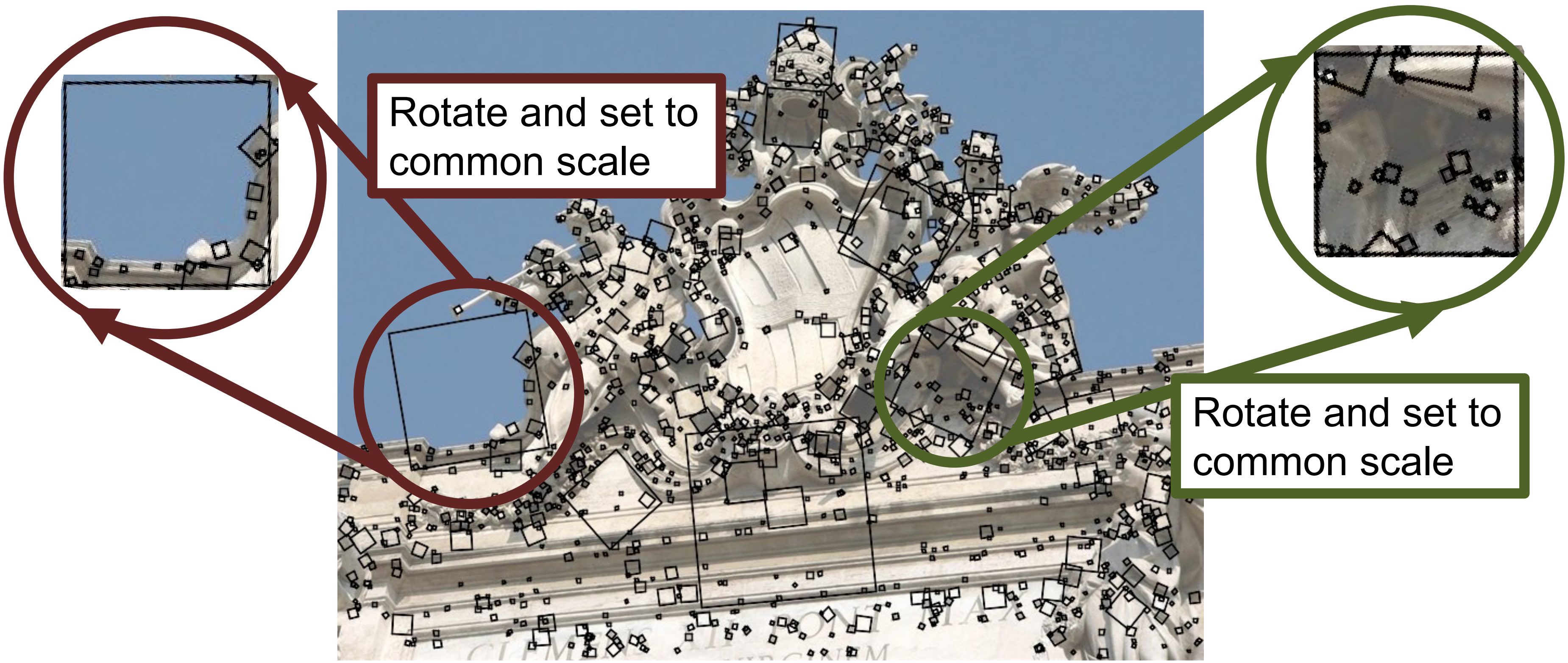
Scale and Rotation

SIFT features at characteristic scales and dominant orientations



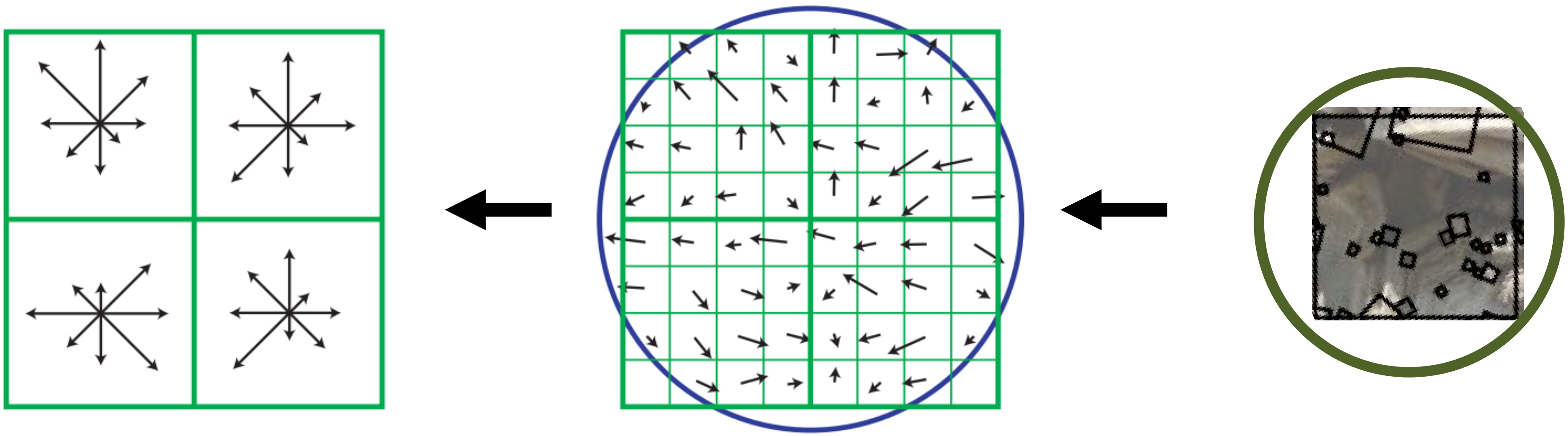
Picture credit: S. Lazebnik. Paper: David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

Scale and Rotation



Picture credit: S. Lazebnik. Paper: David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

SIFT Descriptors



1. Compute gradients
2. Build histogram (2x2 here, 4x4 in practice)

Gradients ignore global illumination changes

Figure from David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

SIFT Descriptors

- In principle: build a histogram of the gradients
- In reality: quite complicated
  - Gaussian weighting: smooth response
  - Normalization: reduces illumination effects
  - Clamping
  - Affine adaptation

Properties of SIFT

- Can handle: up to ~60 degree out-of-plane rotation, Changes of illumination
- Fast and efficient and lots of code available

