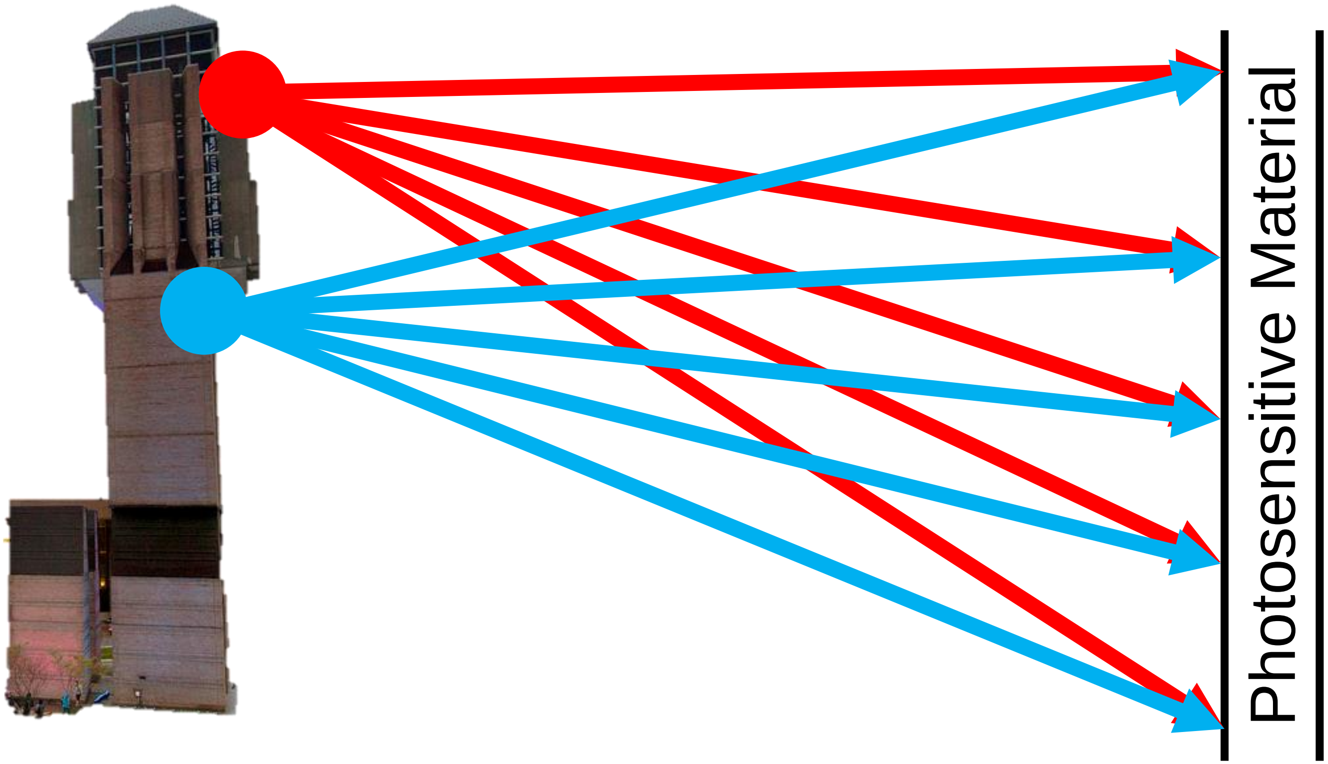
Cameras

EECS 442 – David Fouhey

Fall 2019, University of Michigan

http://web.eecs.umich.edu/~fouhey/teaching/EECS442_F19/

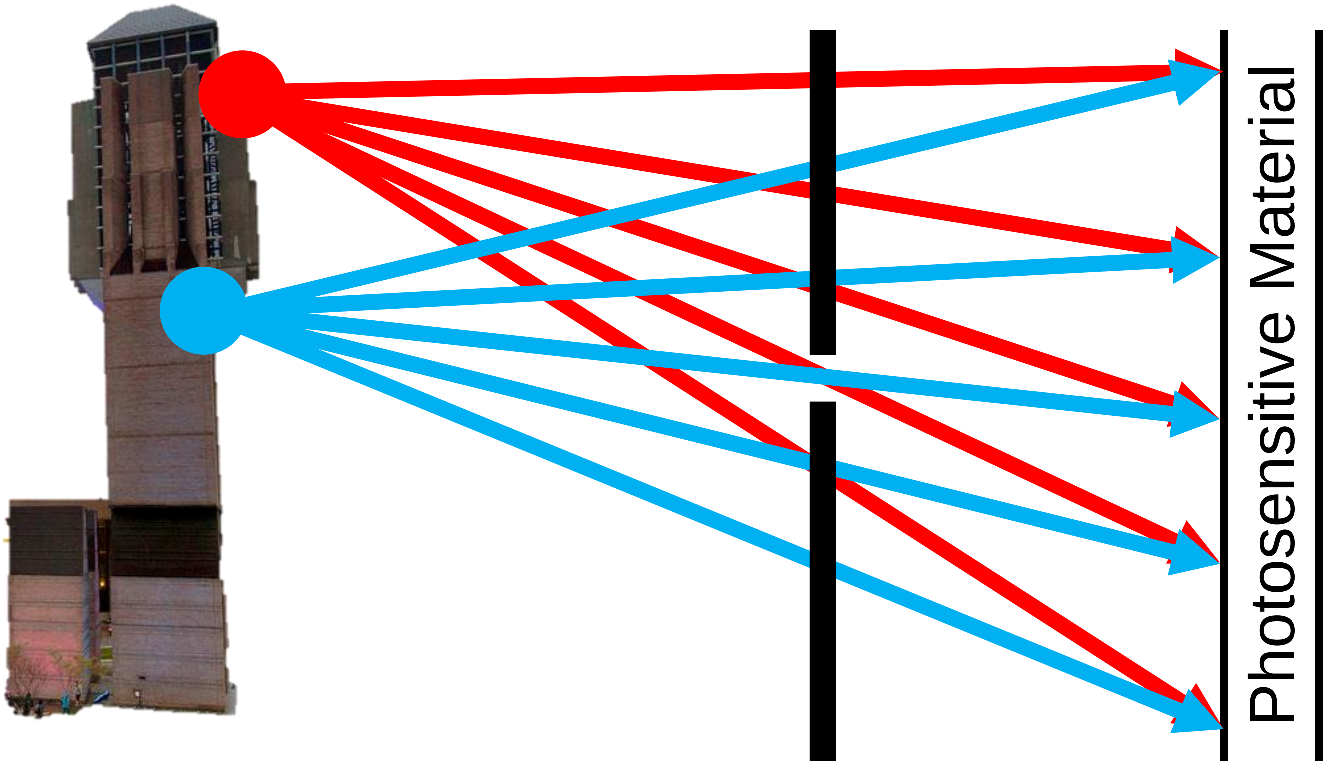
Let’s Take a Picture!



Idea 1: Just use film Result: **Junk**

Slide inspired by S. Seitz; image from Michigan Engineering

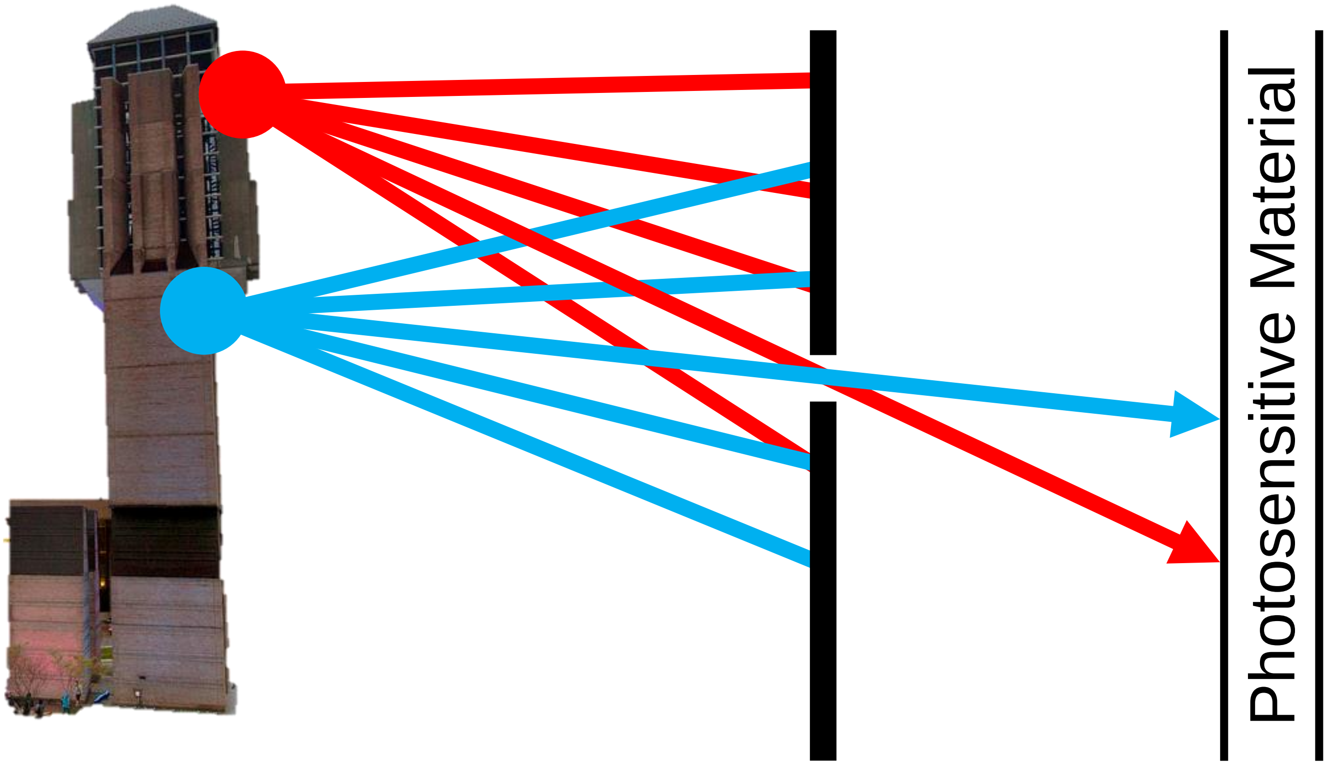
Let’s Take a Picture!



Idea 2: add a barrier

Slide inspired by S. Seitz; image from Michigan Engineering

Let's Take a Picture!



Idea 2: add a barrier

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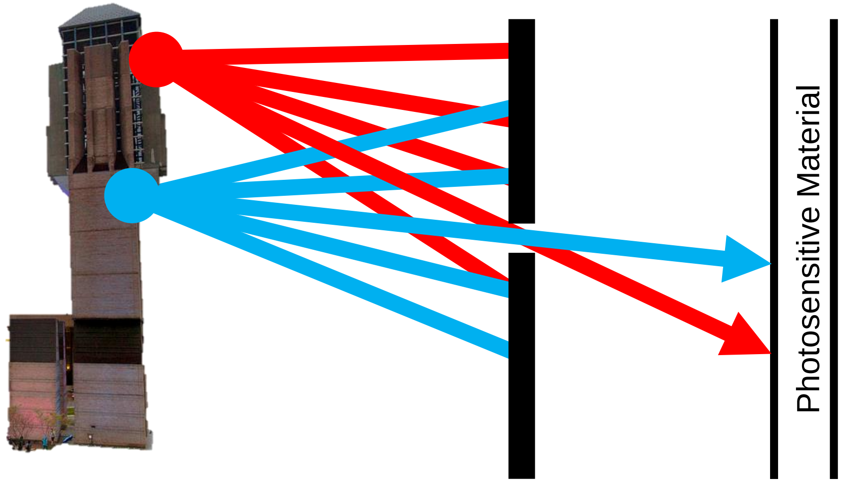
Let’s Take a Picture!

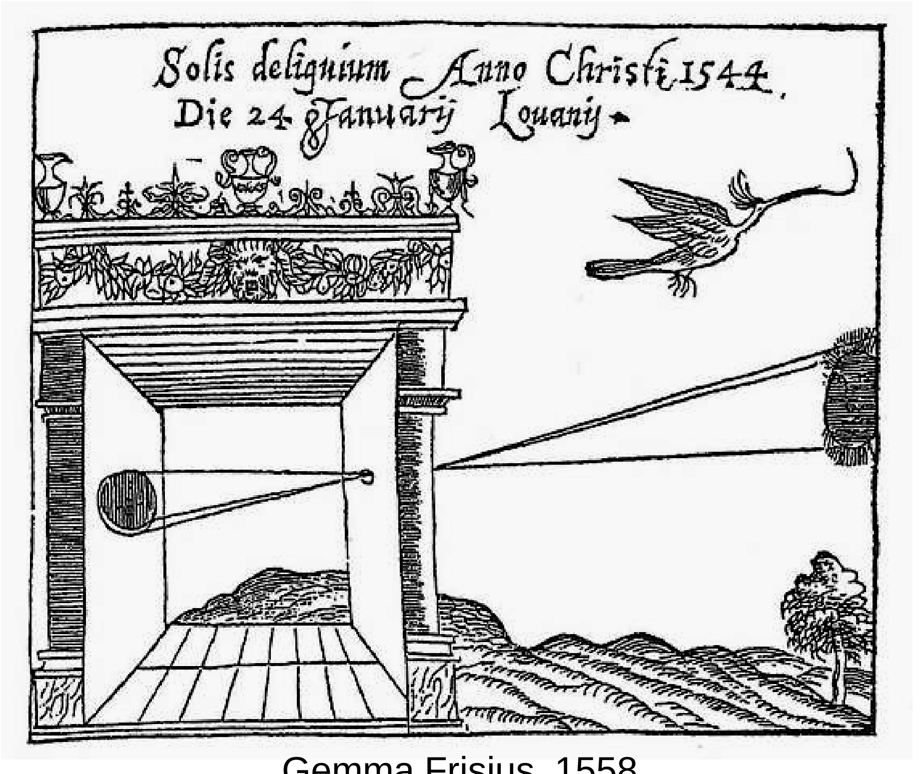


Film captures all the rays going through a point (a p*encil of rays).*

Result: good in theory!

Slide inspired by S. Seitz; image from Michigan Engineering

Camera Obscura

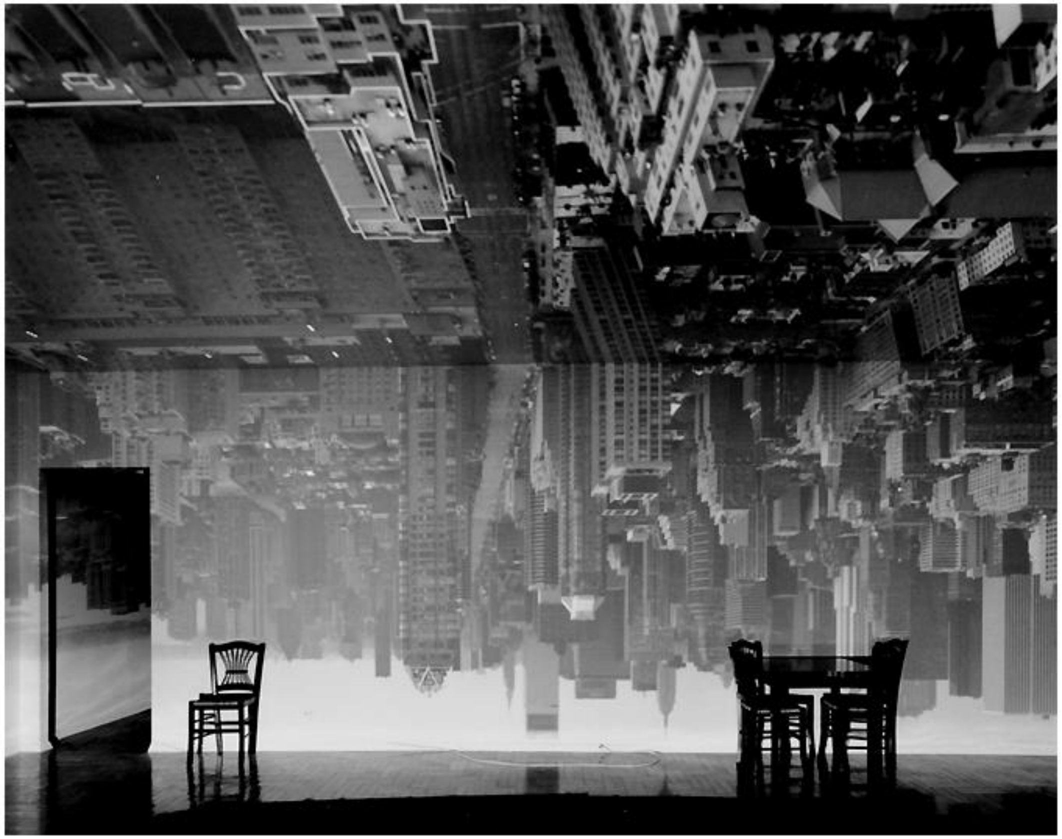


Gemma Frisius, 1558

- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

Source: A. Efros

Camera Obscura



Abelardo Morell, Camera Obscura Image of Manhattan View Looking South in Large Room, 1996

After scouting rooms and reserving one for at least a day, Morell masks the windows except for the aperture. He controls three elements: the size of the hole, with a smaller one yielding a sharper but dimmer image; the length of the exposure, usually eight hours; and the distance from the hole to the surface on which the outside image falls and which he will photograph. He used 4 x 5 and 8 x 10 view cameras and lenses ranging from 75 to 150 mm.

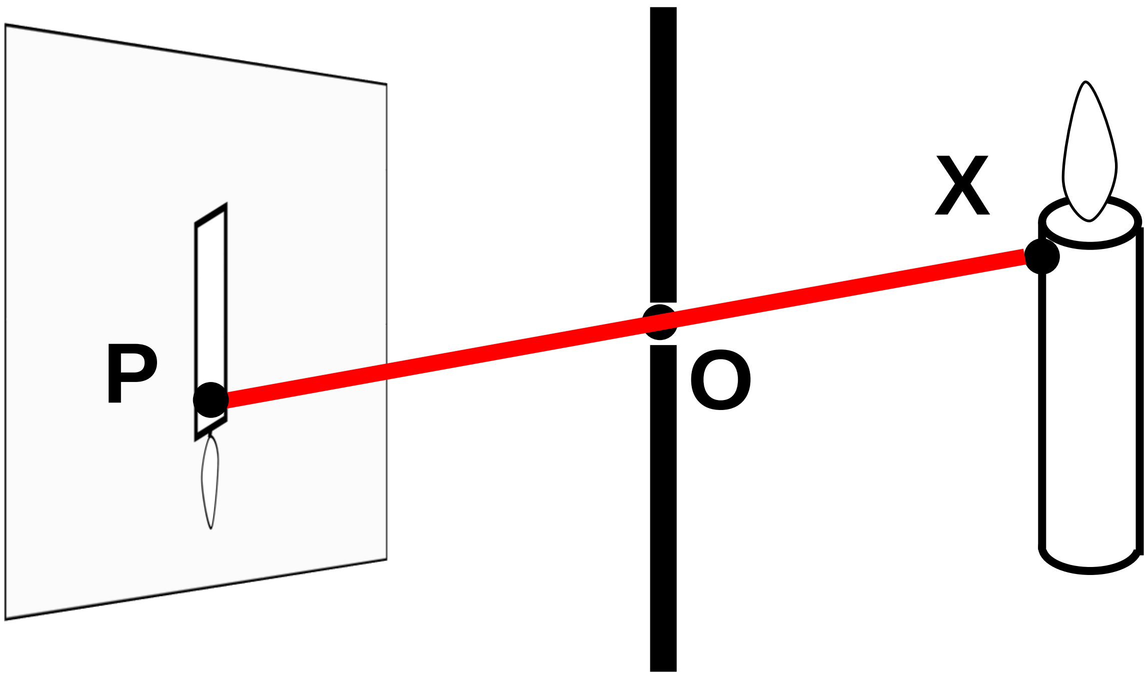
After he's done inside, it gets harder. "I leave the room and I am constantly checking the weather, I'm hoping the maid reads my note not to come in, I'm worrying that the sun will hit the plastic masking and it will fall down, or that I didn't trigger the lens."

