

Write all your answer in the blue-covered Exam Book you were asked to bring to class. Write your name and your TAs name on the front.

You are allowed to use 1 sheet (8 1/2 by 11 inches, both sides) of notes, otherwise the exam is closed book, closed calculator, closed computer, closed PDA, closed cellphone, closed network, open brain.

Answer the first three questions below, by talking to your neighbors. **Then stop and wait until we tell you to turn the page and start the rest of the exam. Do not start reading the rest of the exam until we tell you to start.**

(1) (1 point) What is your name?

(2) (1 point) What is your TA's name?

(3) (1 point) What is the name of the person sitting to your left? ("aisle" is a possible answer.)

(4) (1 point) What is the name of the person sitting to your right? ("aisle" is a possible answer.)

This practice exam has more questions than we will ask in the actual exam.

(Question 1): True or False (with justification): If A and B are n -by- n matrices with entries from F , then $AB = 0$ if and only if $BA = 0$.

(Question 2): True or False (with justification): If A and B are n -by- n matrices with entries from \mathbb{R} , then $AB = 7I_n$ if and only if $BA = 7I_n$.

(Question 3): True or False (with justification): If $x, y \in V$ and $a, b \in F$, then $ax + by = 0$ if and only if x is a scalar multiple of y , or y is a scalar multiple of x .

(Question 4): True or False (with justification): For $A \in M_{m \times n}(F)$, $[L_A]_{\beta}^{\gamma} = A$ if β and γ are the standard bases of F^n and F^m .

(Question 5): Suppose that $T : V \rightarrow V$ is a linear transformation. Prove that $T^2 = 0$ if and only if the range of T is a subspace of the null space of T .

(Question 6): Let $V = P(\mathbb{R})$ be the vector space of polynomials with real coefficients. For each $i \geq 0$, let $f_i \in \mathcal{L}(V, \mathbb{R})$ be the linear transformation that maps a polynomial $p(x)$ to the value $p^{(i)}(0)$ of its i -th derivative at 0. (The 0-th derivative of p is p itself.) Show that the linear transformations f_0, f_1, f_2, \dots are linearly independent vectors in the vectors space $\mathcal{L}(V, \mathbb{R})$.

(Question 7): In the notation of Question 6, show that the linear transformation $g \in \mathcal{L}(V, \mathbb{R})$ where $g(p) = p(1)$ is not in the span of the f_i .

(Question 8): Let $T : V \rightarrow W$ be a linear transformation. Suppose that x_1, \dots, x_r are linearly independent elements of $N(T)$ and that v_1, \dots, v_s are vectors in V such that $T(v_1), \dots, T(v_s)$ are linearly independent. Show that the r_s vectors $x_1, \dots, x_r, v_1, \dots, v_s$ are linearly independent.

(Question 9): Construct a linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that $T(T(T(v))) = 0$ for all $v \in V$ but $T(T(v))$ is nonzero for some v . Do not forget to show that T is a linear transformation.

(Question 10): Let $x = (x_1, \dots, x_n)$ and $y = (y_1; \dots; y_m)$ be given nonzero tuples with rational entries, and define the matrix $A \in M_{m \times n}(\mathcal{Q})$ by $A_{ij} = y_i \cdot x_j$. Let $L_A : \mathcal{Q}^n \rightarrow \mathcal{Q}^m$ be the linear transformation defined by multiplying a vector in \mathcal{Q}^n by the matrix A . Compute the rank and nullity of L_A . Justify your answers.