Slide credit: N. Snavely

Feature Descriptors

Think of feature as some non-linear filter that maps pixels to 128D feature

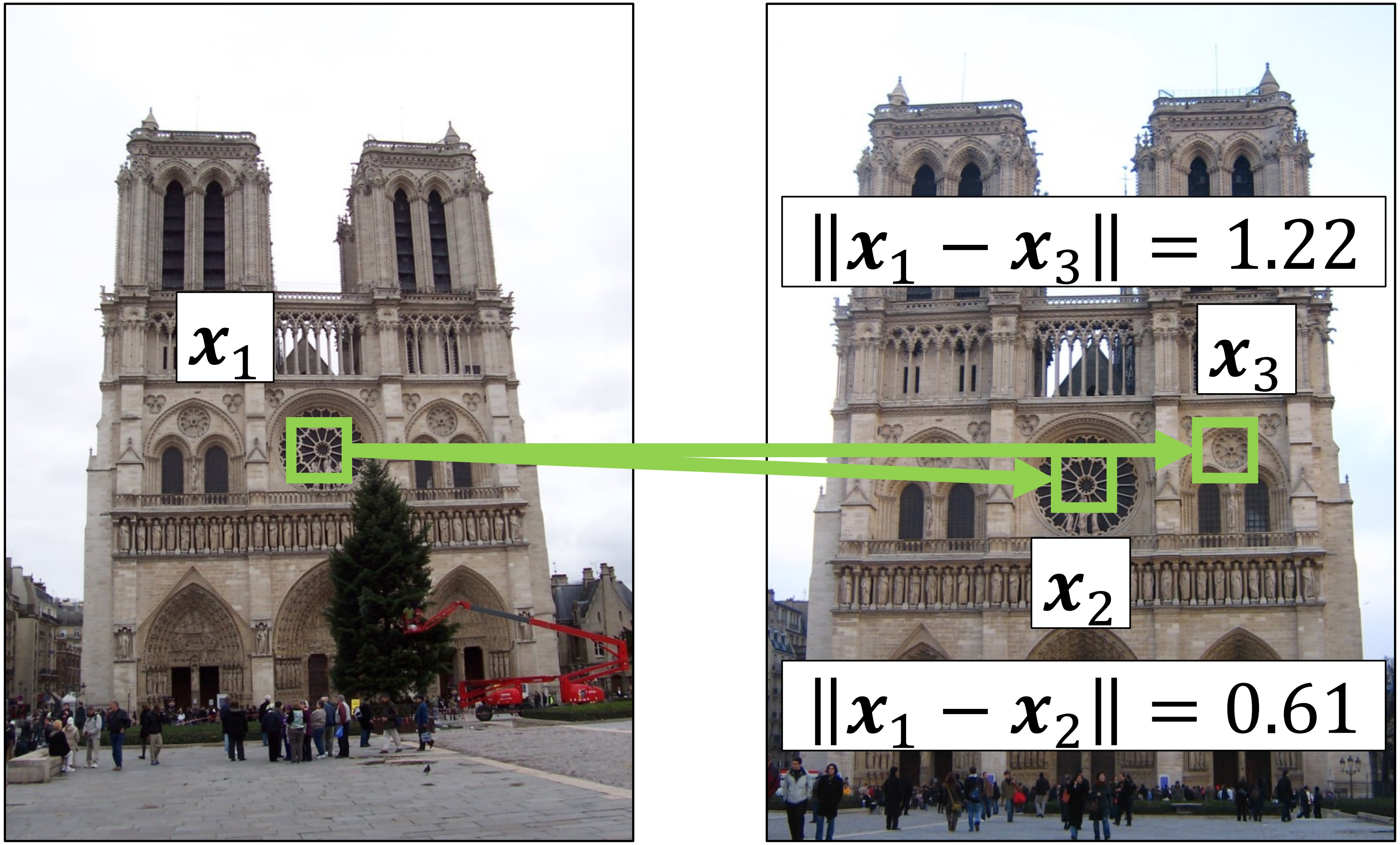
128D vector **x**

Photo credit: N. Snavely

Using Descriptors

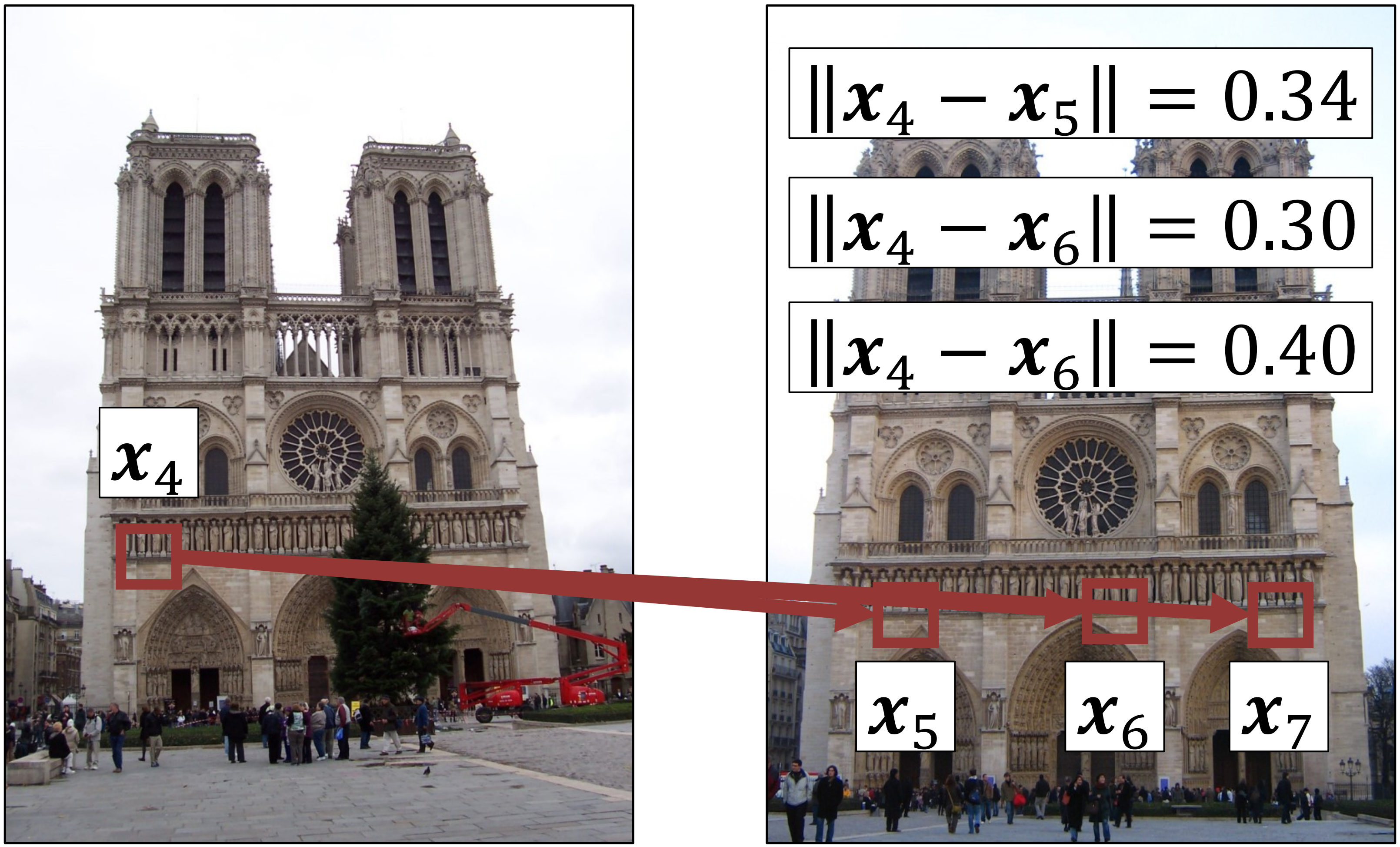
- Instance Matching
- Category recognition

Instance Matching



Example credit: J. Hays

Instance Matching



Example credit: J. Hays

2^{nd} Nearest Neighbor Trick

- Given a feature x, nearest neighbor to x is a good match, but distances can't be thresholded.
- Instead, find nearest neighbor and second nearest neighbor. This ratio is a good test for matches:

$$r = \frac{\left\| \boldsymbol{x}_q - \boldsymbol{x}_{1NN} \right\|}{\left\| \boldsymbol{x}_q - \boldsymbol{x}_{2NN} \right\|}$$