From *Grand Images Through a Tiny Opening*, **Photo District News,** February 2005

http://www.abelardomorell.net/project/camera-obscura/

Abelardomorell.com

Projection

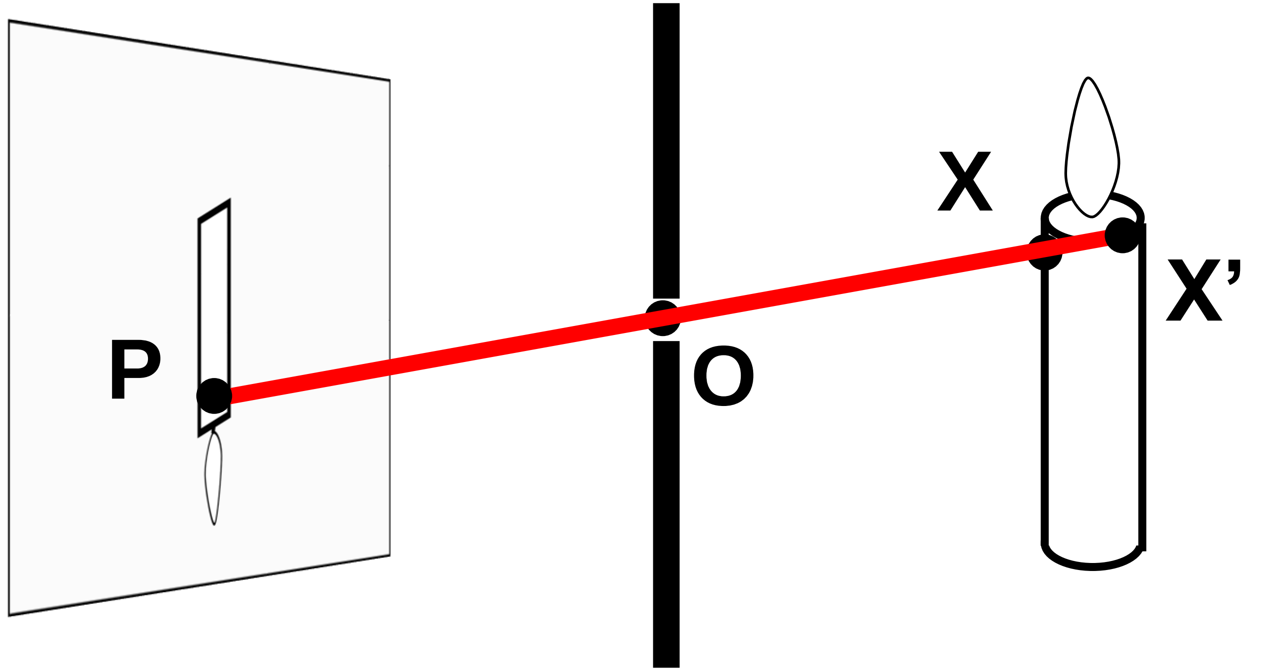


How do we find the projection P of a point X?

Form visual ray from X to camera center and intersect it with camera plane

Source: L Lazebnik

Projection

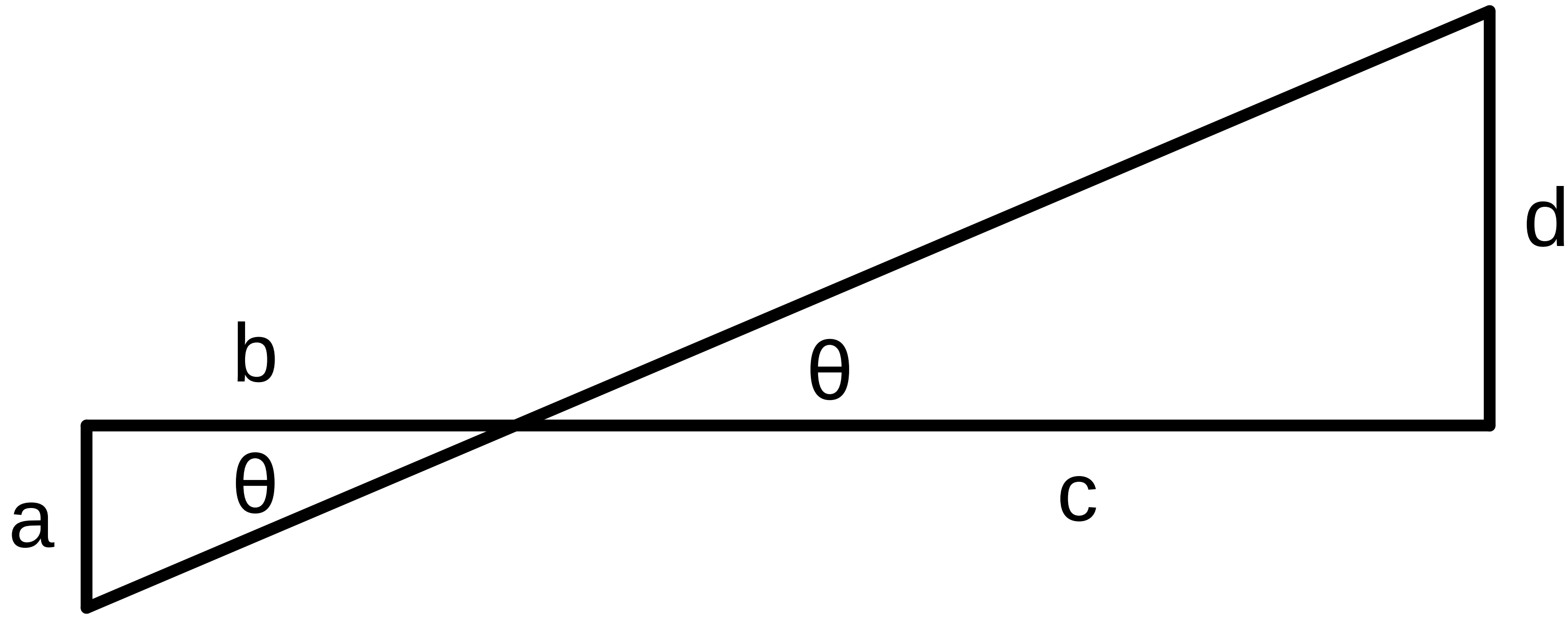


Both X and X' project to P. Which appears in the image?

Are there points for which projection is undefined?

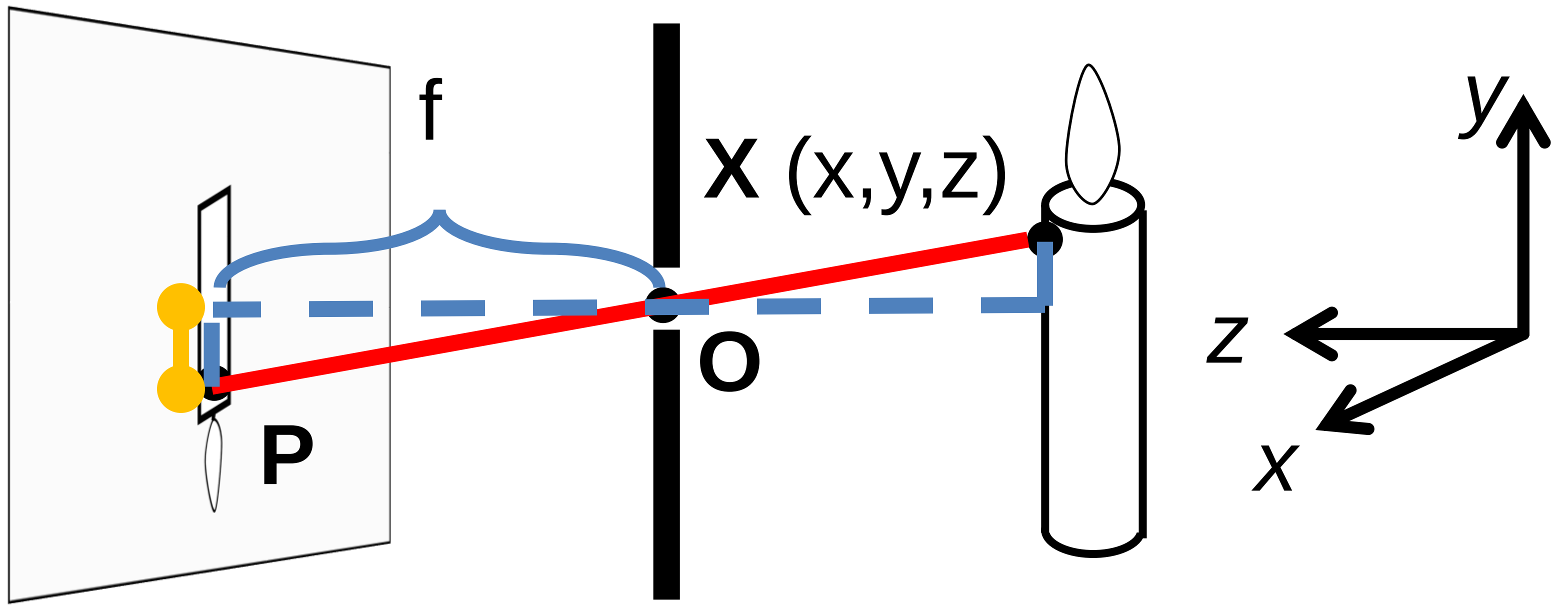
Source: L Lazebnik

Quick Aside: Remember This?



$$\frac{a}{b} = \frac{d}{c} \longrightarrow a = \frac{bd}{c}$$

Projection Equations



Coordinate system: **O** is origin, XY in image, Z sticks out.
XY is image plane, Z is optical axis.

