Math 128a - Homework 8 - Due May 2

1) Problem 8.4.4 (Page 555)

2) Problem 8.4.5 (Page 555)

3) The purpose of this question is to familiarize you with the properties of the ODE solvers in Matlab and their numerical properties. Consider the 0DE

$$x'(t) = c(t) \cdot (x(t) - \sin(t)) + cos(t) = f(x(t), t)$$

with initial conditions $x(0) = 0$. The solution is clearly $x(t) = \sin(t)$ for any continuous $c(t)$. However, we will see that the numerical solvers do not all behave the same way.

The command "options = odeset('AbsTol',1e-4,'RelTol',1e-4)" and then passing options as an argument to the ODE solver means that the estimated LTE at step i should be at most

$$\max(\text{RelTol} \cdot |x_i|, \text{AbsTol}) = 10^{-4} * \max(|x_i|, 1)$$

where $x(i)$ is the i-th computed solution. (The default values of AbsTol and RelTol are 1e-3.)

Solve the above ODE on the interval $[0, 10]$ for each combination of solver, value of AbsTol=RelTol, and $c(t)$ given below ($3 \times 2 \times 6 = 36$ combinations in all):

- ode solver (ode45, ode23, or ode15s)
- RelTol=AbsTol (10^{-4} or 10^{-8}) and
- $c(t)$ (see list below)

For each combination, report

- number of time steps take
- number of seconds taken
- maximum absolute error $\max_i |x_i - sin(t_i)|$, where t_i and x_i are the output arrays of times and solution values at those times.
- maximum relative error $\max_i |x_i - sin(t_i)|/|sin(t_i)|$.

To measure the time taken, measure only the time to solve, not plot or do anything else. Use the command sequence "secs = cputime; [t,x]=ode45(...); secs = cputime - secs" to get the running time in seconds.

Here are the values of $c(t)$ to try for each ODE solver (recall the exact solution of the ODE does not depend on $c(t)$):

- $c(t) = 2$
- $c(t) = 1$
- $c(t) = -1$
- $c(t) = -10$
- $c(t) = -100$
- $c(t) = -10 * \sin(t)$

Comment on your output, saying which method is most accurate, which method is most efficient (measured by the number of steps taken and the time taken and how this depends on $c(t)$. You should plot the solutions "plot(t,y)" to see what is going on but you do not have to turn any plots in. Comment on how the error depends on t for $c(t) = -10 * \sin(t)$.