2nd Nearest Neighbor Trick

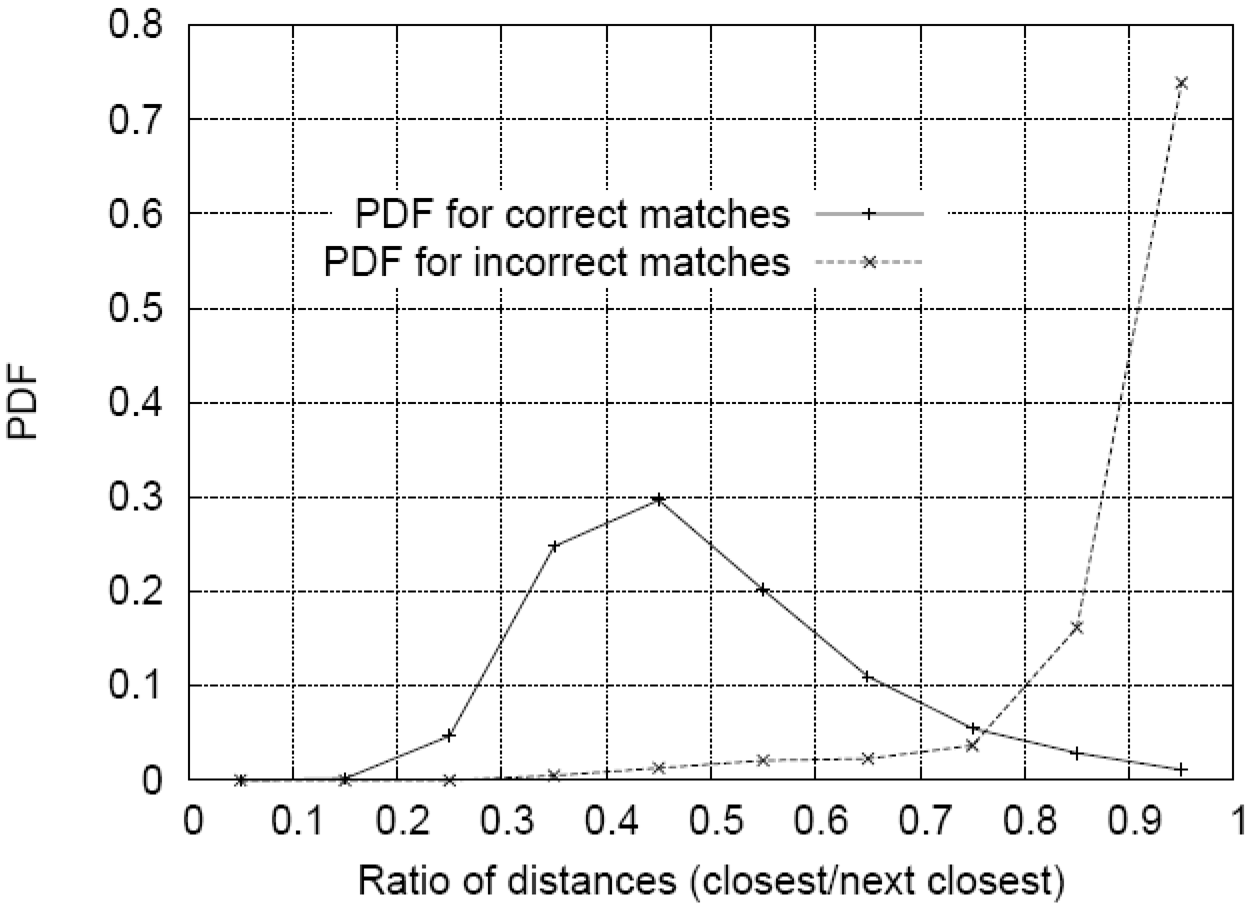


Figure from David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

Extra Reading for the Curious

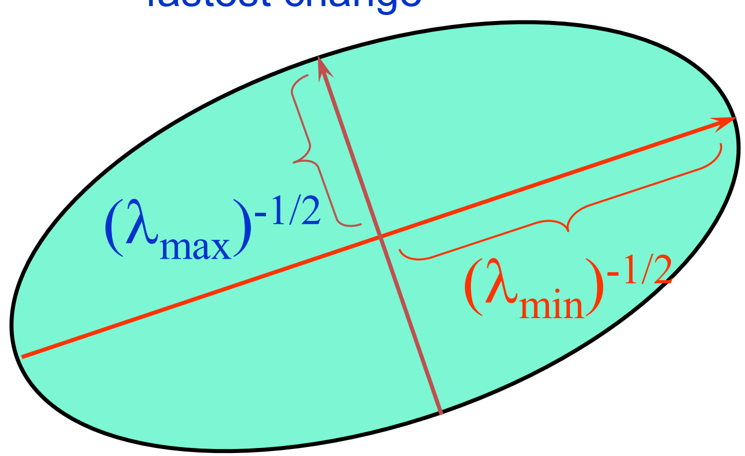
Affine adaptation

Consider the second moment matrix of the window containing the blob:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