(x,y,z) projects to (fx/z,fy/z) via similar triangles

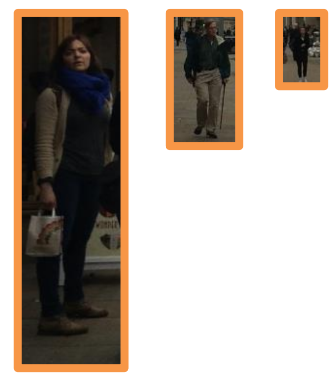
Source: L Lazebnik

Some Facts About Projection

3D lines project to 2D lines

The projection of any 3D parallel lines converge at a vanishing point

Distant objects are smaller

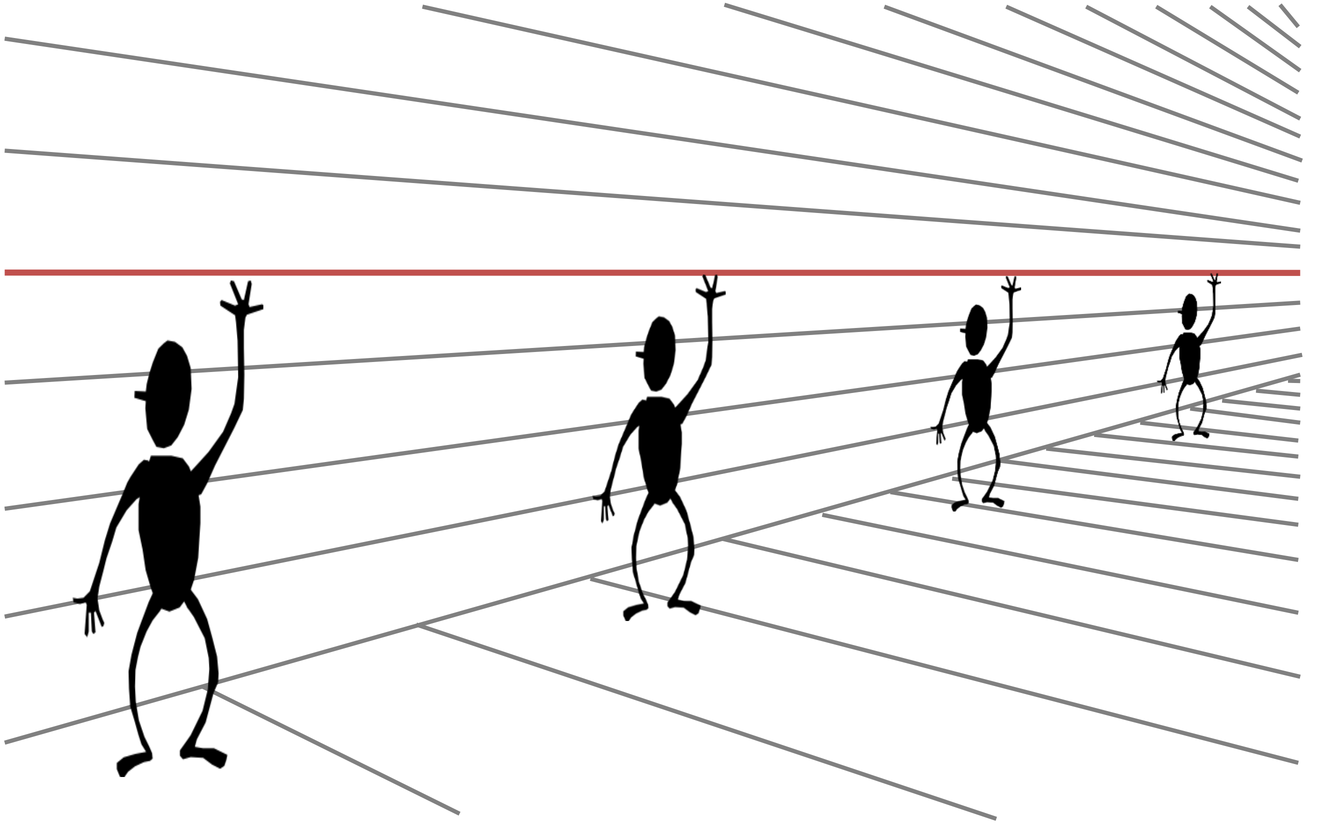


List of properties from M. Hebert

Some Facts About Projection

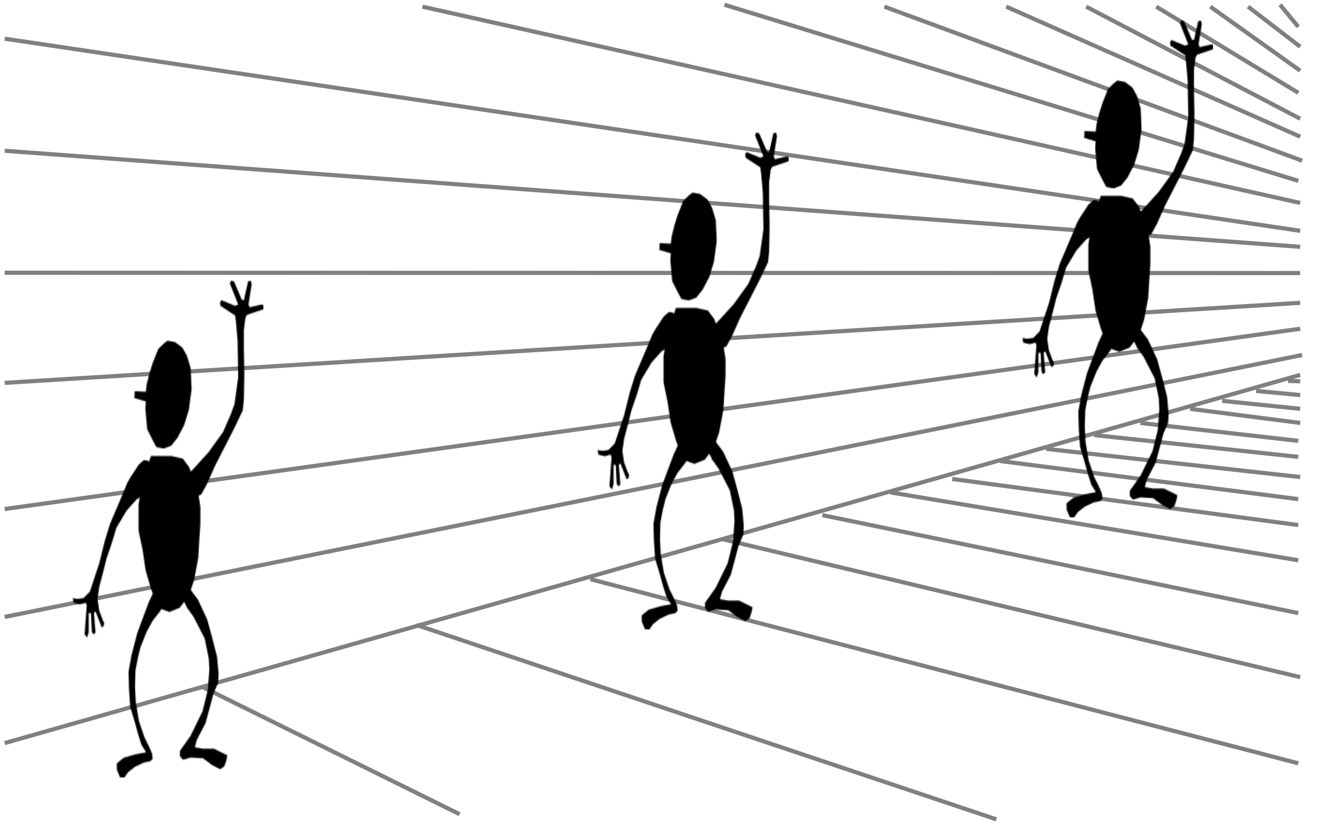
Let’s try some fake images

Some Facts About Projection



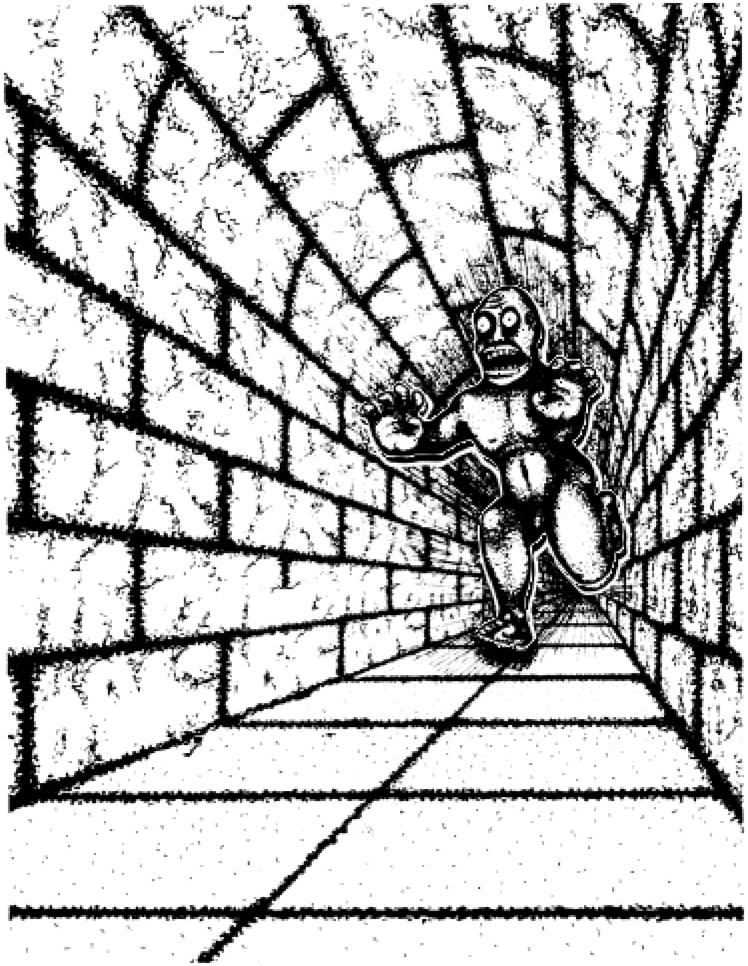
Slide by Steve Seitz

Some Facts About Projection



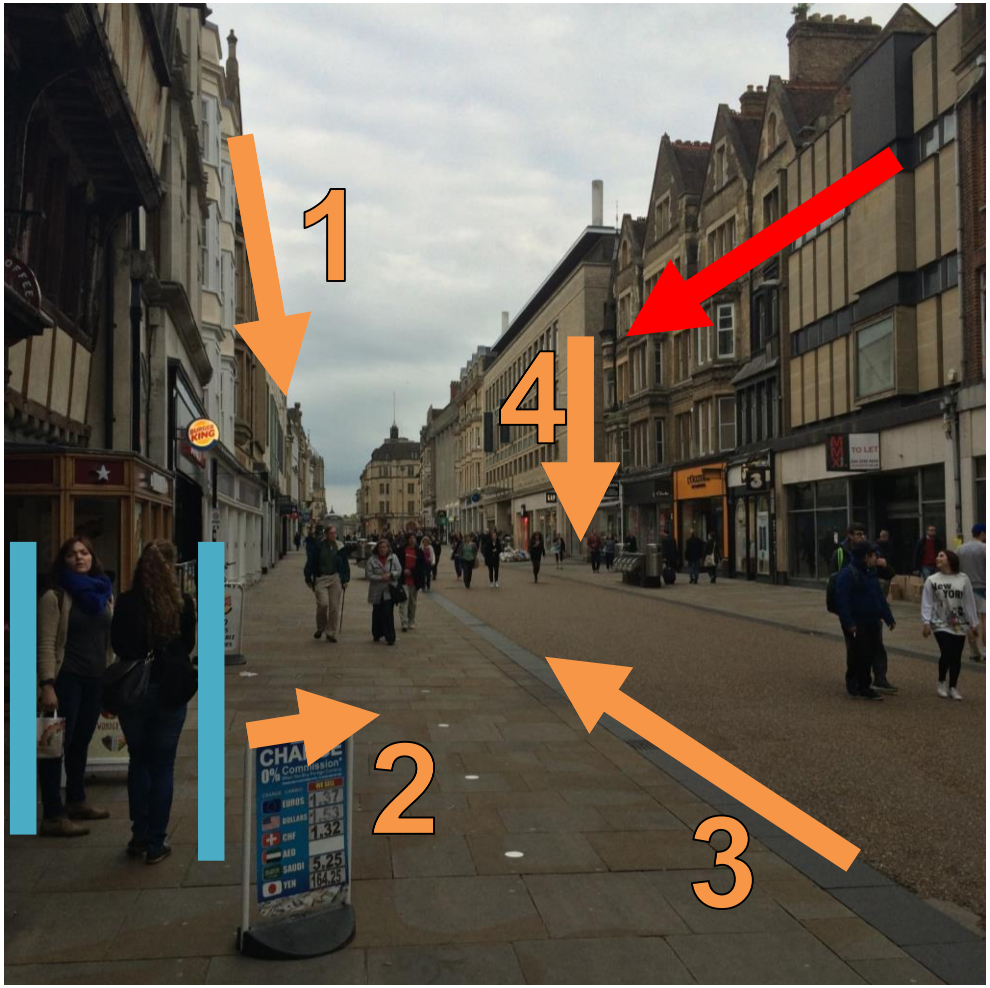
Slide by Steve Seitz

Some Facts About Projection



Illusion Credit: RN Shepard, Mind Sights: Original Visual Illusions, Ambiguities, and other Anomalies

What's Lost?

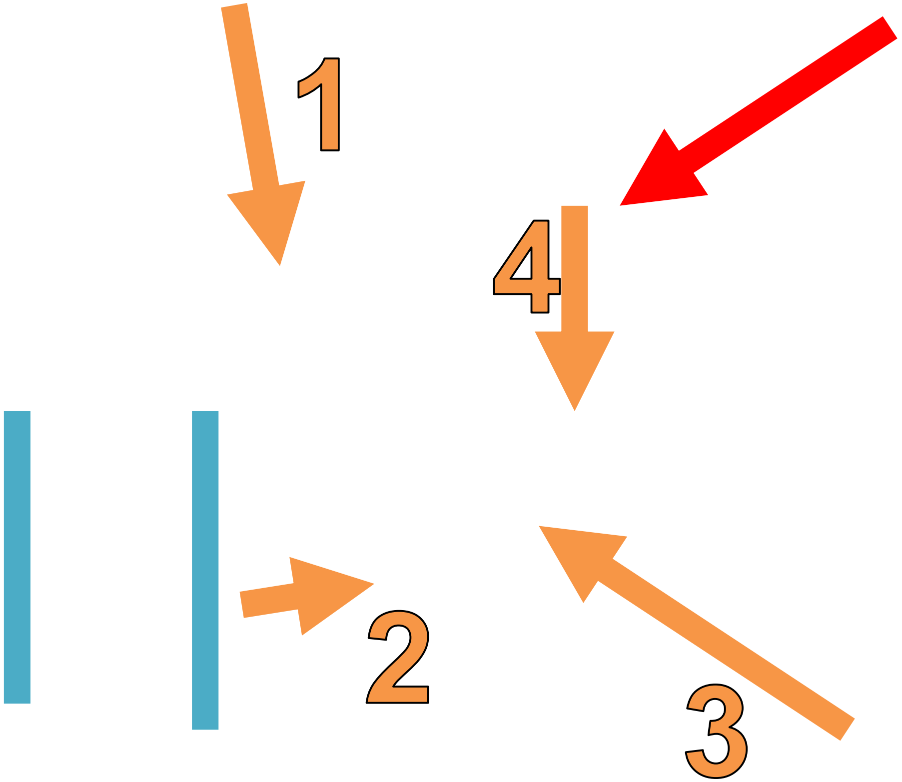


Is she shorter or further away?

Are the orange lines we see parallel / perpendicular / neither to the red line?

Inspired by D. Hoiem slide

What's Lost?



Is she shorter or further away?

Are the orange lines we see parallel / perpendicular / neither to the red line?

Adapted from D. Hoiem slide

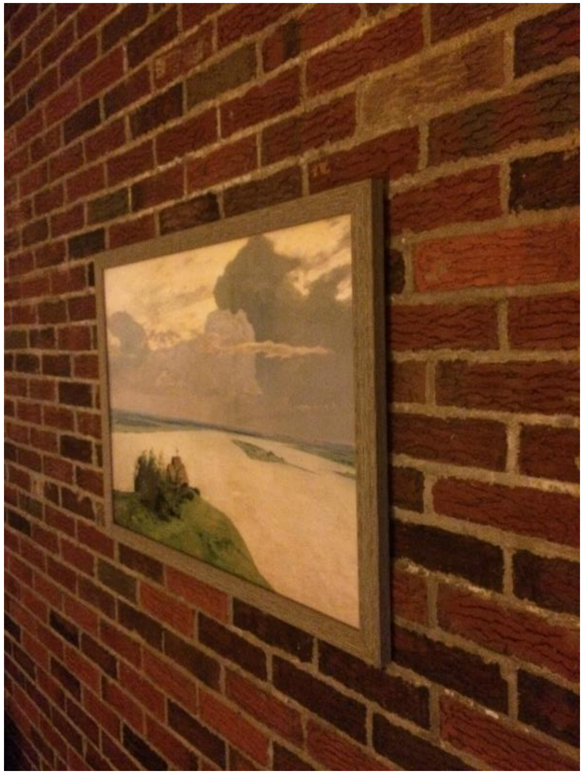
What’s Lost?

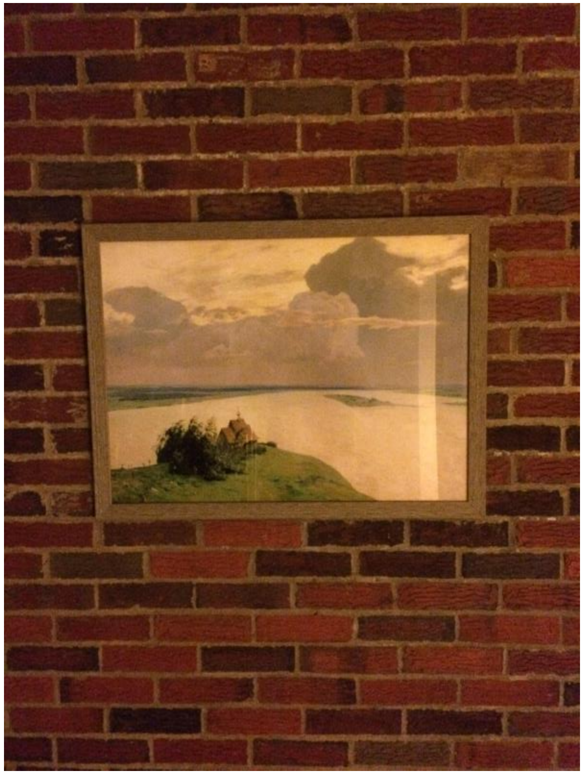
Be careful of drawing conclusions:

- Projection of 3D line is 2D line; NOT 2D line is 3D line.
- **Can you think of a counter-example (a 2D line that is not a 3D line)?**

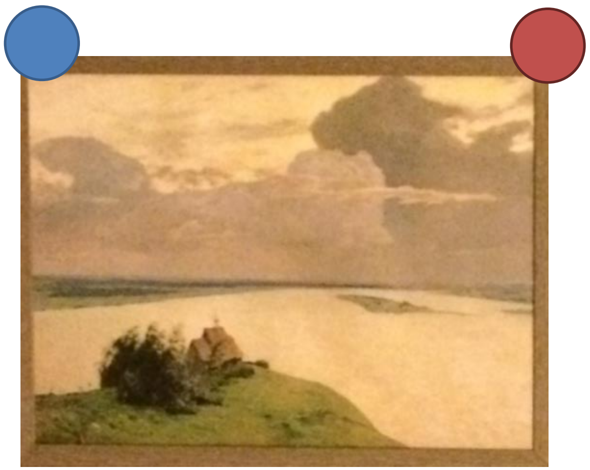
- Projections of parallel 3D lines converge at VP; NOT any pair of lines that converge are parallel in 3D.
- **Can you think of a counter-example?**

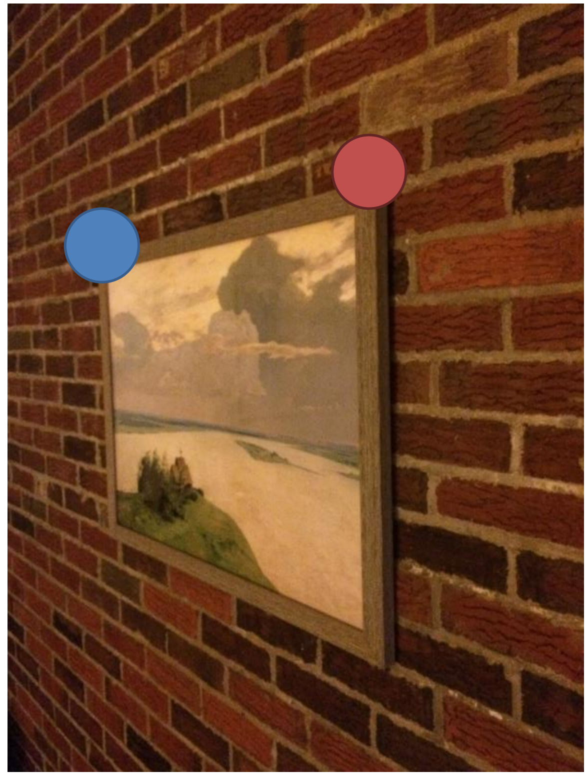
Do You Always Get Perspective?

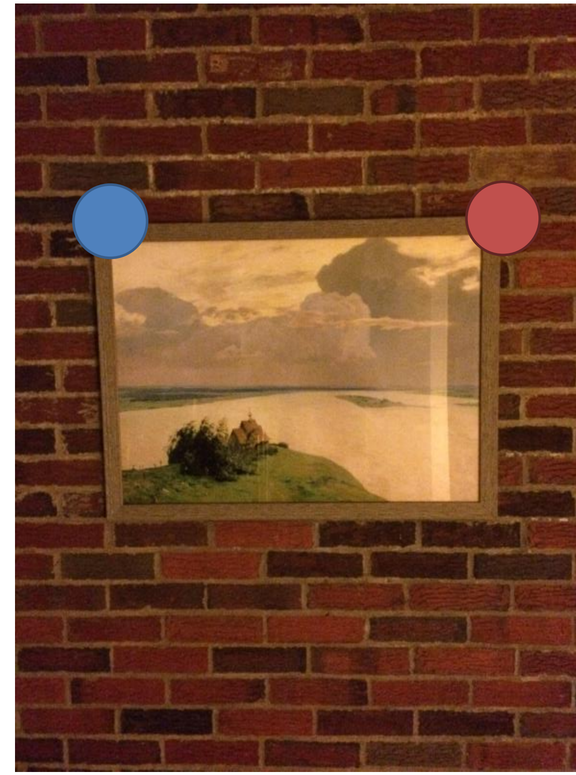




Do You Always Get Perspective?





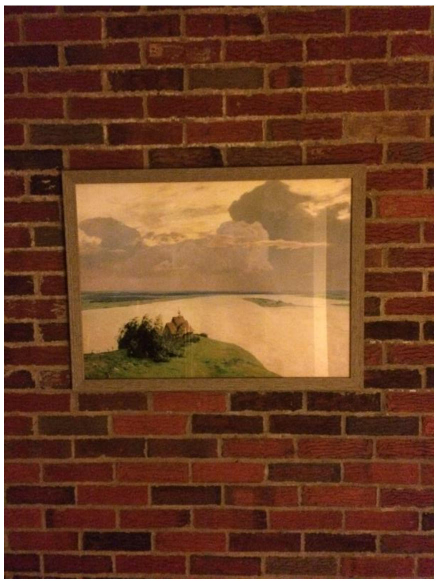


Y location of blue and red dots in image:

$$\frac{fy}{z_2} \qquad \frac{fy}{z_1}$$

$$\frac{fy}{z} \qquad \frac{fy}{z}$$

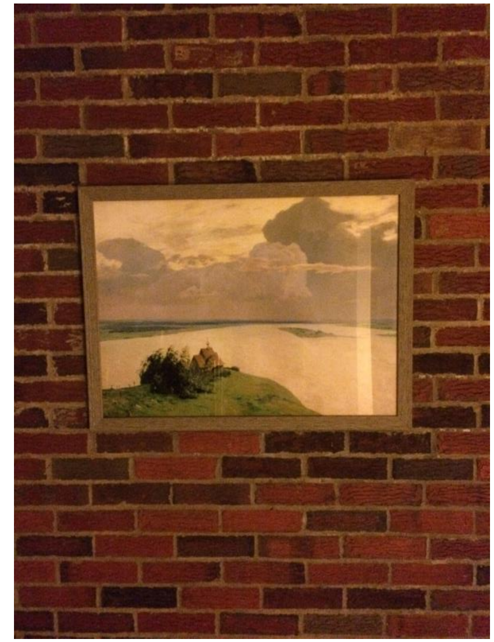
Do You Always Get Perspective?



When plane is fronto-parallel (parallel to camera plane), everything is:

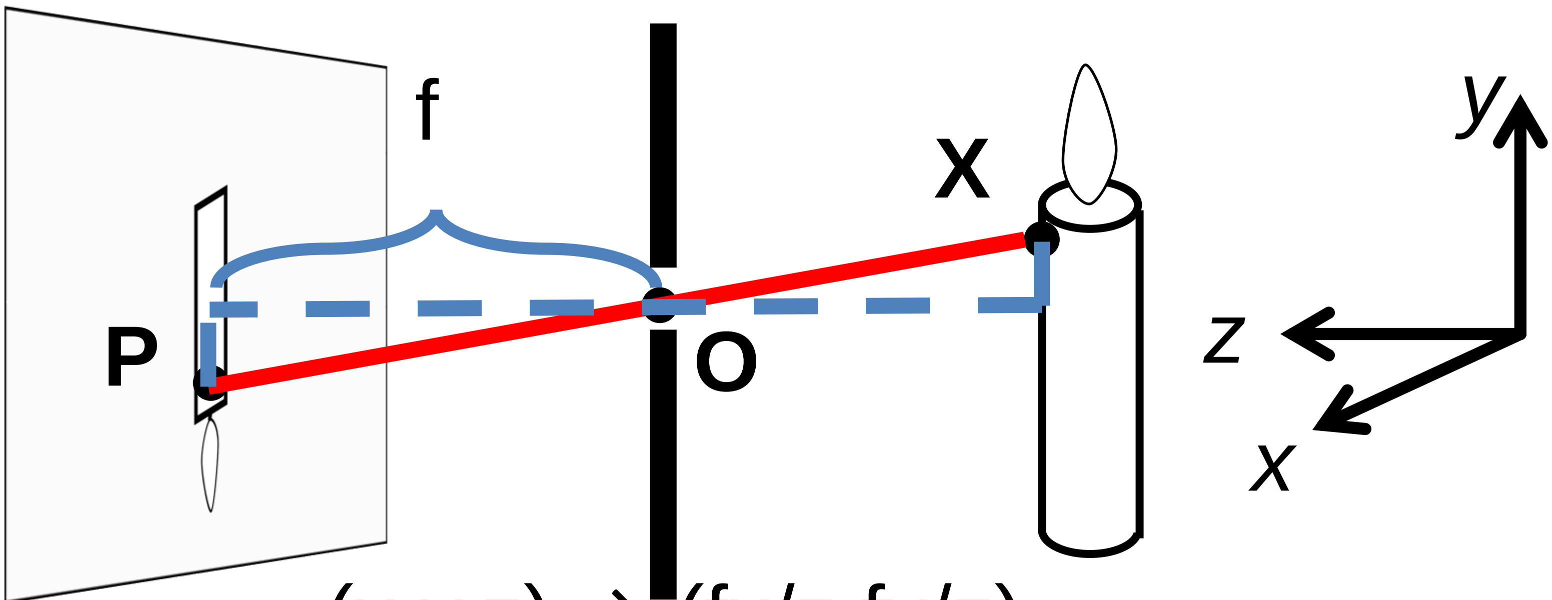
- scaled by f/z
- otherwise is preserved.

What’s This Useful For?



Things looking different when viewed from different angles seems like a nuisance. It’s also a cue. **Why?**

Projection Equation



$(x,y,z) \rightarrow (fx/z, fy/z)$

I promised you linear algebra: is this linear?

Nope: division by z is non-linear

(and risks division by 0)

Adapted from S. Seitz slide

Homogeneous Coordinates (2D)

Trick: add a dimension!

This also clears up lots of nasty special cases

Physical Point		Homogeneous Point		Physical Point
$\begin{bmatrix} x \\ y \end{bmatrix}$	→ Concat w=1	$\begin{bmatrix} u \\ v \\ w \end{bmatrix}$	→ Divide by w	$\begin{bmatrix} u/w \\ v/w \end{bmatrix}$

What if w = 0?

