Math 128a - Homework 6 - Due April 11

1) Let $[a, b]$ be any closed, bounded interval. Let $x_1, ..., x_n$ be the roots of $T_n(x)$, and let $z_i = (x_i + 1) * (b - a)/2 + a$ for $i = 1, ..., n$. Show that of all possible choices of n interpolation points in $[a, b]$, the choice $z_1, ..., z_n$ minimizes the maximum value of the following factor of the interpolation error:

$$\| \prod_{i=1}^{n} (x - z_i) \|_\infty \equiv \max_{a \le x \le b} | \prod_{i=1}^{n} (x - z_i) |$$

and in fact makes

$$\| \prod_{i=1}^{n} (x - z_i) \|_\infty = (b - a)^n 2^{1-2n} \quad .$$

2) Problem 7.1.6.

3) Problem 7.2.1.

4) Problem 7.5.2.

5) Show that

$$\int_1^\infty \frac{\cos(x^2)}{\sqrt{|\sin(x^2)|} \cdot x^{1.1}} dx$$

is finite and compute it as accurately as you can, using Matlab, including the quad command. Explain how you did the integral, and how accurate you think your answer is. Hint: This integral has numerous places ("singularities") where the integrand goes to infinity. Break the interval up into small intervals where the singularities occur at the endpoints, and handle each interval separately. Try changing variables to remove the singulatities. Argue that you can bound the error by integrating from 1 to some large finite value instead of to infinity (subhint: alternating series).