Recall:

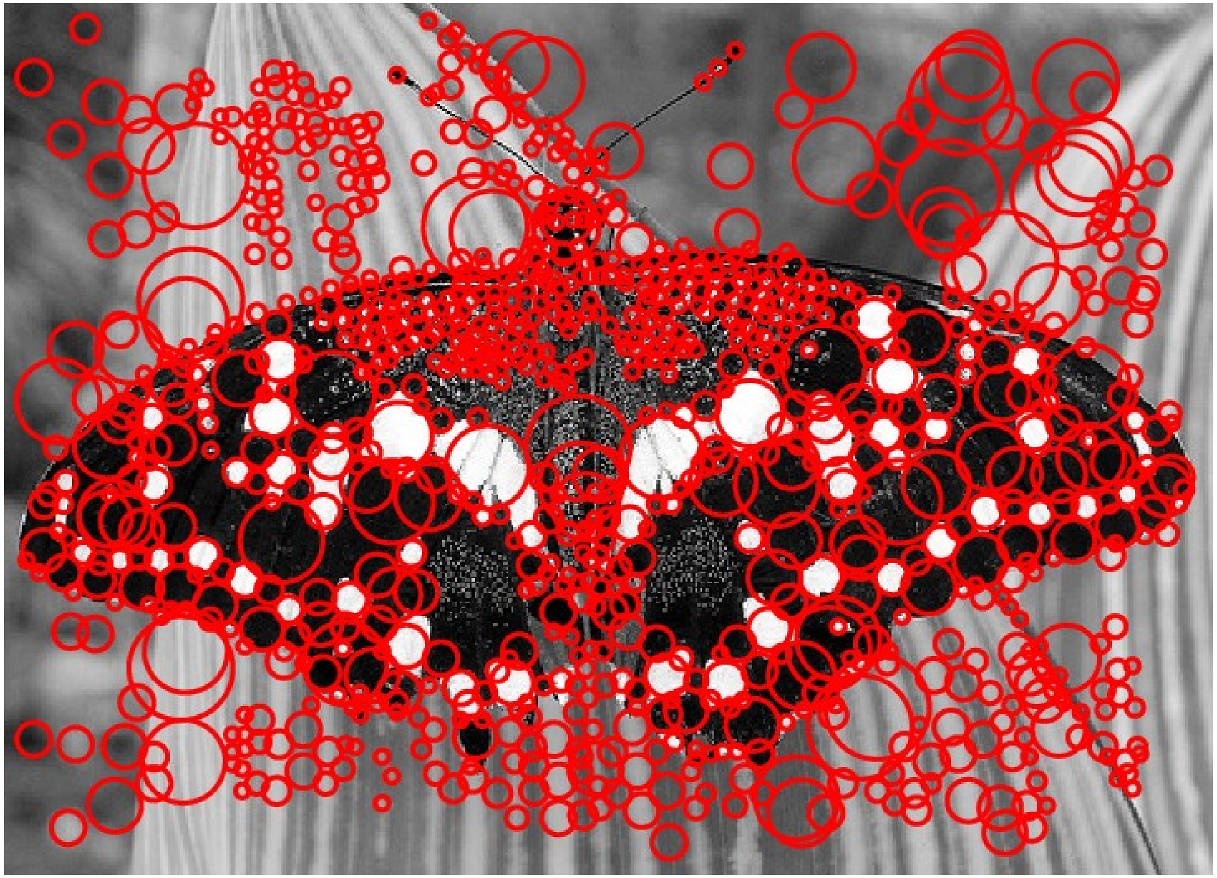
$$[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