Adapted from M. Hebert slide

Homogeneous Coordinates

Triple / Equivalent

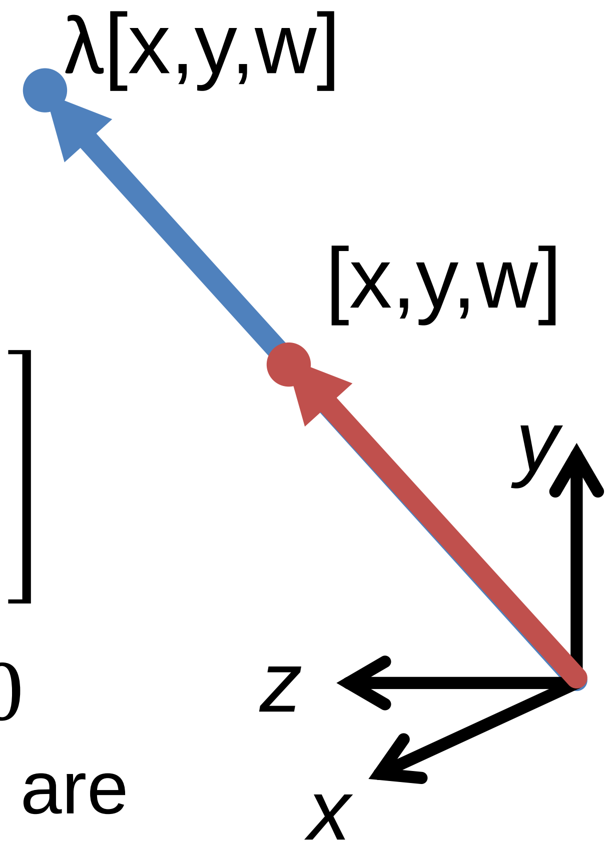
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \boxed{\equiv} \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix}$$

$$\leftrightarrow$$

Double / Equals

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \boxed{=} \lambda \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix}$$

$$\lambda \neq 0$$



Two homogeneous coordinates are **equivalent** if they are proportional to each other. **Not = !**

Benefits of Homogeneous Coords

General equation of 2D line:

$$ax + by + c = 0$$

Homogeneous Coordinates

$$\boldsymbol{l}^T \boldsymbol{p} = 0, \qquad \boldsymbol{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \boldsymbol{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

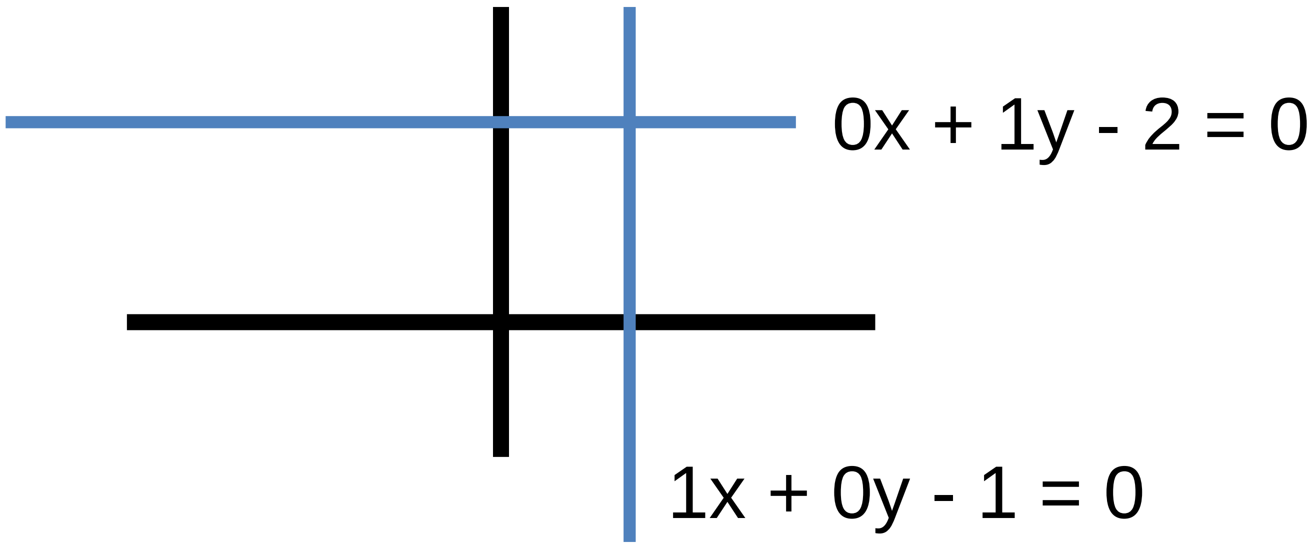
Slide from M. Hebert

Benefits of Homogeneous Coords

- Lines (3D) and points (2D → 3D) are now the same dimension.
- Use the *cross* (x) and *dot product* for:
  - Intersection of lines **l** and **m**: **l** x **m**
  - Line through two points **p** and **q**: **p** x **q**
  - Point **p** on line **l**: $\mathbf{l}^T\mathbf{p}$
- Parallel lines, vertical lines become easy (compared to y=mx+b)

Benefits of Homogeneous Coords

What's the intersection?



$$[0,1,-2] \times [1,0,-1] = [-1,-2,-1]$$

Converting back (divide by -1)

(1,2)

Cameras

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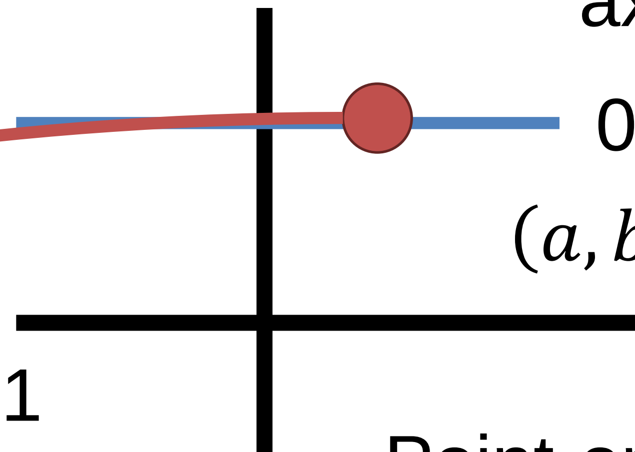
http://web.eecs.umich.edu/~fouhey/teaching/EECS442_F19/

Recap: Homogeneous Coords



Line of y=2 in ax+by+c=0:

0x + 1y - 2 = 0

$(a, b, c) = (0,1,-2)$

$\color{red}{(x, y)} = (1,2)$

↓ Append 1

$\color{red}{(u, v, w)} = (1,2,1)$

↓ Divide by w

$\color{red}{(x, y)} = (1,2)$

Point-on-line test: $\mathbf{l}^T\mathbf{p}$

$(a, b, c)^T(u, v, w) =$

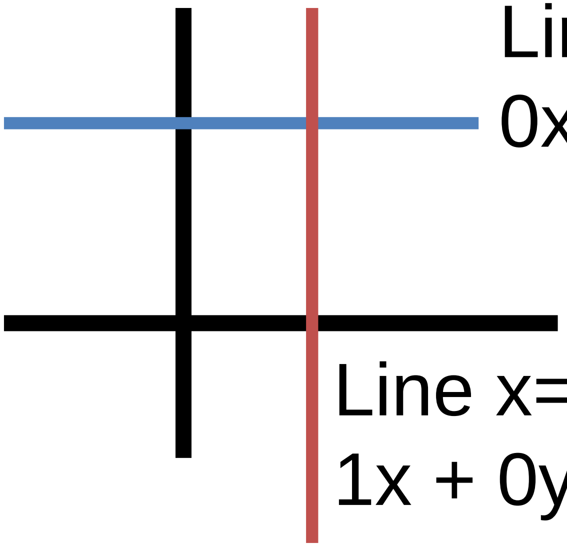
$(0,1,-2)^T(1,2,1) = 0$

Recap: Homogeneous Coords

$(a_1, b_1, c_1) = (0,1,-2)$

$(a_2, b_2, c_2) = (1,0,-1)$

Intersection: l_1 x l_2



Line y=2

0x + 1y - 2 = 0

Line x=1

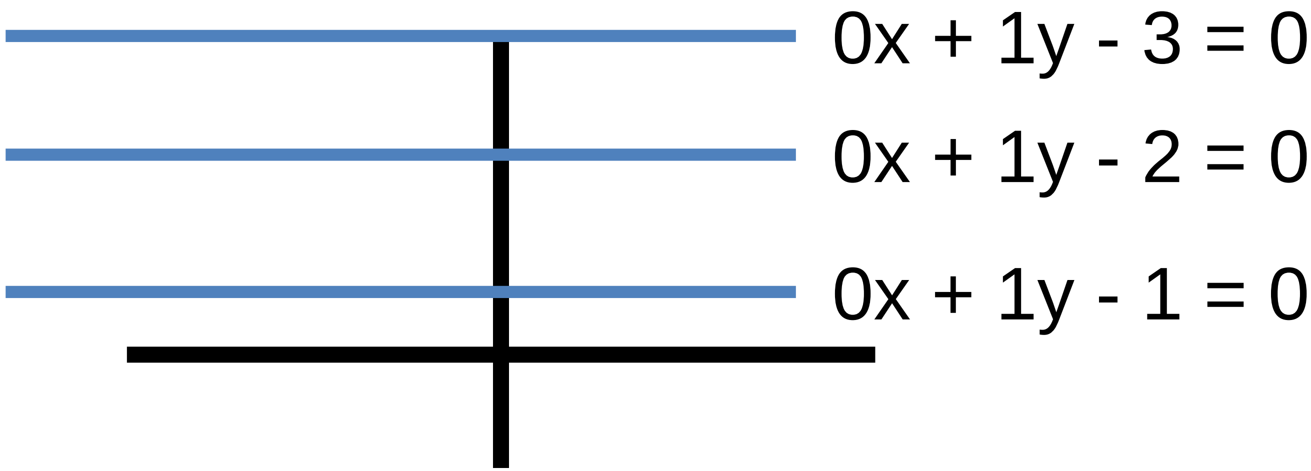
1x + 0y - 1 = 0

[0,1,-2] x [1,0,-1] = [-1,-2,-1]

Converting back (divide by -1)

(1,2)

Benefits of Homogeneous Coords



Intersection of y=2, y=1

[0,1,-2] x [0,1,-1] = [1,0,0]

Does it lie on y=3? Intuitively?

$[0,1,-3]^T[1,0,0] = 0$

Benefits of Homogeneous Coords

Translation is now linear / matrix-multiply

If w = 1 $$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} u + t_x \\ v + t_y \\ 1 \end{bmatrix}$$

Generically $$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} u + wt_x \\ v + wt_y \\ w \end{bmatrix}$$

Rigid body transforms (rot + trans) now linear

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

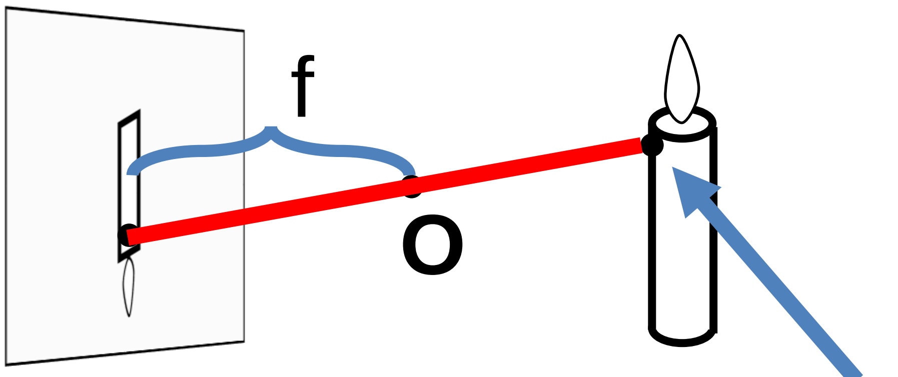
3D Homogeneous Coordinates

Same story: add a coordinate, things are equivalent if they're proportional

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} u \\ v \\ w \\ t \end{bmatrix} \rightarrow \begin{bmatrix} u/t \\ v/t \\ w/t \end{bmatrix}$$

Projection Matrix

Projection (fx/z, fy/z) is matrix multiplication

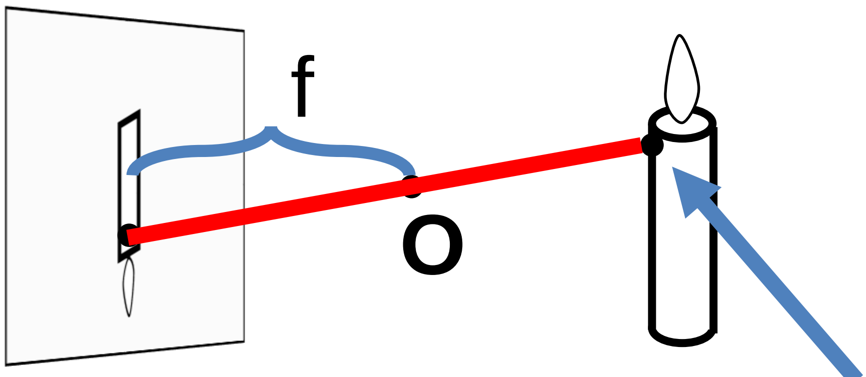


$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspired from L. Lazebnik

Projection Matrix

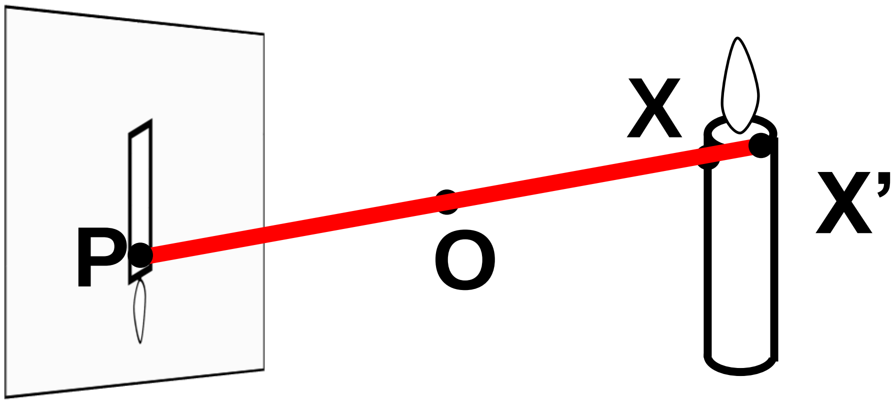
Projection (fx/z, fy/z) is matrix multiplication



$$\begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \equiv \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} fx/z \\ fy/z \end{bmatrix}$$

Slide inspired from L. Lazebnik

Why ≡ ≠ =



Project X and X' to the image and compare them

YES $\begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \equiv \begin{bmatrix} fx' \\ fy' \\ z' \end{bmatrix}$

NO $\begin{bmatrix} fx \\ fy \\ z \end{bmatrix} = \begin{bmatrix} fx' \\ fy' \\ z' \end{bmatrix}$

Typical Perspective Model

P: 2D homogeneous point (3D)

X: 3d homogeneous point (4D)

$\boldsymbol{P} \equiv$ $\boldsymbol{X}_{4x1}$

Typical Perspective Model

R: rotation between world system and camera

t: translation between world system and camera

$$\boldsymbol{P} \equiv \qquad [\boldsymbol{R}_{3x3} \quad \boldsymbol{t}_{3x1}] \quad \boldsymbol{X}_{4x1}$$

Typical Perspective Model

f focal length

u0,v0: principal point (image coords of camera origin on retina)

$$\boldsymbol{P} \equiv \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} [\boldsymbol{R}_{3x3} \quad \boldsymbol{t}_{3x1}] \quad \boldsymbol{X}_{4x1}$$

Typical Perspective Model

Intrinsic Matrix K **Extrinsic Matrix [R,t]**

$$\boldsymbol{P} \equiv \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{R}_{3x3} & \boldsymbol{t}_{3x1} \end{bmatrix} \boldsymbol{X}_{4x1}$$

$$\boldsymbol{P} \equiv \boldsymbol{K}[\boldsymbol{R}, \boldsymbol{t}]\boldsymbol{X} \equiv \boldsymbol{M}_{3x4}\boldsymbol{X}_{4x1}$$

Other Cameras – Orthographic

Orthographic Camera (z infinite)

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \boldsymbol{X}_{3x1}$$

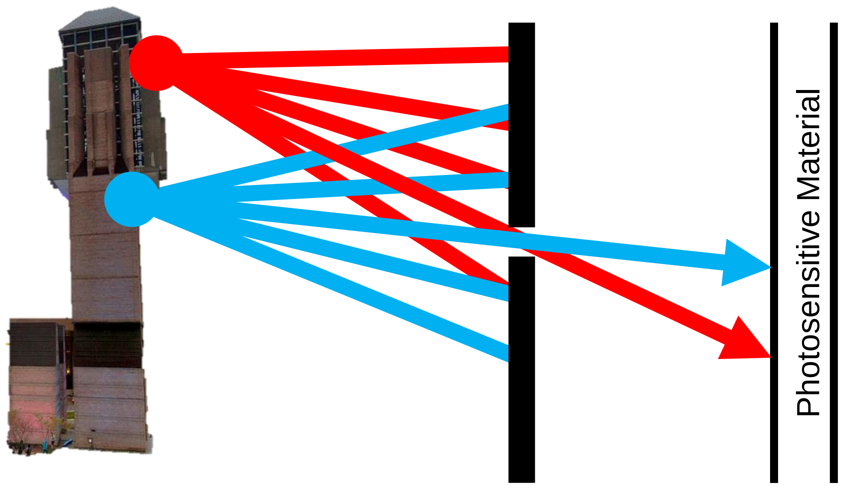
Image Credit: Wikipedia

Other Cameras – Orthographic

Why does this make things easy and why is this popular in old games?

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The Big Issue



Film captures all the rays going through a ***point*** (a p*encil of rays).*

How big is a point?

