Math 110 *Practice* Second Midterm

You are allowed to use 1 sheet (8 1/2 by 11 inches, both sides) of notes. Otherwise this is a closed book, closed notes, closed calculator, closed computer, closed PDA, closed cellphone, closed network, open brain exam.

You get one point each for filling in the 4 lines below All other questions are worth 10 points.

Fill in the questions below. **Then stop and wait until we tell you to turn the page and start the rest of the exam. Do not start reading the rest of the exam until we tell you to start.**

After you start, read all the questions on the exam before you answer any of them, so you do the ones you find easier first.

Write all your answers on this exam. If you need scratch paper, ask for it, write your name on each sheet, and attach it when you turn it in (we have a stapler).

(1) (1 point) What is your name?

(2) (1 point) What is your TA's name?

(3) (1 point) What is the name of the person sitting to your left? ("aisle" is a possible answer.)

(4) (1 point) What is the name of the person sitting to your right? ("aisle" is a possible answer.)

This practice exam has more questions than we will ask in the actual exam.

(Question 1): Let V be a finite dimensional vector space with subspace W, such that $W \neq \{0\}$ and $W \neq V$. Show that there is a nonzero linear functional $f \in V^*$ such that $f(w) = 0$ for all $w \in W$.

(Question 2): Let A be an n by n invertible matrix. Prove that there is a factorization $A = P_L \cdot L \cdot U$ where P_L is an n by n permutation matrix, L is an n by n unit lower triangular matrix, and U is an n by n upper triangular matrix. In other words, when A is invertible, we can assume the permutation $P_R = I$ in the LU factorization.

(Question 3) Suppose $A = L \cdot U$, where A, L and U are all n by n and invertible, L unit lower triangular and U upper triangular. Let A_i denote the i by i matrix lying in rows 1 through i and columns 1 through i of A. Show that $U_{11} = A_{11}$ and $U_{ii} = \det(A_i)/\det(A_{i-1})$.

(Question 4) True or false (with justification): A matrix with real entries is diagonalizable when considered as an element of $M_{n\times n}(\mathbb{C})$ if and only if it is diagonalizable when considered as an element of $M_{n\times n}(\mathbb{R})$.

(Question 5) True or false (with justification): When A is a square matrix over F, the system of linear equations $Ax = b$ has exactly one solution if and only if the corresponding homogeneous system $Ax = 0$ has exactly one solution.

(Question 6) True or false (with justification): An upper triangular matrix with distinct diagonal entries is diagonalizable.

(Question 7) True or false (with justification): If A is an n by n matrix satisfying $0 = (A-2I)\cdot(A-3I)\cdot(A-\pi I)$, then at least one of the numbers 2, 3 and π is an eigenvalue of A.

(Question 8) Use mathematical induction to establish a formula for

$$C = \det\left(\begin{bmatrix} 0 & 0 & \cdots & 0 & c_1 \\ 0 & 0 & \cdots & c_2 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & c_{n-1} & \cdots & 0 & 0 \\ c_n & & \cdots & 0 & 0 \end{bmatrix}\right)$$

that has zero entries except possibly along the "anti-diagonal" from the bottom left corner to the upper right corner.

(Question 9) Suppose that A is an n by n invertible and diagonalizable matrix over F. Show that there are constants c_0,...,c_{n-1} in F such that

$$A^{-1} = c_{n-1}A^{n-1} + c_{n-2}A^{n-2} + \cdots + c_1 A + c_0 I$$

(Question 10) Let V be the real vector space $P_3(\mathbb{R})$ of polynomials of degree at most 3 with real coefficients. Let $\beta = \{1, x, x^2, x^3\}$ be the standard ordered basis of V. Let $T : V \to V^*$ be the linear transformation that sends $p \in V$ to f_p, where $f_p(q) = \int_0^1 p(x)q(x)dx$. Find $[T]_\beta^{\beta^*}$, where β^* is the dual basis of β.

(Question 11) Let $T : V \to W$ be an isomorphism of vector spaces, and let W' be a nonzero subspace of W. Show that there is a subspace V' of V such that $T : V' \to W'$ is an isomorphism.

(Question 12) Find an LU decomposition of $A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 2 & 3 \\ 5 & 6 & 7 \end{bmatrix}$, and use it to compute $det(A)$ and solve $Ax = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

(Question 13) Find the eigenvalues of $B = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 7 & 0 \\ 5 & 6 & 4 \end{bmatrix}$. Is B diagonalizable?