This ellipse visualizes the “characteristic shape” of the window

Slide: S. Lazebnik

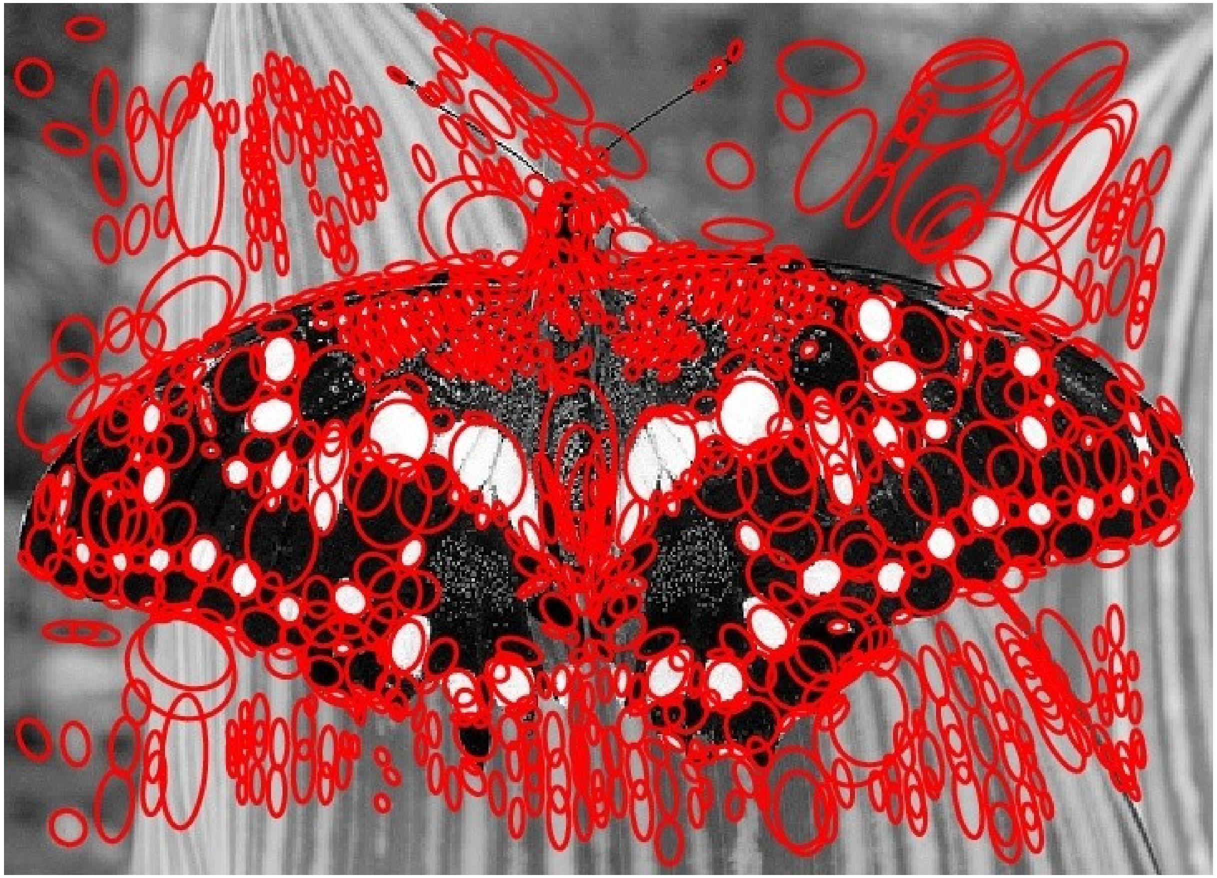
Affine adaptation example



Scale-invariant regions (blobs)

Slide: S. Lazebnik

Affine adaptation example



Affine-adapted blobs

Slide: S. Lazebnik