Slide inspired by S. Seitz; image from Michigan Engineering

Math vs. Reality

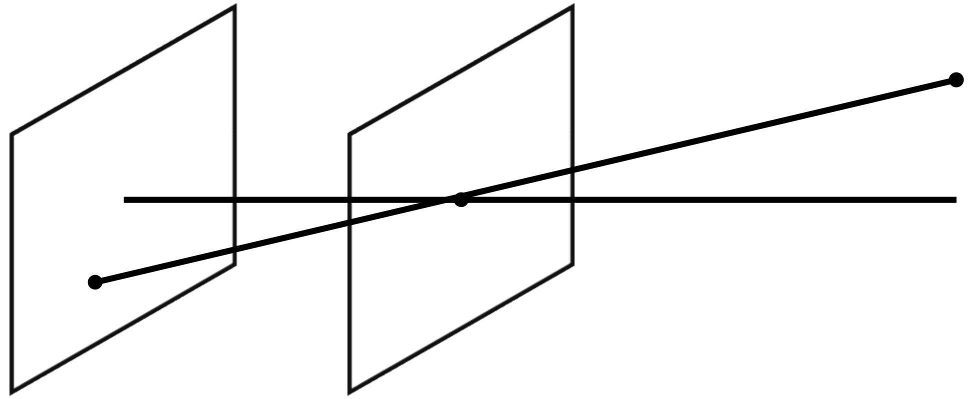
- Math: Any point projects to one point
- Reality (as pointed out by the class)
  - Don’t image points behind the camera / objects
  - Don’t have an infinite amount of sensor material
- Other issues
  - Light is limited
  - Spooky stuff happens with infinitely small holes

Limitations of Pinhole Model

Ideal Pinhole

-1 point generates 1 image

-Low-light levels

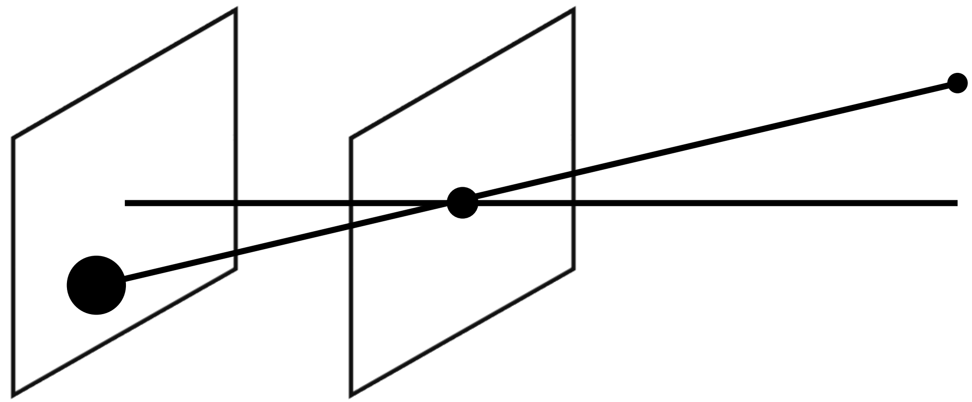


Finite Pinhole

-1 point generates region

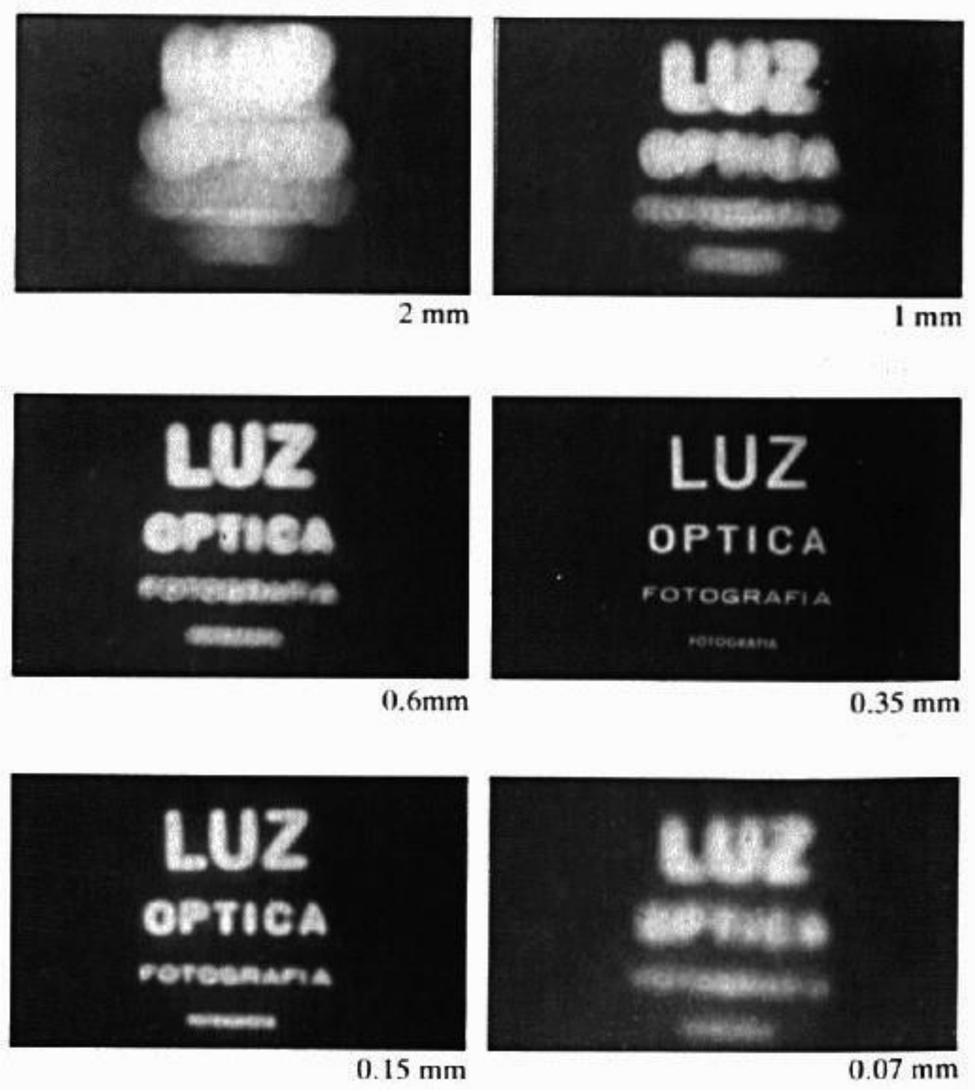
-Blurry.

Why is it blurry?



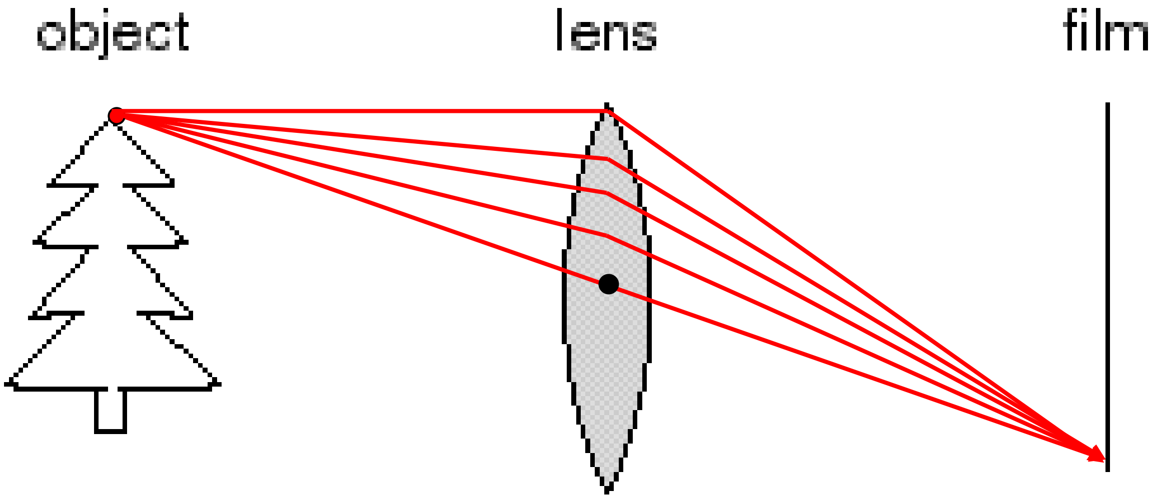
Slide inspired by M. Hebert

Limitations of Pinhole Model



Slide Credit: S. Seitz

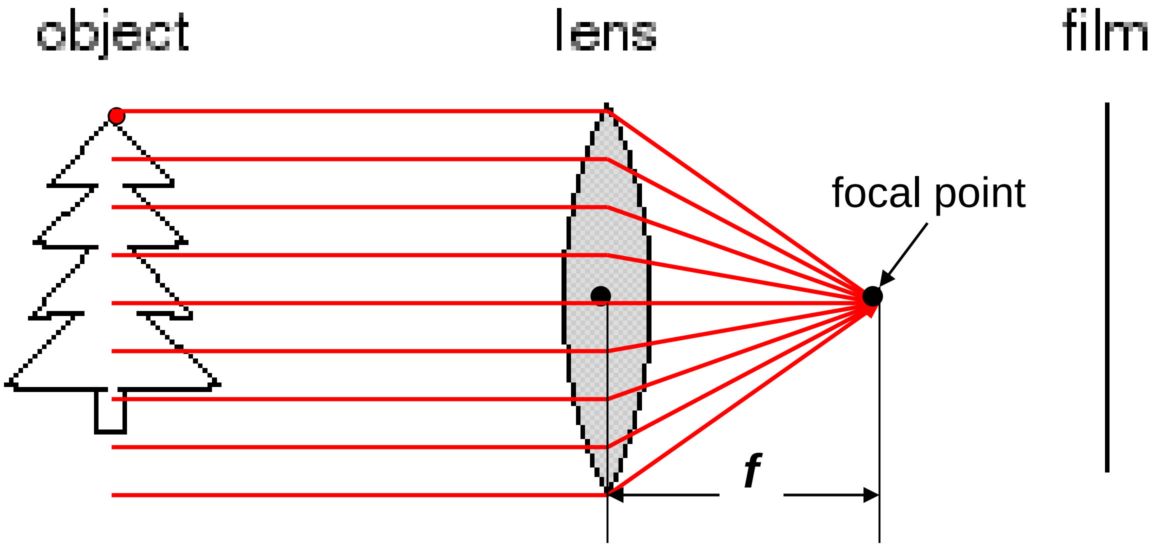
Adding a Lens



- A lens focuses light onto the film
- Thin lens model: rays passing through the center are not deviated (pinhole projection model still holds)

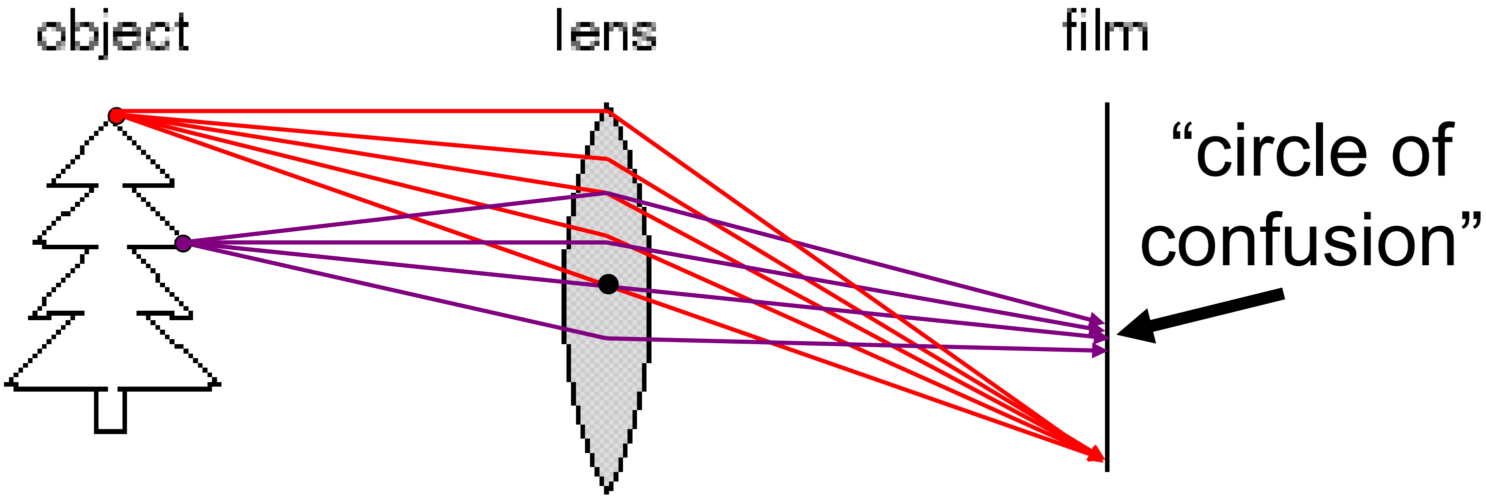
Slide Credit: S. Seitz

Adding a Lens



- All rays parallel to the optical axis pass through the *focal point*

What's The Catch?



- There's a distance where objects are "in focus"
- Other points project to a "circle of confusion"

Slide Credit: S. Seitz

Thin Lens Formula

We care about images that are in focus.

When is this true? Discuss with your neighbor.

When two paths from a point hit the same image location.

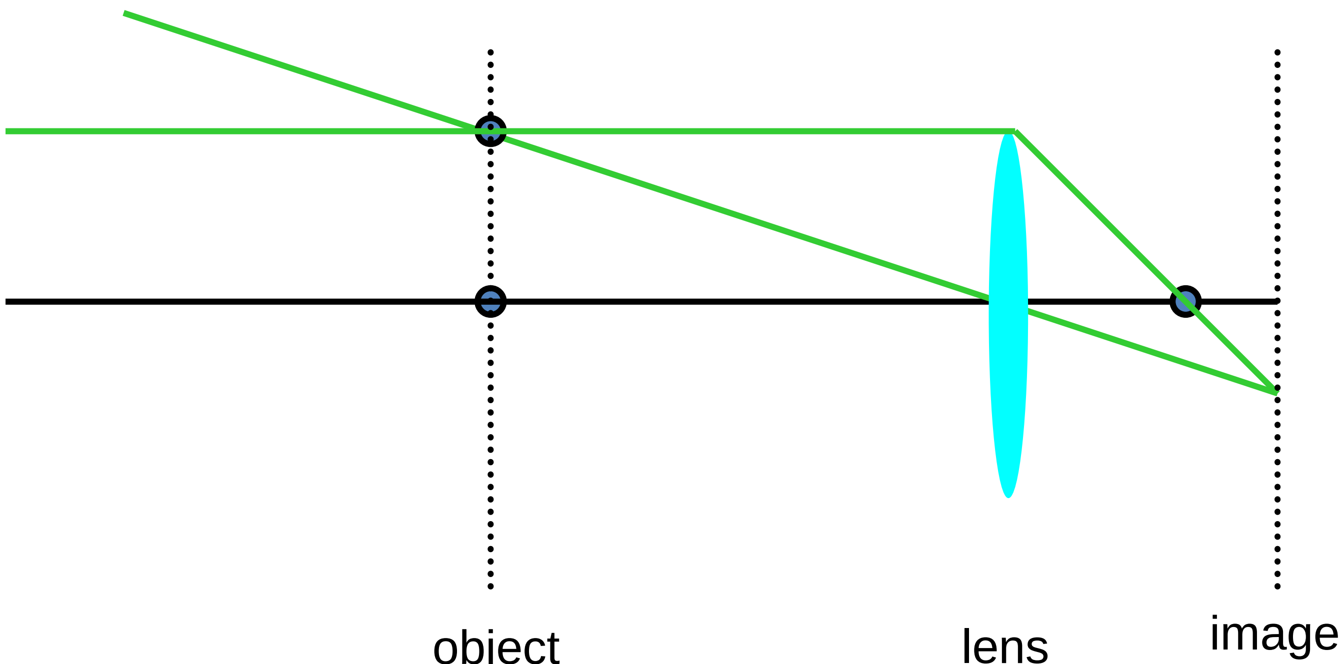


Diagram credit: F. Durand

Thin Lens Formula

Let's derive the relationship between object distance D, image plane distance D', and focal length f.

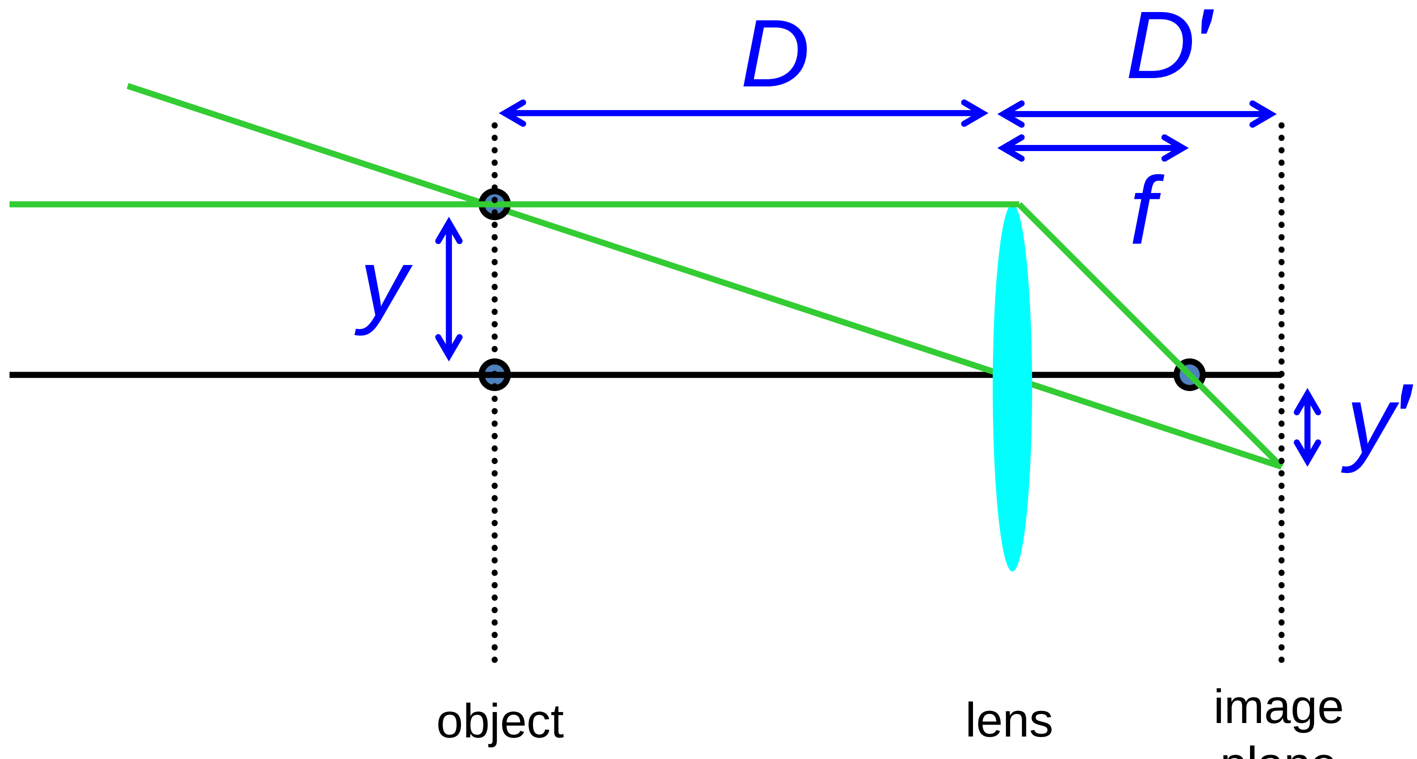


Diagram credit: F. Durand

Thin Lens Formula

One set of similar triangles:

$$\frac{y'}{D'-f} = \frac{y}{f} \longrightarrow \frac{y'}{y} = \frac{D'-f}{f}$$

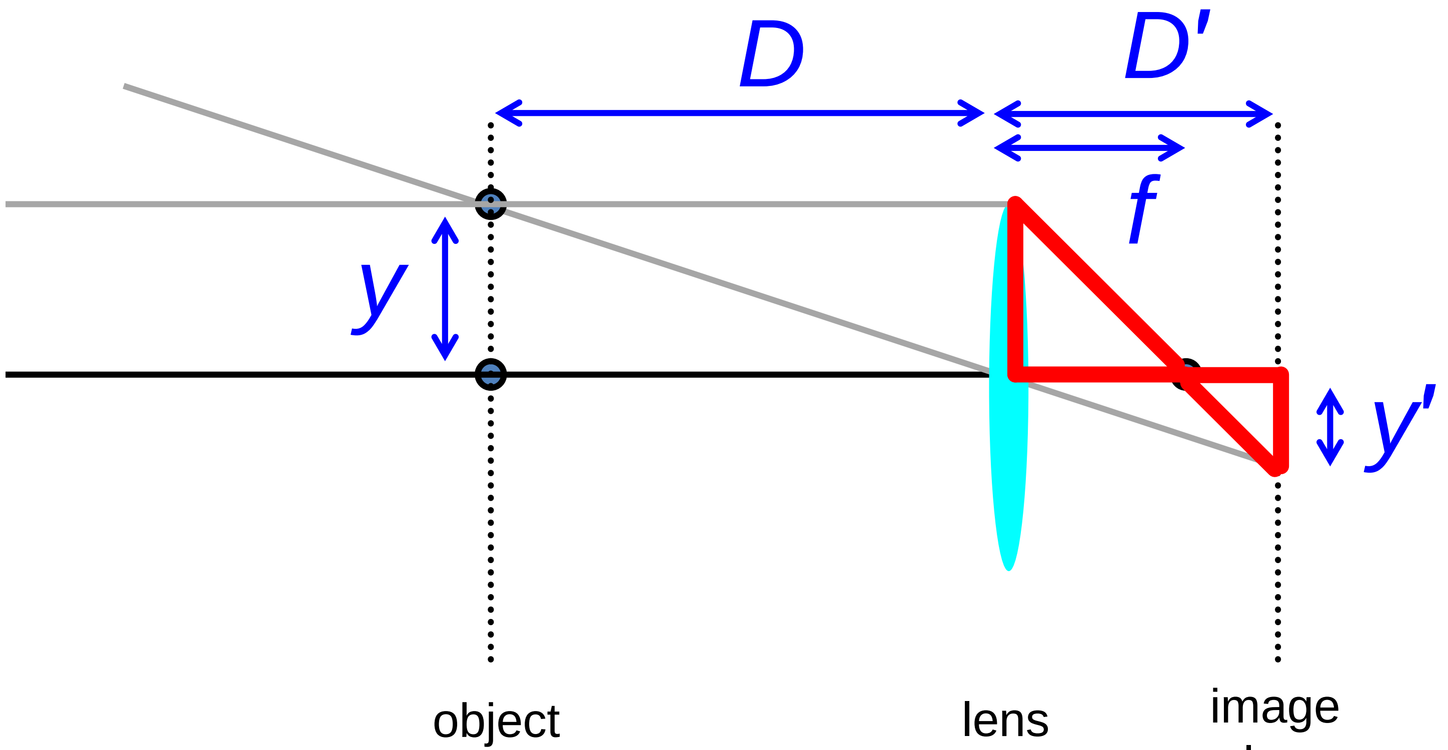


Diagram credit: F. Durand

Thin Lens Formula

Another set of similar triangles:

