Due Mar 11, 8:00am

You may work in groups of up to 3 people (no larger!). Please read the group collaboration policies on bSpace or www.cs.berkeley.edu/~demmel/cs70_Spr11 before beginning group work. You must write up the solution set entirely on your own. You must never look at any other students’ solutions (from any semester, not even a draft), nor share your own solutions (not even a draft).

Please begin your answer to each question on a new sheet of paper, and make sure that each sheet is labeled with your name, GSI name, the assignment number, the question number, and “CS70-Spring 2011”.

Turn in each question in a different box in 283 Soda Hall: Question 1 in the box labeled “CS70 - 1”, Question 2 in the box labeled “CS70 - 2”, etc. Reason: Different problems will be graded in parallel by different readers.

Warning: You risk receiving no credit, or losing points, for any homework that does not conform to the above regulations! Please take the time to write clear and concise solutions; we will not grade messy or unreadable submissions. No late homeworks will be accepted. We will drop the lowest two homework scores.

1. (20 pts.)

1. How many different 13 card bridge hands are there (from a 52 card deck; order irrelevant)
2. What is the probability that a random 13 card bridge hand contains no 2s or 3s?
3. What is the probability that a random 13 card bridge hand contains all the 2s and 3s in the deck?
4. What is the probability that a 13 card bridge hand contains exactly two 2s or exactly two 3s, but not exactly two 2s and exactly two 3s?
5. What is the probability that a 13 card bridge hand contains exactly 5 clubs?

2. (20 pts.)

1. How many ways can a car race end, when there are 5 cars, and arbitrary ties are possible? For example, if there were just two cars, A and B, there would be 3 ways for the race to end: A wins, B wins, and they tie.
2. If all outcomes are equally likely, what is the probability that at least 3 cars are involved in a tie?

3. (20 pts.)

We throw 12 identical balls randomly into 9 bins. What is the probability that no bin is empty? Assume the bins are distinguishable, i.e. numbered 1 through 9.
4. (20 pts.)
Let \( n_1, \ldots, n_t \) be positive integers. Show that if \( n_1 + n_2 + \ldots + n_t - t + 1 \) balls are placed into \( t \) bins, then for some \( i \in \{1, 2, \ldots, t\} \), the \( i \)-th bin contains at least \( n_i \) balls.

5. (20 pts.)
In the game of Set, you have a deck of cards. Each card has 1 or more objects drawn on it, which have the following properties:

- Each object has one of 3 shapes: diamond, oval or squiggle.
- Each object has one of 3 colors: purple, red or green.
- Each object has one of 3 shadings: solid, striped or empty.
- There can be 1, 2 or 3 objects drawn on each card (if there is more than 1 object on a card, all have the identical shape, color and shading).

To play the game, some of the cards are put face up on the table, and all players look for 3 cards that "match", where to match the three cards must

- all have the same shaped objects, or must have 3 different shapes
- all have the same color objects, or must have 3 different colors
- all have the same shaded objects, or must have 3 different shadings
- all have the same number of objects, or must have 3 different numbers

Whenever a player sees matching cards, they yell "match!", pick up the three matching cards, and replace them with 3 new cards. The game continues until no player can find any more matches, and the player who found the most matches wins.

1. If the deck of cards has exactly one of each kind of possible card, how many cards are there?
2. How many different subsets of 3 cards are there in the deck that match?

6. (20 pts.) Based on a 1st grade homework assignment from a local school.

1. Suppose you can have 9 pieces of fruit, which can be either apples or oranges. How many ways can you have 9 pieces of fruit (for example, you could have 4 apples and 5 oranges, or 0 apples and 9 oranges, etc.)?
2. Answer part 1 for \( n \) pieces of fruit.
3. Suppose you can have 9 pieces of fruit, which can be apples, oranges or pears. How many ways can you have 9 pieces of fruit (actual 1st grade assignment)?
4. Answer part 3 for \( n \) pieces of fruit.
5. Suppose you can have \( n \) pieces of fruit, and there are \( m \) different kinds of fruit. How many ways can you have \( n \) pieces of fruit?