$$\frac{y'}{D'} = \frac{y}{D} \longrightarrow \frac{y'}{y} = \frac{D'}{D}$$

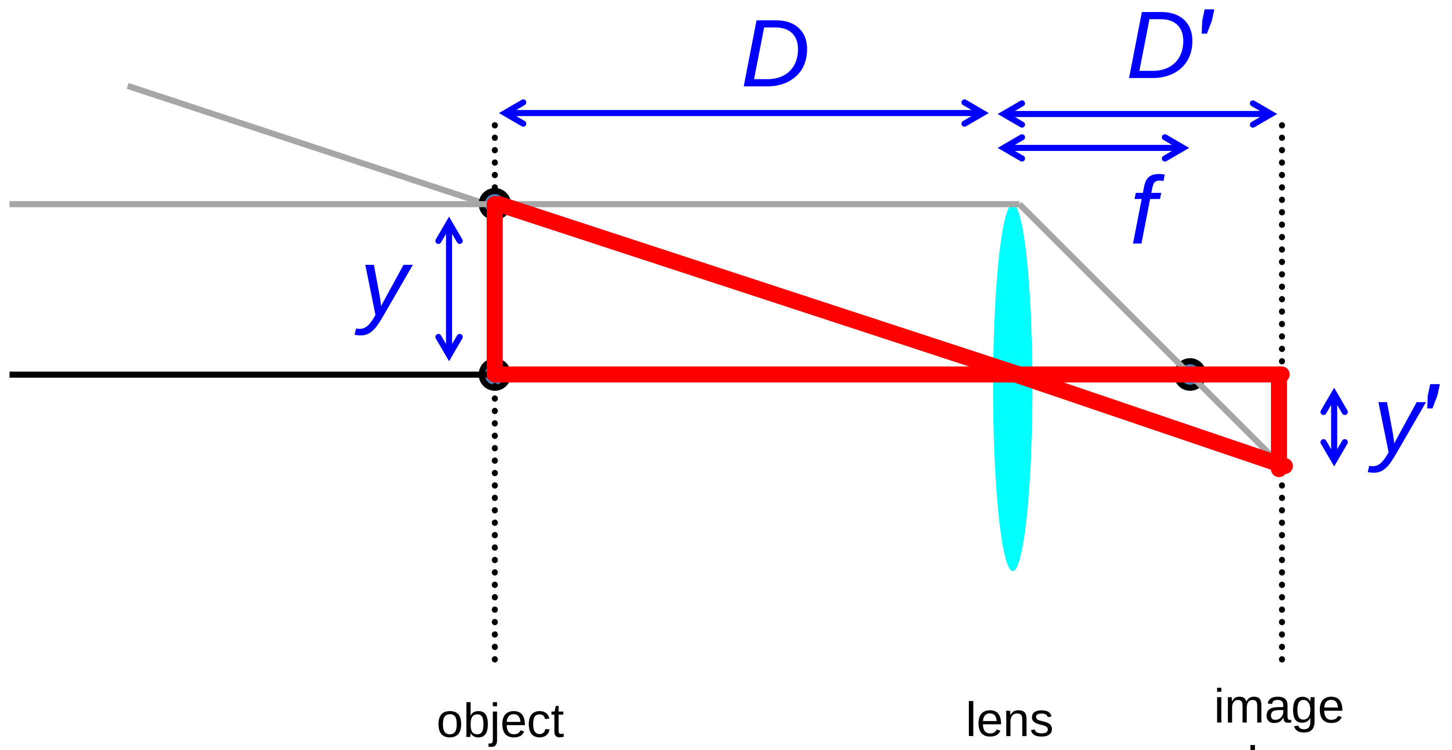


Diagram credit: F. Durand

Thin Lens Formula

Set them equal:

$$\frac{D'}{D} = \frac{D-f}{f} \quad \longrightarrow \quad \frac{1}{D} + \frac{1}{D'} = \frac{1}{f}$$

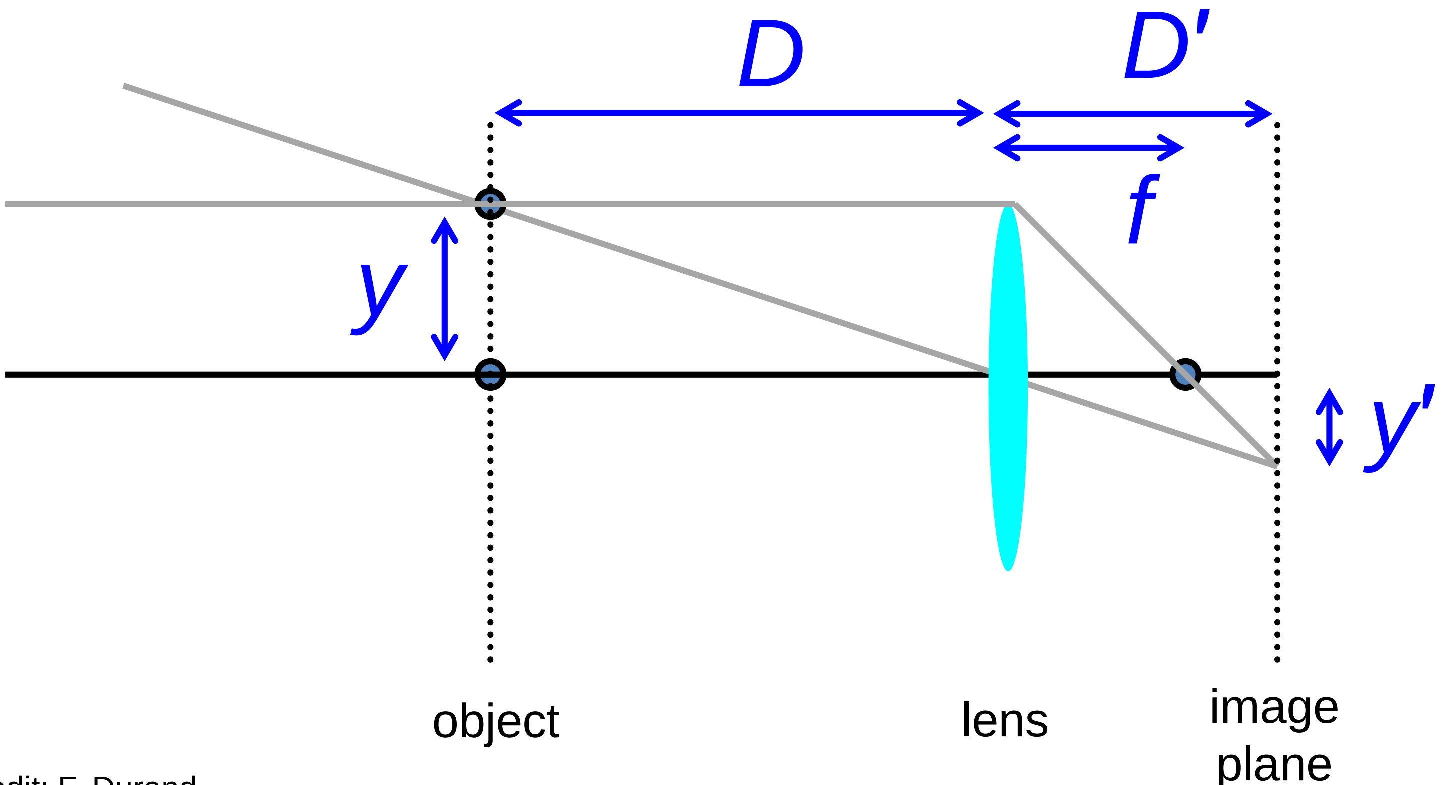


Diagram credit: F. Durand

Thin Lens Formula

Suppose I want to take a picture of a lion with D big?
Which of D, D', f **are fixed?**

How do we take pictures of things at different distances?

$$\frac{1}{D} + \frac{1}{D'} = \frac{1}{f}$$

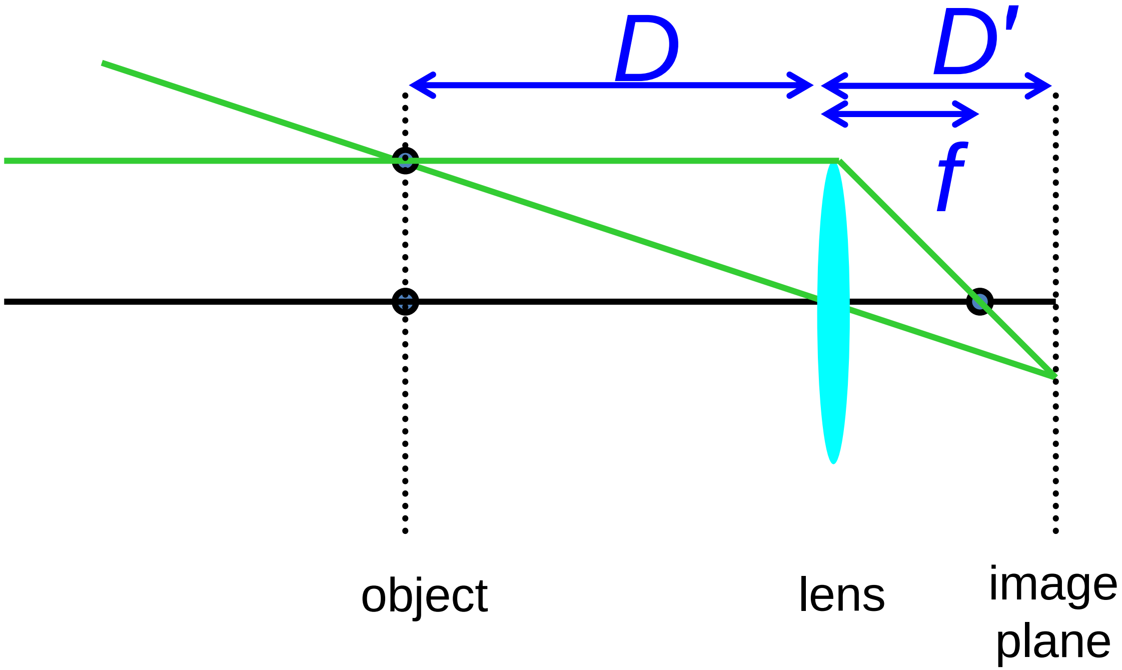
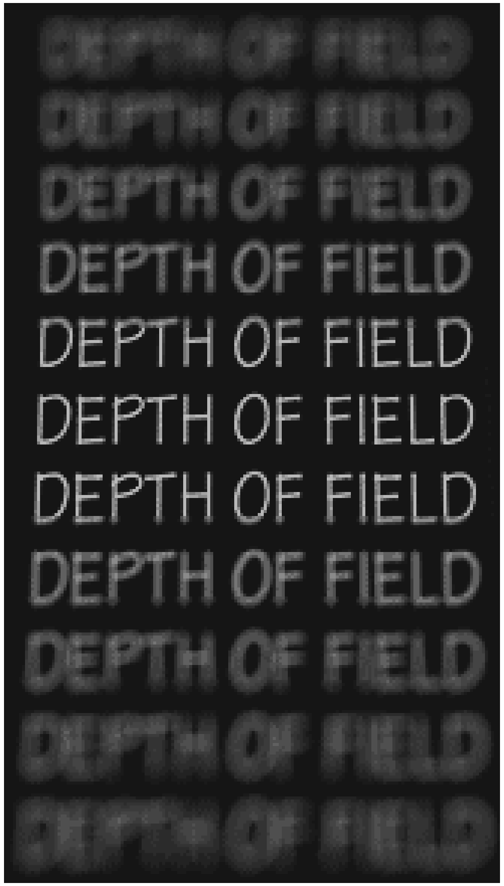


Diagram credit: F. Durand

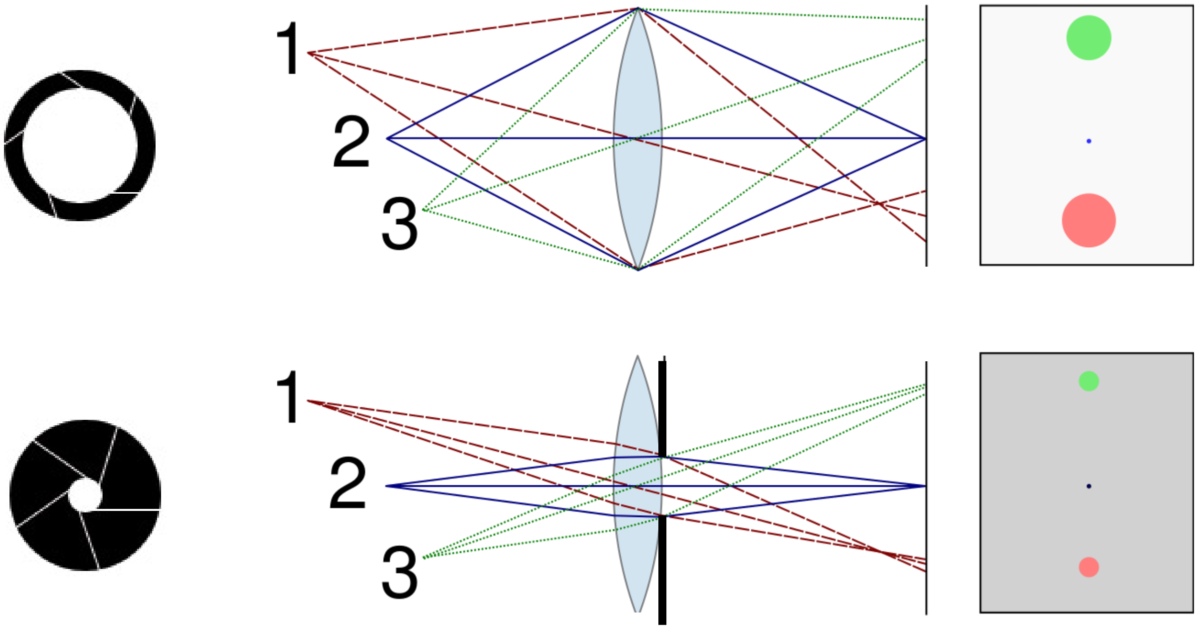
Depth of Field



http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Slide Credit: A. Efros

Controlling Depth of Field

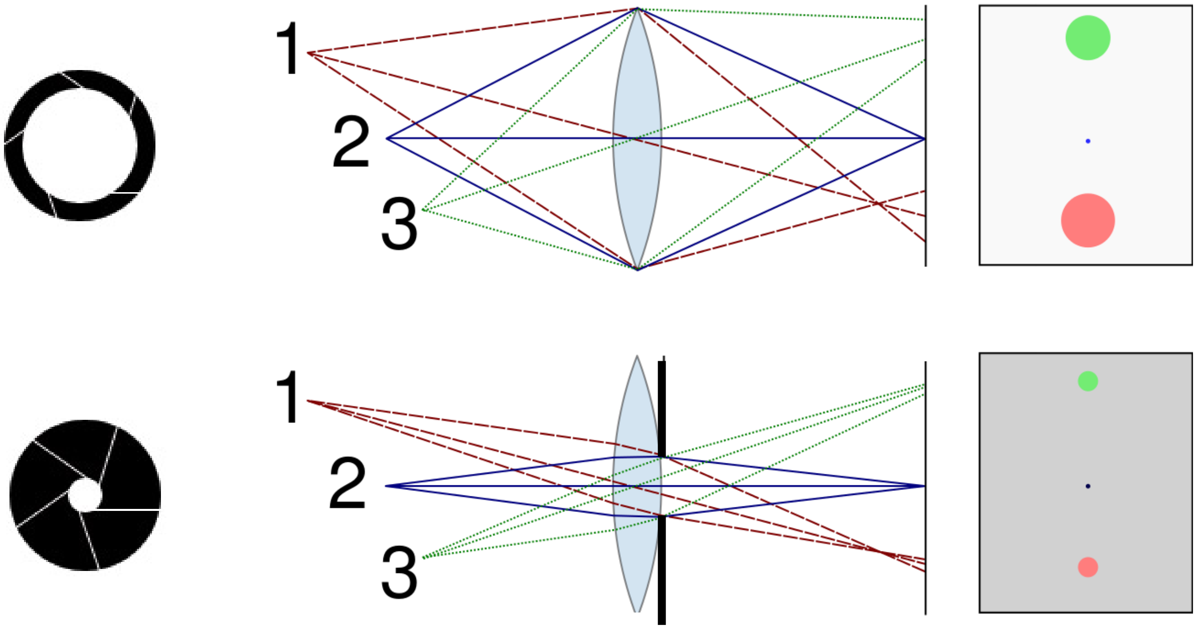


Changing the aperture size affects depth of field

A smaller aperture increases the range in which the object is approximately in focus

Diagram: Wikipedia

Controlling Depth of Field



If a smaller aperture makes everything focused, why don’t we just always use it?

Diagram: Wikipedia

Varying the Aperture

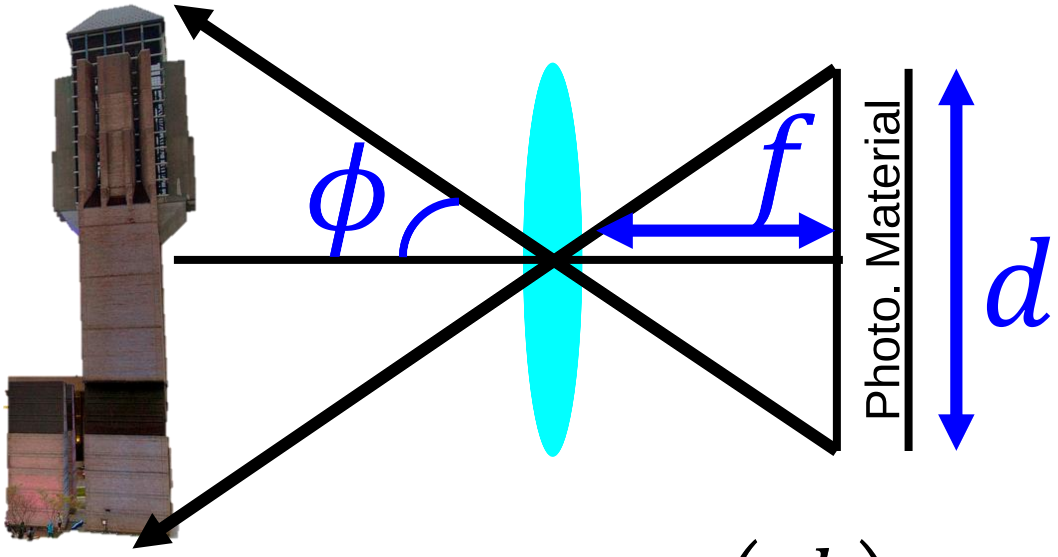
Small aperture = large DOF

Large aperture = small DOF

Slide Credit: A. Efros, Photo: Philip Greenspun

Varying the Aperture

Field of View (FOV)

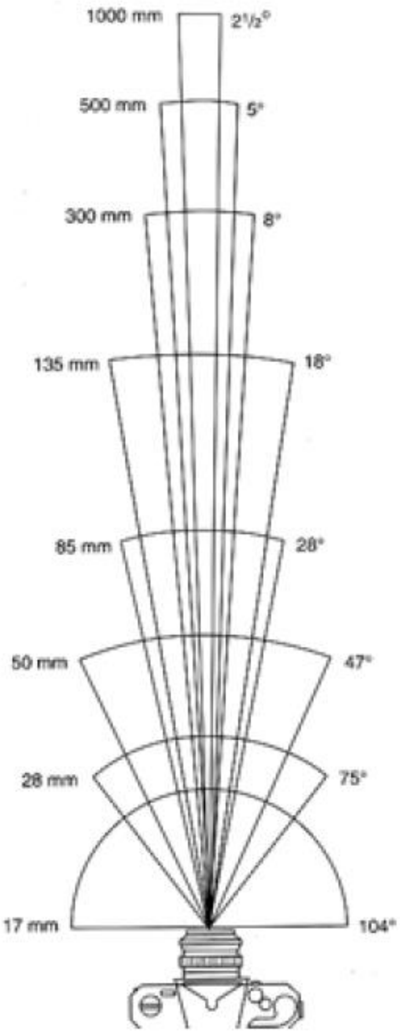


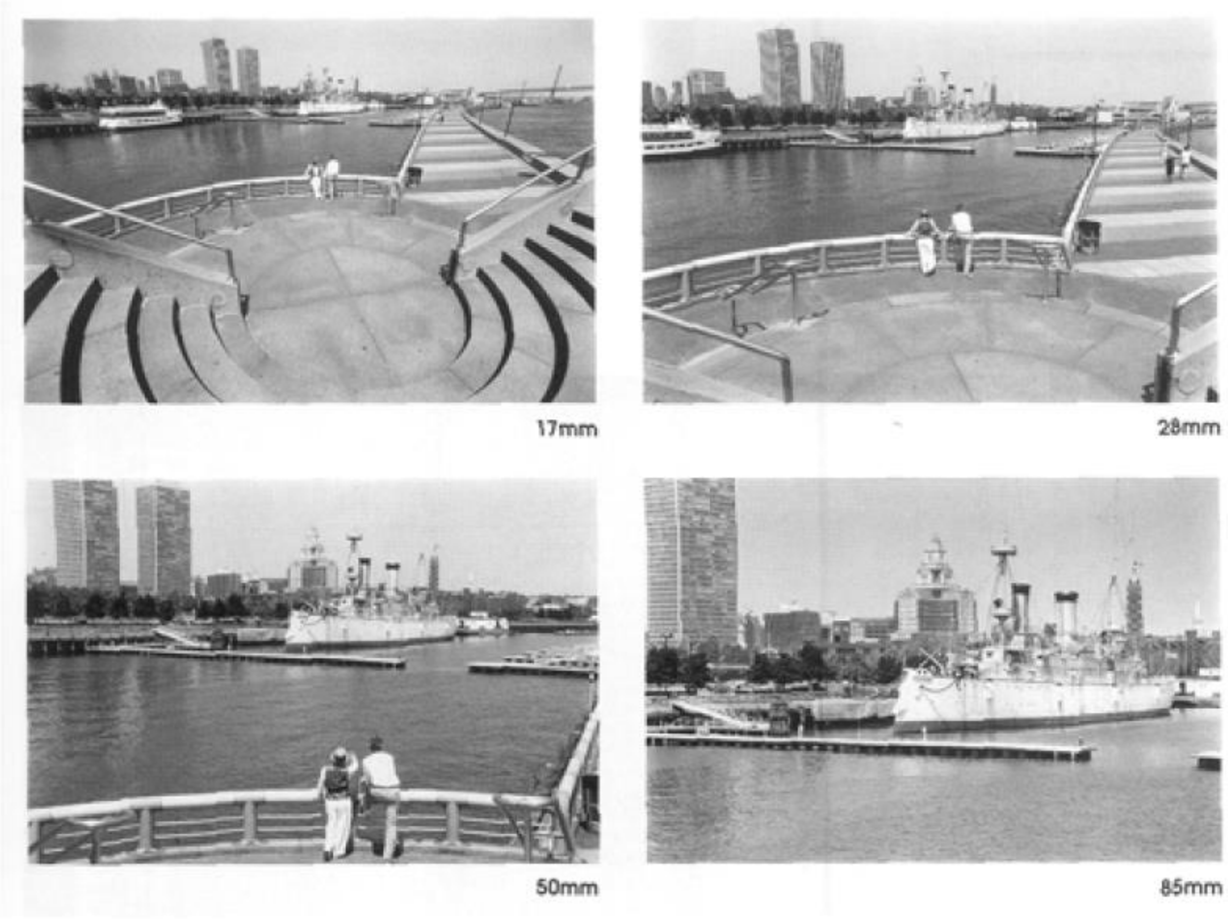
$$\phi = \tan^{-1}\left(\frac{d}{2f}\right)$$

$\tan^{-1}$ is monotonic increasing.

How can I get the FOV bigger?

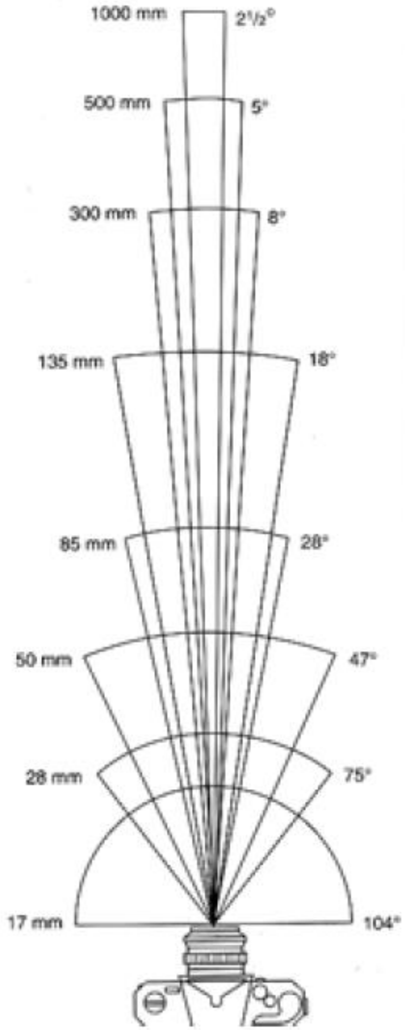
Field of View

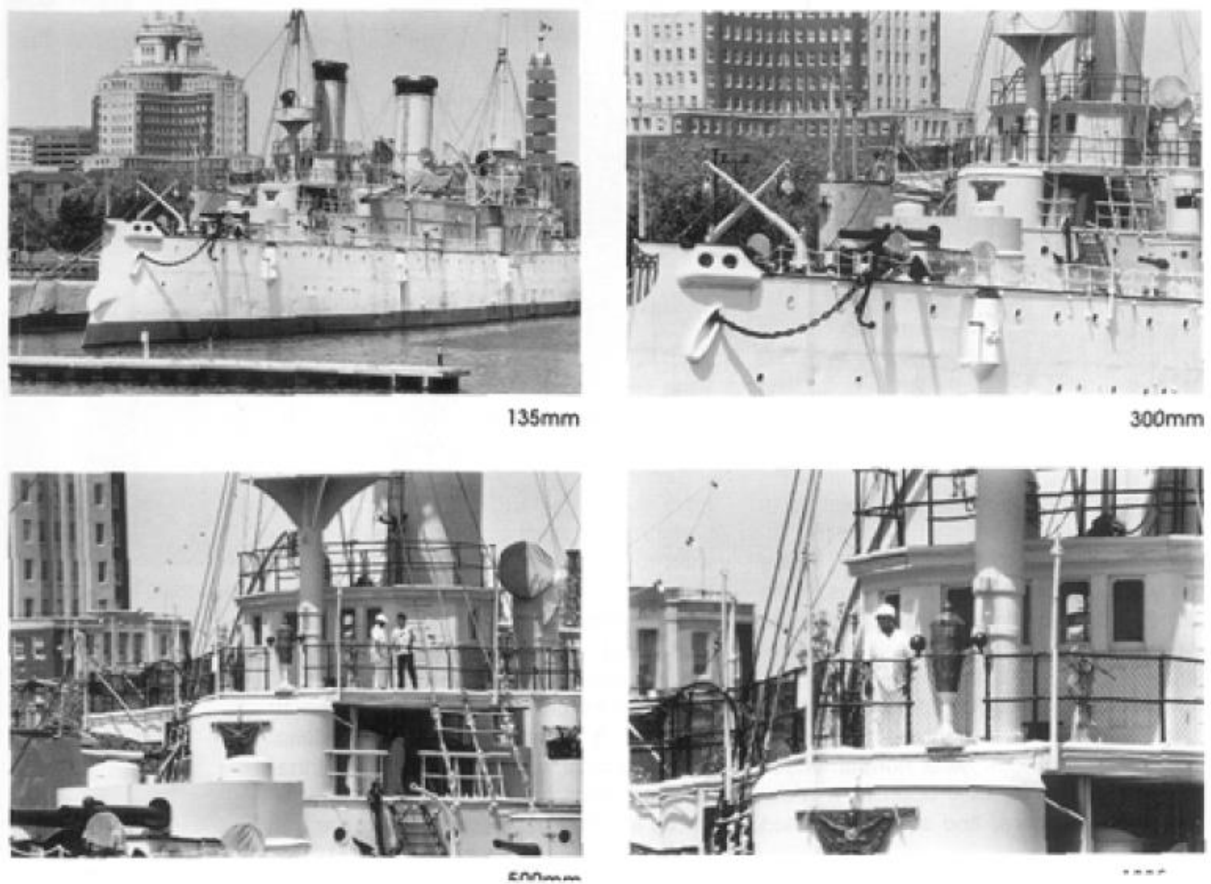




Slide Credit: A. Efros

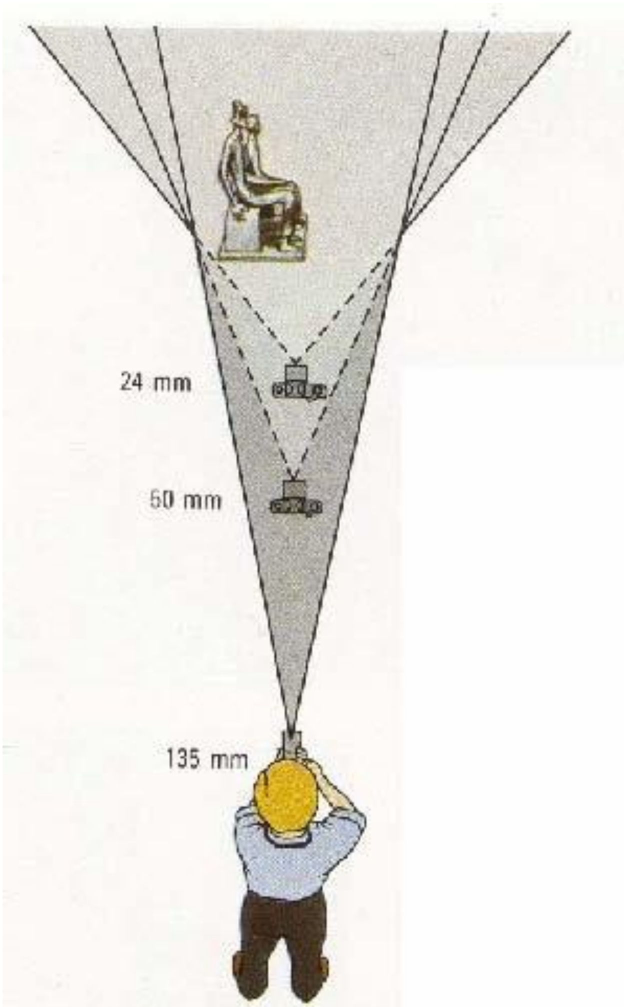
Field of View





Slide Credit: A. Efros

Field of View and Focal Length



Large FOV, small *f*
Camera close to car

Small FOV, large *f*
Camera far from the car

Slide Credit: A. Efros, F. Durand

Field of View and Focal Length

wide-angle

standard

telephoto

Slide Credit: F. Durand

Dolly Zoom

Change f and distance at the same time

Video Credit: Goodfellas 1990

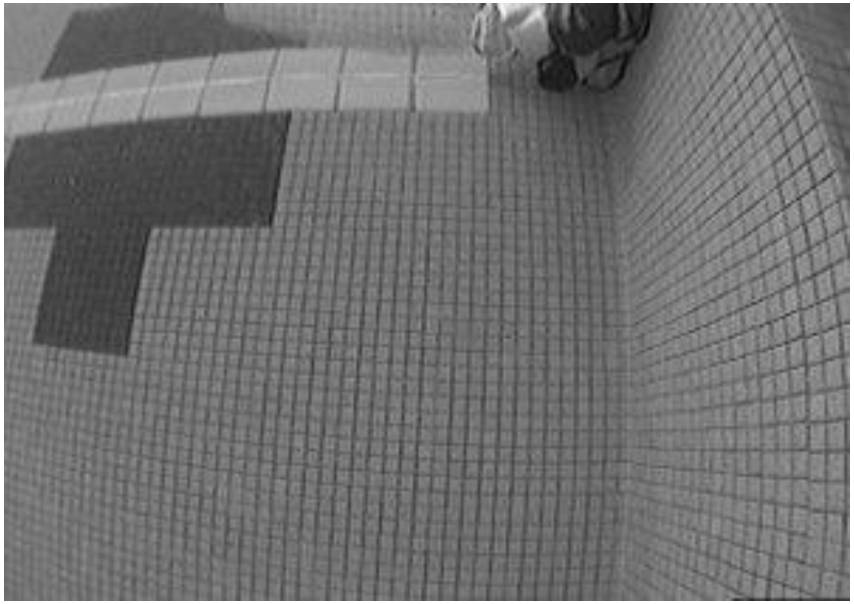
More Bad News!

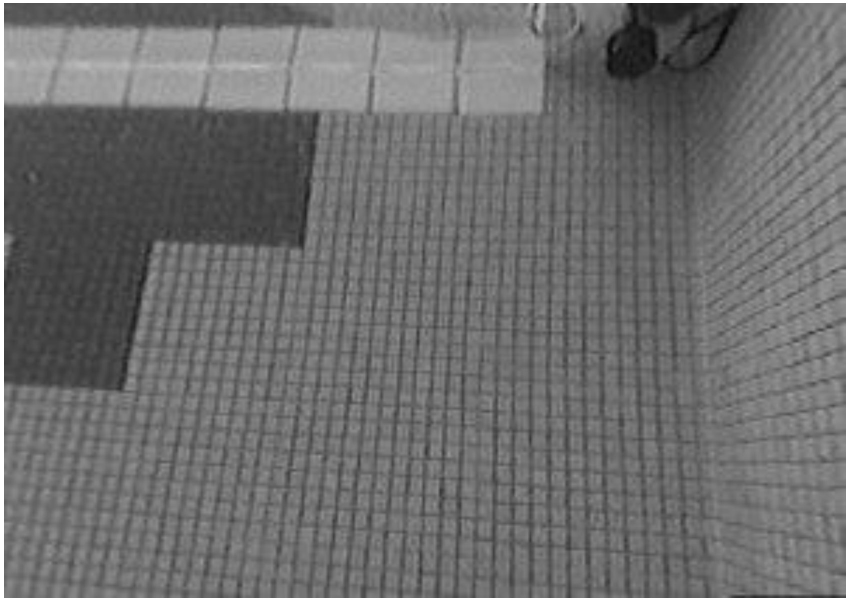
- First a pinhole…
- Then a thin lens model….

Slide: L. Lazebnik

Lens Flaws: Radial Distortion

Lens imperfections cause distortions as a function of distance from optical axis

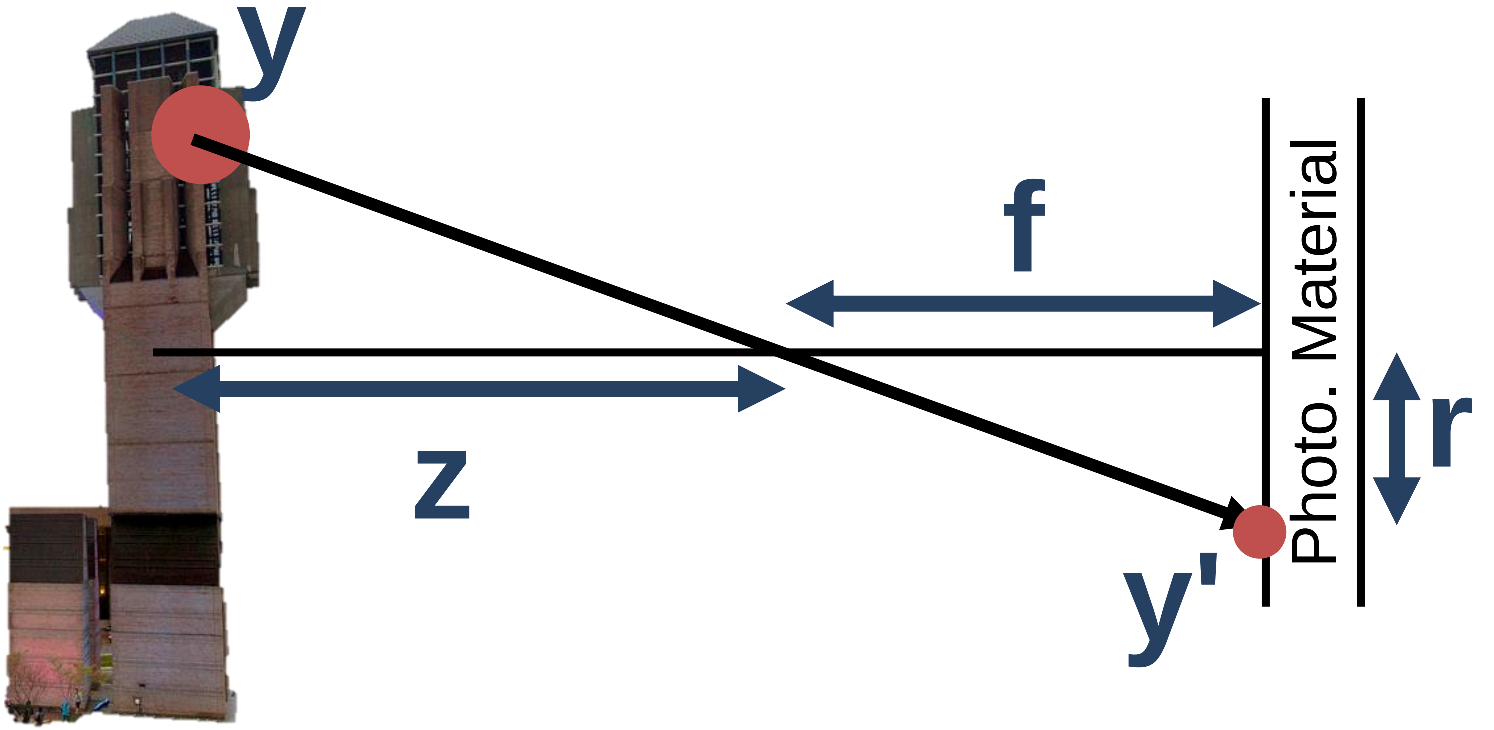




Less common these days in consumer devices

Photo: Mark Fiala, U. Alberta

Radial Distortion Correction



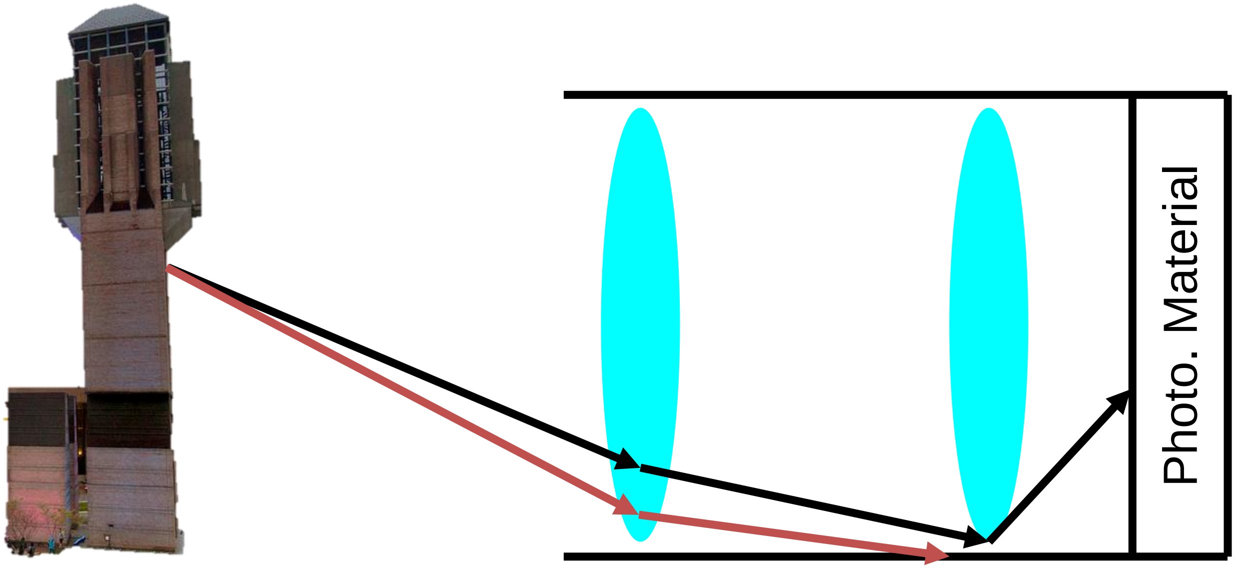
Ideal

$$y' = f\frac{y}{z}$$

Distorted

$$y' = (1 + k_1 r^2 + \cdots)\frac{y}{z}$$

Vignetting



What happens to the light between the black and red lines?

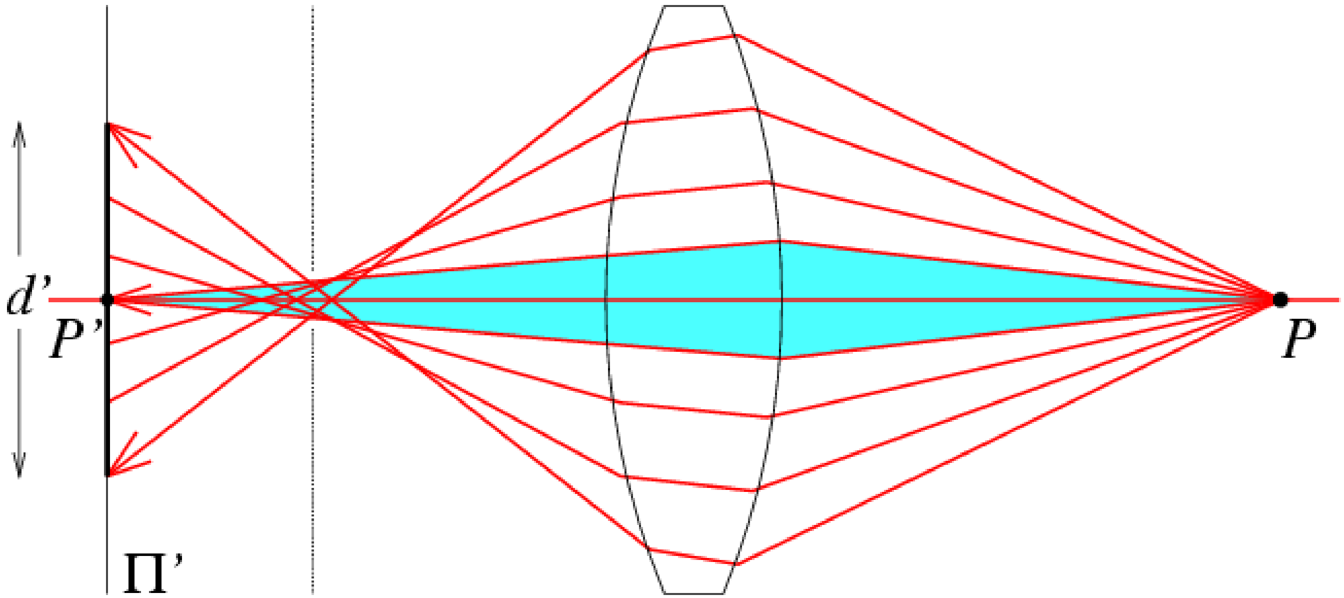
Slide inspired by L. Lazebnik Slide

Vignetting

Photo credit: Wikipedia (https://en.wikipedia.org/wiki/Vignetting)

Lens Flaws: Spherical Abberation

Lenses don't focus light perfectly!
Rays farther from the optical axis focus closer



Slide: L. Lazebnik

Lens Flaws: Chromatic Abberation

Lens refraction index is a function of the wavelength. Colors “fringe” or bleed

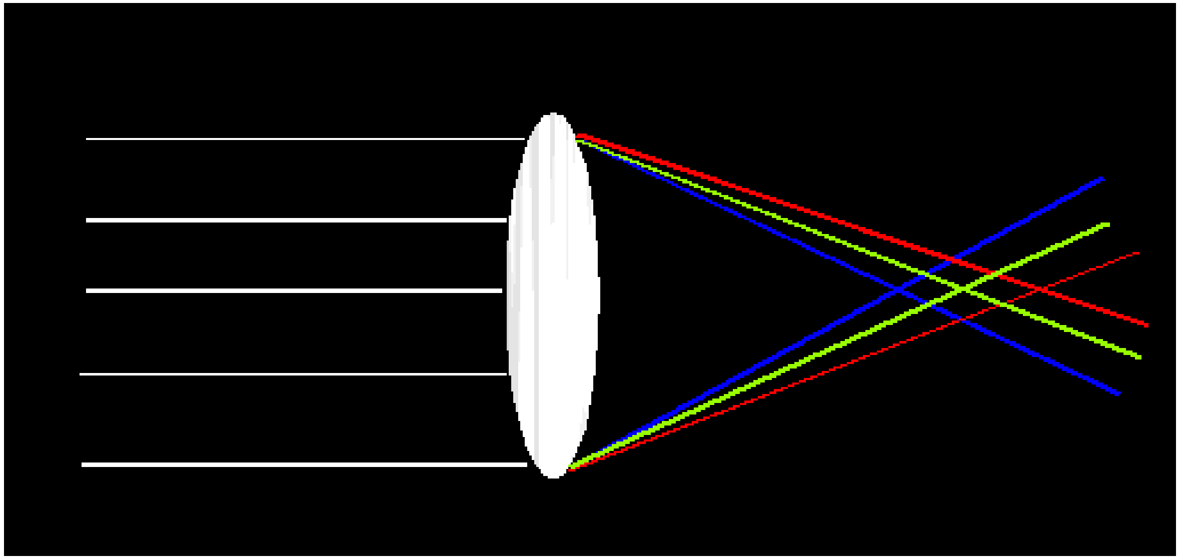
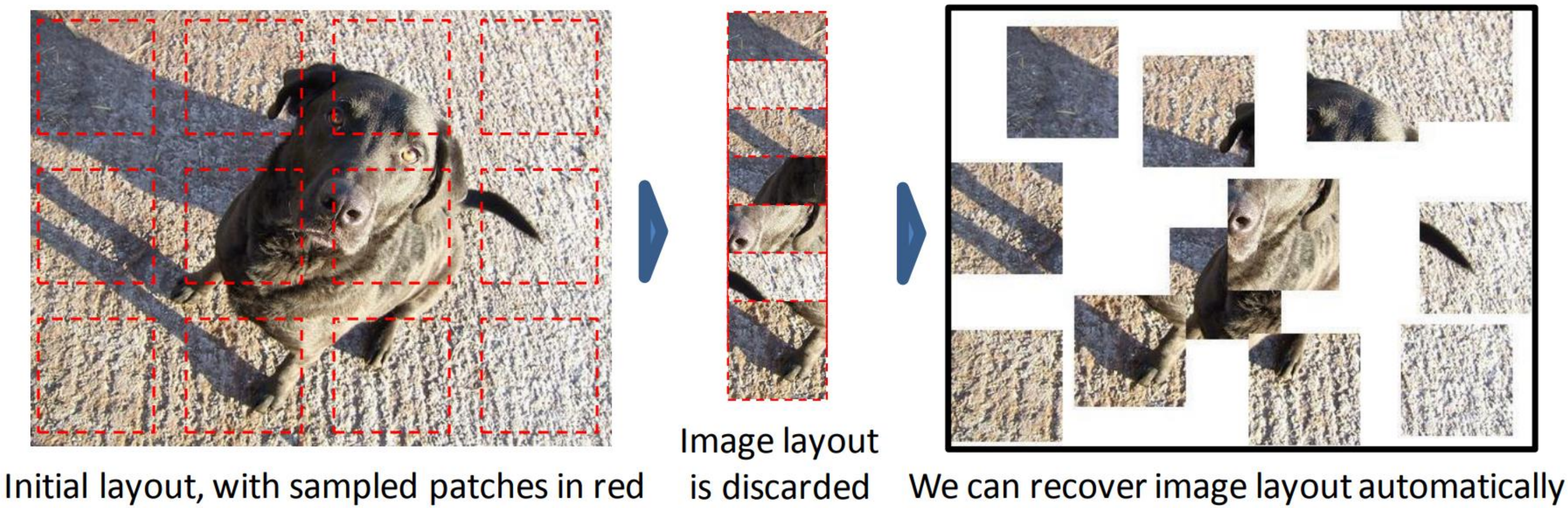


Image credits: L. Lazebnik, Wikipedia

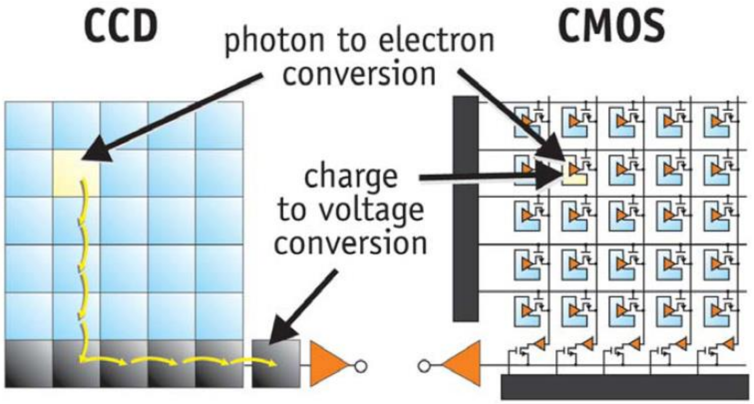
Lens Flaws: Chromatic Abberation

Researchers tried teaching a network about objects by forcing it to assemble jigsaws.



Slide Credit: C. Doersch

From Photon to Photo



CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.

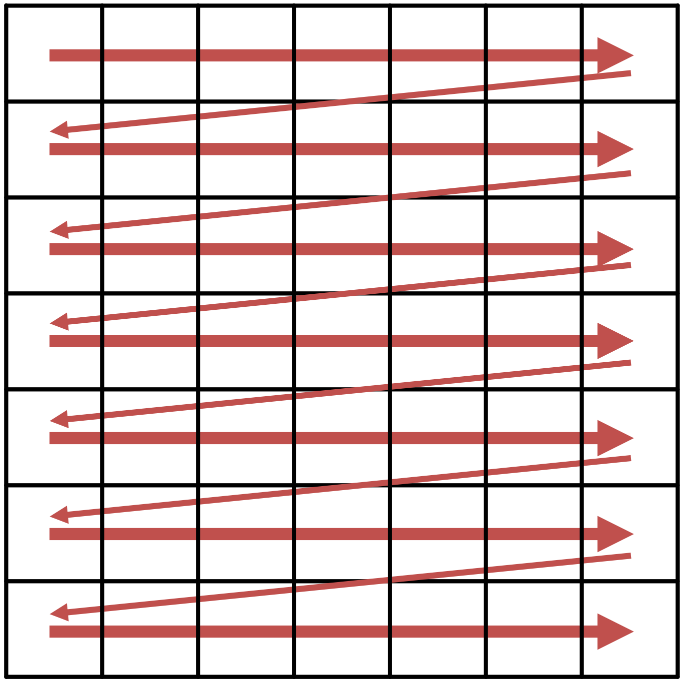
- Each cell in a sensor array is a light-sensitive diode that converts photons to electrons
  - Dominant in the past: **Charge Coupled Device (CCD)**
  - Dominant now: **Complementary Metal Oxide Semiconductor (CMOS)**

Slide Credit: L. Lazebnik, Photo Credit: Wikipedia, Stefano Meroli

From Photon to Photo

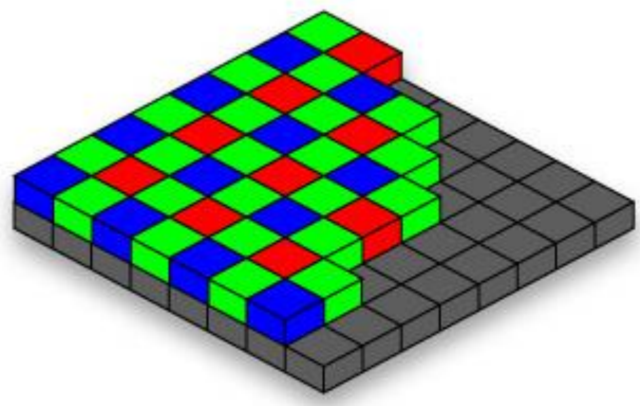
Rolling Shutter: pixels read in sequence

Can get global reading, but **$$$**

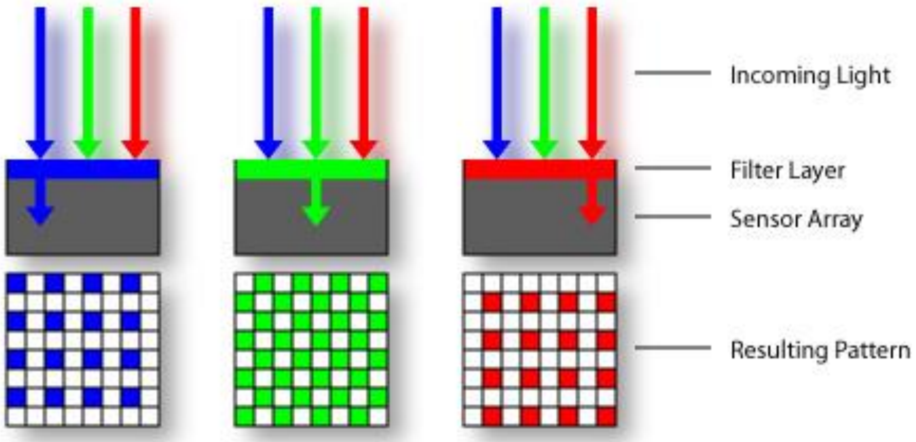


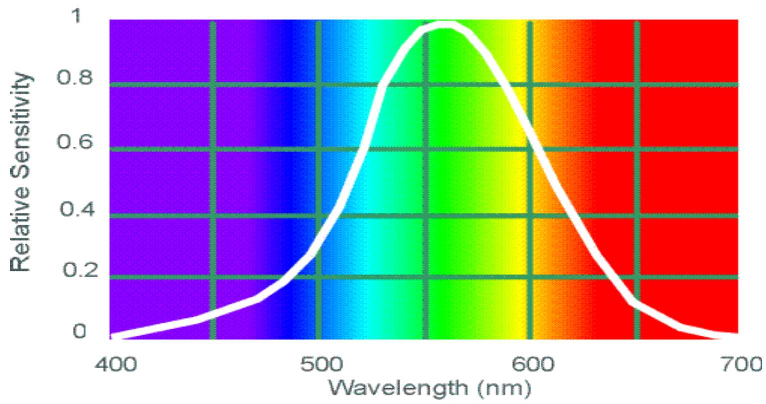
Preview of What's Next

Bayer grid



Demosaicing:
Estimation of missing components from neighboring values



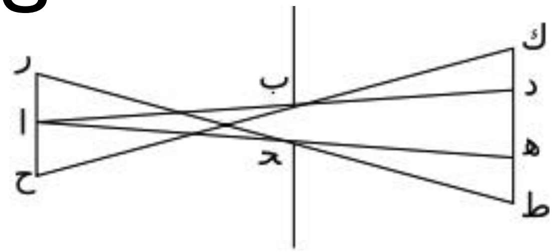


Slide Credit: S. Seitz

Human Luminance Sensitivity Function

Historic milestones

- **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- **Camera obscura:** Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo:** Joseph Nicephore Niepce (1822)
- **Daguerréotypes** (1839)
- **Photographic film** (Eastman, 1889)
- **Cinema** (Lumière Brothers, 1895)
- **Color Photography** (Lumière Brothers, 1908)
- **Television** (Baird, Farnsworth, Zworykin, 1920s)
- **First consumer camera with CCD** Sony Mavica (1981)
- **First fully digital camera:** Kodak DCS100 (1990)



Alhacen's notes

Niepce, "La Table Servie," 1822

Old television camera

Slide Credit: S. Lazebnik

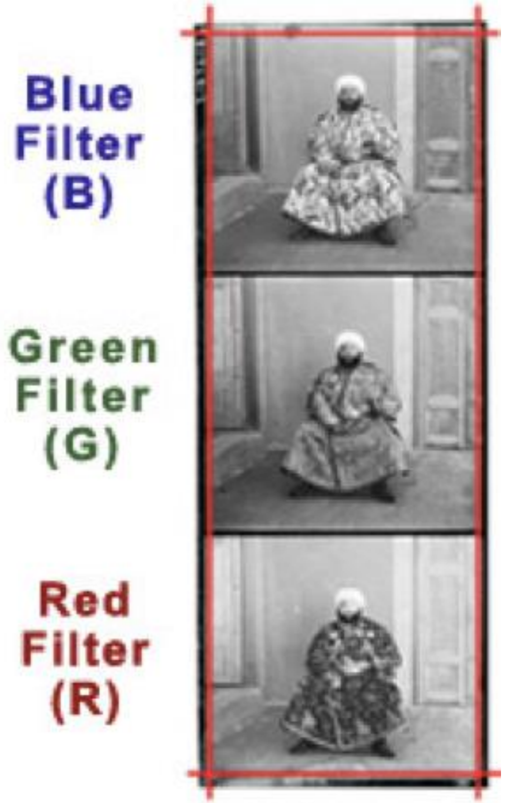
First digitally scanned photograph

- 1957, 176x176 pixels

Slide Credit: http://listverse.com/history/top-10-incredible-early-firsts-in-photography/

Historic Milestone

Sergey Prokudin-Gorskii (1863-1944)
Photographs of the Russian empire (1909-1916)



Slide Credit: S. Maji

Historic Milestone

Slide Credit: S. Maji

Future Milestone

Your job in homework 1:
Make the left look like the right.